FISCAL SUSTAINABILITY UNDER A TIME-VARING FISCAL REACTION RULE: A STATE SPACE APPROACH

Christopher Biolsi*

H. Youn Kim**

August 14, 2018

A PRELIMINARY VERSION

* Department of Economics; Western Kentucky University; Bowling Green, KY 42101, USA; christopher.biolsi@wku.edu
** Department of Economics; Western Kentucky University; Bowling Green, KY 42101, USA; youn.kim@wku.edu

FISCAL SUSTAINABILITY UNDER A TIME-VARYING FISCAL REACTION RULE: A STATE SPACE APPROACH

Abstract

This study proposes a time-varying fiscal reaction function to assess the dynamics of U.S. government debt over 1916-2016. We employ a state space model by treating the reaction coefficient as an unobservable stochastic process. The estimated reaction coefficient exhibits significant time variation -- contrary to the prevailing view. The U.S. government had been more proactive about constraining increases in debt during the 20th century than previously thought. Since the year 2000, however, policy makers have become much less aggressive about containing the debt process. The U.S. government debt was sustainable for most of the period except during Depression, World War II, and in the years surrounding Great Recession.

Keywords: Intertemporal government budget constraint, Fiscal reaction function, Debt-GDP ratio, Stability condition, State space model

JEL Numbers: E62, H62, H63

I. Introduction

The U. S. federal government has run budget deficits in most fiscal years since the 1980s. These persistent deficits have given rise to a mounting debt burden for Americans. In particular, the federal debt[1] rose from about 24 percent of GDP (gross domestic product) in 1980 to 47 percent of GDP in 1996 – an event unprecedented in peacetime. Although the federal government ran surpluses in the late 1990s, it has run deficits in each fiscal year since 2002, and the federal debt reached 72 percent of GDP in 2016. As of that year, each adult American's share of the federal government debt was about $56,000. Clearly, this is an alarming number that merits attention. The problem is likely to worsen as the large baby-boom generation is beginning to retire in large numbers and starting to draw on government benefits for the elderly. The Congressional Budget Office has projected debt-GDP ratios nearing 100 percent by 2028.

Given the gravity of the government debt problem facing the U.S. and many other countries, a good number of empirical studies have been devoted to test the sustainability of fiscal policy.[2] Results are inconclusive. Bohn (1998) calls into question the validity of the traditional analysis of fiscal sustainability based on the transversality condition of the government budget constraint. He argues that previous studies fail to account for the fact that the government takes corrective action for budget deficits by increasing taxes and/cutting spending in response to a rising or higher debt-GDP ratio. This idea, described in the form of the "fiscal reaction" function relating the primary surplus to the debt level, should be incorporated into the government budget constraint to test for fiscal sustainability. Bohn (1998) estimates the fiscal reaction function for 1916-1995 and finds that U.S. fiscal policy historically has been sustainable despite extended periods of primary deficits.

The fiscal reaction function has become a standard tool in modern analysis of fiscal sustainability (see, e.g., Bohn, 2008; Canzoneri, Cumby, and Diba, 2001; Mendoza and Ostry, 2008; Collignon, 2012).[3] This function, though widely used, is based on the implicit assumption that the reaction coefficient, which measures the degree of the government's fiscal willingness to respond to changes in debt, is constant. This needs not necessarily the case. In fact, the fiscal reaction function with a constant coefficient may not provide an adequate description of the government's fiscal behavior toward debt and hence may be too limited to properly assess fiscal sustainability. We postulate that the government's fiscal response to changes in debt is likely influenced by economic conditions as well as the political environment it is facing. Importantly, its response may not be fixed; instead it varies over time.

To provide some motivating evidence, take the case of the financial crisis of 2007-2009. In the wake of the crisis, a number of prominent economists have advocated for further fiscal stimulus in spite of the fact that output was slowly returning to its potential levels. For example, Larry Summers has suggested that, to combat "secular stagnation," policy makers should stop being overly concerned with debt.[4] A similar argument has been made by Paul Krugman.[5] In its Fiscal Year 2019 budget request, the Trump Administration projected deficits as a share of GDP in excess of 4 percent through fiscal 2020, in spite of the highest debt-GDP ratios experienced since the end of World War II. These results indicate that a different economic environment (such as the financial crisis) calls for a different fiscal response to changes in debt. On the other hand, a number of studies have pointed to political determinants of fiscal surpluses and deficits (e.g., Alesina and Tabellini, 1990; Goff and Tollison, 2002). Mendoza and Ostry (2008) find a significant difference in the reaction coefficient for industrial and emerging countries. Although they do not provide an adequate explanation, this is clearly a reflection of differences in the economic and political environment facing different countries. Moreover, even as far back as Barro (1979), there existed the notion that debt might evolve randomly over time (as opposed to tending towards some target value), implying a time-varying aspect to the fiscal reaction coefficient. This discussion supports our claim that the existing model based on a constant reaction coefficient is too limiting to provide an adequate description of the government's fiscal behavior toward debt, demonstrating the need for a more general framework to address the question of fiscal sustainability.

To formulate the above argument, we develop a generalized fiscal reaction function with a time-varying reaction coefficient that is allowed to vary with the interest rate and growth rate as well as other unobservable factors. This enables us to analyze the dynamic behavior of government debt and deficits in a more comprehensive manner, relative to earlier studies. With this work, we intend to make contributions on both theoretical and empirical grounds. On the theoretical side, with the generalized fiscal reaction function, we provide a full analysis of debt dynamics by deriving the stability condition that ensures a convergent path of the debt-GDP ratio toward a long-run or steady state value. Then we examine the effects of the interest rate and growth rate on the steady state debt-GDP ratio. We find that a constant reaction coefficient implies that an increase in the economy's growth rate lowers the steady state debt-GDP ratio, but an increase in the interest rate raises the steady state debt-GDP ratio. When the reaction coefficient evolves endogenously with growth rates or the interest rate, this needs no longer hold. Further, we analyze the long-run relationship between the primary surplus or deficit and the debt-GDP ratio, which describes the fraction of GDP that the government needs to save, i.e., the primary budget surplus, in order to service a debt in the steady state. An important finding is that at low interest rates relative to growth rates, budget deficits are consistent with a sustainable debt.

In the empirical analysis, we present a state space model to estimate the generalized fiscal reaction function by treating the reaction coefficient as an unobservable stochastic process. Use of a state space approach is also justified by the argument that the government's corrective action with respect to increasing debt is based on the natural rate of interest and the growth rate of potential output, which are not observable. The observed real interest rate is specified as the sum of the natural rate of interest and a transitory or cyclical component. It is correlated with the growth rate of potential output as well as other determinants (such as consumers' preferences) that may evolve as a random walk. The cyclical component of the interest rate is specified as an autoregressive process. Observed real GDP series is defined as the sum of unobservable trend and transitory or cyclical components. The trend follows a random walk with a drift, and we interpret this drift as the natural rate of output growth. The state space model is estimated using the Kalman filter via Bayesian techniques with the use of prior information, since the model is fairly heavily parameterized and the sample period is somewhat short.

We fit the model to the data on U.S. debt-GDP ratios over 1916 to 2016. Our results reveal that the federal government was actually more aggressive about containing the debt-GDP ratio for much of the postwar 20th century than would be suggested by the estimates of Bohn (1998). This increased aggressiveness is offset by the fact that since the year 2000, policy makers have become progressively laxer about addressing the debt question. As our model incorporates dynamics associated with recessions and military build-ups as a result of wars, this increasing laxness cannot be attributed to the Great Recession or the recent conflicts in Afghanistan or Iraq.

At the end of our sample period in 2016, the reaction coefficient is estimated to be negative, albeit close to zero. The stability condition that we derive is close to being violated as of 2016, an unprecedented occurrence in a relatively peaceful period of moderate growth.

These results help us to obtain deeper insight into the forces driving the debt-GDP ratio in the US. Although a number of policy makers and commentators have expressed concern about the trajectory of debt, our findings provide an additional layer of structure, by establishing quantitatively that the reaction function governing fiscal policy at the federal level has, in fact, materially changed in the last fifteen years. While we continue to conclude, as did Bohn (1998), that the debt-GDP ratio is mean-reverting, our results suggest that the reversion is now slower and the mean is higher than at the time of writing for Bohn (1998). The process for the debt-GDP ratio is also currently much closer to violating the stability condition that helps determine sustainability.

The rest of the paper proceeds as follows. We discuss our theoretical framework in Section 2. We introduce our empirical model in Section 3 and present estimation methods and report results in Section 4. In Section 5, we compare our results with those of other popular specifications, and we offer conclusions in Section 6.

II. Dynamics of Government Debt under a Generalized Fiscal Reaction Rule

When conducting fiscal policy, the government faces a budget constraint in each period *t* (*t* = 0, 1, ..., *n*), which is expressed as

$$D_{t+1} = (1+r)(D_t + G_t - T_t) = (1+r)(D_t - S_t), \ \forall t \geq 0, \qquad (1)$$

where D_t is the stock of an interest-bearing government bond outstanding at the beginning of period *t*, *r* is the rate of interest, G_t is government spending net of interest payments during period *t*, T_t is government revenue during period *t*, and $S_t \equiv T_t - G_t$ is the "primary" budget balance or surplus at time *t*, which is the difference between the government's tax revenue and spending net of interest payment. For ease of exposition, the interest rate is assumed to be constant over the period or stationary with a constant mean, but this assumption will be relaxed in the empirical analysis. In the United States, the importance of seigniorage or money financing as a government revenue source is small, so it is not explicitly considered, although it can be regarded as being part of tax revenue.

A growing economy with a high GDP has relatively more resources available to pay the principal and interest on its debts, so a pertinent measure of the government's indebtedness is not the level of debt itself, but the debt as a percent of GDP, i.e., the debt-GDP ratio. Suppose that the economy, as measured by GDP (denoted by Y), grows at a rate of g percent per period so that $Y_{t+1} = (1+g)Y_t, \ \forall t \geq 0$. Then divide both sides of (1) by GDP to express the government budget constraint in terms of the debt-GDP ratio:

$$d_{t+1} = (1+\delta)\left(d_t - s_t\right), \forall t \geq 0, \qquad (2)$$

with $\lim_{n\to\infty} d_{n+1} \geq 0$, where $d_t \equiv D_t / Y_t$ is the debt-GDP ratio, $s_t \equiv S_t / Y_t$ is the surplus-GDP ratio, and $1+\delta \equiv (1+r)/(1+g)$, which gives $\delta \approx r - g$, the growth-adjusted interest rate.

Taxes and spending, and hence surpluses, are not exogenously given in the government budget constraint. Rather, they are government choice or policy variables determined by a political process, thereby affecting the surpluses or deficits. In particular, we assume that the government takes corrective action for its budget by increasing taxes and/or cutting spending in response to a higher debt-GDP ratio. Bohn (1998), forcefully argues that this idea, described by the fiscal reaction function, should be incorporated into the government budget constraint (2) to test for fiscal sustainability.

Following Bohn (1998), consider the following linear fiscal reaction function:

$$s_t = \rho d_t + \mu_t, \forall t \geq 0, \qquad (3)$$

where ρ is the reaction coefficient and μ_t captures the effects of other explanatory variables influencing the surplus-GDP ratio.[6] If the government takes corrective measures by increasing taxes or cutting spending in response to a higher debt-GDP ratio, we expect that the surplus-GDP ratio will be positively related to the debt-GDP ratio, with the value $\rho \geq 0$.

While the fiscal reaction function of the form in (3) has been useful in analyzing fiscal sustainability (see, e.g., Bohn, 1998; Canzoneri, Cumby, and Diba, 2001; Mendoza and Ostry, 2008; Collignon, 2012), it, by and large, rests on the implicit assumption that the value of ρ is constant.[7] However, ρ measures the degree of fiscal responsiveness to changes in debt, and we argue that its value needs not be fixed, but instead varies over time, possibly with changes in the interest rate and output growth rate, i.e., $\rho = f(r, g)$. For a given debt level, rising interest rates may likely force the government into further fiscal adjustment in the form of tax hikes or spending cuts to pay for higher debt service, thus necessitating larger budget surpluses. On the other hand, a higher growth rate can increase a country's fiscal capacity and allow the government to have smaller surpluses in order to maintain a given debt level.

Thus we propose that the reaction coefficient ρ takes the following form:

$$\rho = \alpha_0 + \alpha_1 r + \alpha_2 g, \tag{4}$$

where $\alpha_0, \alpha_1,$ and α_2 are coefficients to be estimated.[8] We hypothesize that $\alpha_0 > 0$, $\alpha_1 > 0$, and $\alpha_2 < 0$. A positive value of α_1 can help explain Mendoza and Ostry's (2008) conjecture that in countries with riskier financial and fiscal environments (such as developing countries), where interest rates may be higher, the fiscal reaction coefficient is also higher, implying larger budget surpluses or smaller budget deficits required for these countries in relation to GDP. However, with a negative value of α_2, high economic growth experienced by developing countries can make the reaction coefficient smaller than that of industrial countries, implying smaller budget surpluses or larger budget deficits affordable by developing countries in relation to GDP.

Substituting (4) into (3), we obtain

$$s_t = \alpha_0 d_t + \alpha_1 (r d_t) + \alpha_2 (g d_t) + \mu_t, \forall t \geq 0, \tag{5}$$

which can be rewritten as

$$s_t = \alpha_0 d_t + \mu_t^m, \forall t \geq 0, \tag{6}$$

where $\mu_t^m \equiv \alpha_1 (r d_t) + \alpha_2 (g d_t) + \mu_t, \forall t \geq 0$. Clearly, Bohn's fiscal reaction function (3) with a constant ρ is a special case of the generalized fiscal reaction function (5) with an endogenous ρ if we let $\alpha_0 = \rho$ and $\alpha_1 = \alpha_2 = 0$. In this case, we have the fiscal reaction function (6) that is of the same in form as (3). This implies that when the fiscal reaction coefficient is endogenously determined, estimation of (3) instead of (5) likely produces a biased estimate of the reaction coefficient. Notably, equation (5) allows us to examine the effect of the interest rate and growth rate on the primary surplus. That is, $\partial s_t / r = \alpha_1 d_t > 0$ and $\partial s_t / g = \alpha_2 d_t < 0$. This suggests that policy makers may alter their behavior in response to changes in the interest rate or growth rate, which can affect the budget surplus or deficit. From now on, we will assume that the reaction coefficient ρ in (3) takes the form in (4).

To investigate dynamic properties of the debt-GDP ratio, substitute (3) into (2) to get

$$d_{t+1} = \kappa d_t + w_t, \forall t \geq 0, \quad (7)$$

or, equivalently,

$$\Delta d_{t+1} \equiv d_{t+1} - d_t = -(1-\kappa) d_t + w_t, \forall t \geq 0, \quad (8)$$

where $\kappa \equiv (1+\delta)(1-\rho)$ and $w_t \equiv -(1+\delta)\mu_t$ with $\mu_t < 0$. Equation (7) is a first-order difference equation with the forcing variable w, and the stability condition requires that $-1 < \kappa < 1$. It is common to use an approximate expression for κ with $\delta = r - g$ (Bohn, 1998; Mendoza and Ostry, 2008; Collignon, 2012). Further, for small values of δ and ρ, $\kappa \approx 1 + \delta - \rho$. Then the stability condition gives the result that $r - g < \rho$ (see Bohn, 1998). In the interests of a more precise analysis, we instead use an exact expression for κ with $1 + \delta = (1+r)/(1+g)$. We then have the stability condition of the form: $((r-g)/(1+r)) < \rho$.[9] From this discussion, we state:

PROPOSITION 1. *The viable condition for fiscal sustainability requires that* $((r-g)/(1+r)) < \rho$. *For* $r < g$, *this condition is satisfied for a positive value of* ρ. *When this condition is satisfied, the debt-GDP ratio has a stable, convergent path toward a steady state, and the government's debt or fiscal position is sustainable. If the fiscal position is not sustainable, the government debt is explosive and will not converge to a steady state.*

Thus, the viable sustainability condition does not depend on the $r > g$ condition, which is required in the conventional analysis based on the transversality condition of the intertemporal government budget constraint (see Hamilton and Flavin, 1986). It can be valid even if $r < g$ as long as $((r-g)/(1+r)) < \rho$.

The stability condition ensures a convergent path of the debt-GDP ratio toward a long-run or steady-state value. Its value can be obtained by setting $\Delta d_{t+1} = 0$ in (8) and solving it for d. Assuming that the value of w_t is constant at w with μ over time, the steady-state value of the debt-GDP ratio $(\bar{d})$ is given by

$$\bar{d} = \frac{w}{1-\kappa} = \frac{-(1+r)\mu}{(1+g)-(1+r)(1-\rho)}. \tag{9}$$

For a stable solution, a positive steady-state debt-GDP ratio requires that $\mu < 0$.[10] With this steady-state debt-GDP ratio, the solution of (7) is

$$d_t - \bar{d} = \kappa^t \left(d_0 - \bar{d} \right), \forall t \geq 0. \tag{10}$$

This equation describes the trajectory of the debt-GDP ratio over time toward the steady state.[4] Substituting the value of d_t from (10) into (5), we also obtain the time path of primary surplus over time. Equation (10) reveals that the debt-GDP ratio has a mean-reverting property with the mean taken to be its steady-state value. Note that $(d_t - \bar{d})$, $\forall t \geq 0$, is the deviation of the debt-GDP ratio from its steady state value. A fraction of this deviation or gap tends to decay each period with the government's fiscal action, absent further shock, at a constant rate ϕ given by $\phi \equiv 1-\kappa$. Whether a rise in the interest rate or growth rate increases or decrease the rate of convergence to the steady state depends on its effect on the steady-state debt-GDP ratio,

All else constant, a higher reaction coefficient ρ reduces the steady-state debt-GDP ratio because the government responds with larger primary budget surpluses, instead of further borrowing, as debt rises. It is commonly claimed that high economic growth lowers the debt-GDP ratio or low economic growth raises the debt-GDP ratio (see Bohn, 1995, page 265, fn. 9; Collignon, 2012, page 545). To assess the effect of economic growth on debt dynamics, we can evaluate the steady-state debt-GDP ratio (9) with respect to the fiscal reaction function with a constant coefficient ρ. Taking derivatives of this expression with respect to the interest rate and growth rate, we have

$$\frac{\partial \bar{d}}{\partial r} = \frac{-(1+g)\mu}{\{(1+g)-(1+r)(1-\rho)\}^2} > 0 \tag{11}$$

and

$$\frac{\partial \bar{d}}{\partial g} = \frac{(1+r)\mu}{\{(1+g)-(1+r)(1-\rho)\}^2} < 0, \tag{12}$$

since $\mu < 0$. Equation (11) shows that an increase in the interest rate raises the steady-state debt-

GDP ratio. Equation (12) clearly supports the claim that high economic growth lowers the debt-GDP ratio (Collignon, 2012, page 545).

This claim, however, is based on a constant reaction coefficient and fails to account for the effect of the growth rate on the fiscal reaction coefficient as described in (4). In particular, high economic growth lowers the reaction coefficient, resulting in a higher debt-GDP ratio. Thus to ascertain the effect of a higher growth rate on the debt-GDP ratio, we need to compare the higher debt-GDP ratio from a lower reaction coefficient, with the lower debt-GDP ratio obtained by assuming the constant reaction coefficient. A generalized fiscal reaction function with an endogenous reaction coefficient allows us to undertake a more comprehensive examination of this issue. With ρ of the form in (4), there are two main variables that drive the steady-state debt-GDP ratio $(\bar{d})$: *r* and *g*. To examine the effect of the interest rate and growth rate on the steady-state debt-GDP ratio, we take a general form of the fiscal reaction coefficient, i.e., $\rho = f(r,g)$, and derive implications for the linear form assumed in (4). Taking derivatives of $\bar{d}$ in (9) with respect to *r* and *g*, we get

$$\frac{\partial \bar{d}}{\partial r} = \frac{\left[-(1+g)+(1+r)^2 f_r(r,g)\right]\mu}{\{(1+g)-(1+r)(1-f(r,g))\}^2} \tag{13}$$

and

$$\frac{\partial \bar{d}}{\partial g} = \frac{(1+r)\left[1+(1+r)f_g(r,g)\right]\mu}{\{(1+g)-(1+r)(1-f(r,g))\}^2}. \tag{14}$$

Note that equations (11) and (12) are special cases of (13) and (14) when $f_r(r,g) = f_g(r,g) = 0$. Unlike equations (11) and (12), however, the signs of the derivatives in (13) and (14) are not known. Since $\mu < 0$, evaluating them requires knowing whether the bracket term in the numerator is positive or negative. This gives us the following condition for *r*:

$$\frac{\partial \bar{d}}{\partial r} > 0 \text{ according as } f_r(r,g) < \frac{1+g}{(1+r)^2} \tag{15a}$$

or

$$\frac{\partial \bar{d}}{\partial r} < 0 \text{ according as } f_r(r,g) > \frac{1+g}{(1+r)^2}. \tag{15b}$$

We obtain the similar condition for *g*:

$$\frac{\partial \bar{d}}{\partial g} > 0 \text{ according as } f_g(r,g) < \frac{-1}{1+r} \tag{16a}$$

or

$$\frac{\partial \bar{d}}{\partial g} < 0 \text{ according as } f_g(r,g) > \frac{-1}{1+r}. \tag{16b}$$

Evaluating the condition in (15a) and (15b), since $(1+g)/(1+r)^2 > 0,$ this implies that $f_r(r,g) > 0.$ However, since $(1+g)/(1+r)^2 \approx 1,$ if $f_r(r,g)$ lies between 0 and 1, an increase in the interest rate will raise the steady-state debt-GDP ratio. If $f_r(r,g)$ is sufficiently greater than one, an increase in the interest rate will lower the steady-state debt-GDP ratio. For the condition in (16a) and (16b), we can provide a similar explanation. Since $1/(1+r) > 0,$ this implies that $f_g(r,g) < 0.$ Further, since $1/(1+r) \approx 1,$ if $f_g(r,g)$ lies between 0 and 1 in absolute value, an increase in the growth rate will lower the steady-state debt-GDP ratio. If $f_g(r,g)$ is sufficiently greater than one in absolute value, an increase in growth rate will raise the steady-state debt-GDP ratio.

To derive implications of these conditions, we can look at the generalized fiscal reaction function (4). Since we hypothesize that $f_r(r,g) = \alpha_1 > 0,$ it, in effect, measures the effect of the interest rate on the primary surplus for a given debt. And since we also suppose that $f_g(r,g) = \alpha_2 < 0,$ it measures the effect of the growth rate on the primary surplus for a given debt. The above discussion suggests that how the government adjusts its primary surplus in response to a change in the interest rate or growth rate determines the steady-state debt-GDP ratio. These results allow us to state an important finding of our analysis:

PROPOSITION 2. *An increase in growth rate can raise or lower the steady-state debt-GDP ratio. This is in sharp contrast to the common view that an increase in growth rate lowers the* steady-state *debt-GDP ratio. Whether an increase in growth rate raises or lowers the* steady-state *debt-GDP ratio depends on the degree of fiscal responsiveness of the primary surplus for a given debt level with respect to an increase in the growth rate; a higher degree of fiscal* *responsiveness allows a country to have a higher debt-GDP ratio. Similarly, an increase in the interest rate can raise or lower the* steady-state *debt-GDP ratio. Whether an increase in the interest rate raises or lowers the* steady-state *debt-GDP ratio depends on the degree of fiscal responsiveness of the primary surplus for a given debt level with respect to an increase in the interest rate; a lower degree of fiscal responsiveness allows a country to have a higher debt-GDP ratio.*

A change in the interest rate or growth rate moves the steady-state debt-GDP ratio accompanied by a movement in its transitional path to a new steady state. Figure 1 illustrates the time path of the debt-GDP ratio subsequent to an increase in growth rate on the basis of the assumption that it induces a high fiscal response to the government. The lower graph describes the debt-GDP ratio path associated with the economy's growth rate of g_1, while the upper graph describes the debt-GDP ratio path with a higher growth rate of g_2. The economy begins with the debt-GDP ratio of d_0, and it increases gradually until it reaches the steady state. As the growth rate increases, the debt-GDP ratio path moves up. With a higher growth rate, however, the speed of convergence decreases and it takes more time to reach the new steady state.

This discussion underscores the importance of economic growth for a country's fiscal position. A country with a high growth rate can afford to sustain a high debt. In contrast, a country with a low growth rate may not sustain high debt-GDP ratios. Interestingly, this provides some insight into the widely cited Reinhart and Rogoff (1990) analysis about the relationship between government debt and GDP growth. They investigate this relationship among 20 advanced economies in the post-war period and find that there is a debt threshold or tipping point, i.e., a government debt-GDP ratio beyond which there are negative effects on growth. Many studies have followed to reexamine Reinhart and Rogoff's (1990) analysis (see, e.g., Checherita-Westphal and Rother, 2012; Herndon, Ash, and Pollin, 2014). Our analysis reveals that there is a simultaneity between debt and growth. A high debt affects growth rate, but high growth may induce a country to take on more debt.

While an increasing debt-GDP ratio can be sustainable as long as the stability condition is satisfied, a constant or steady-state debt-GDP ratio is always sustainable because it is stationary and the debt will never explode. When the debt-GDP ratio reaches its steady-state value, we can find the primary surplus that supports it. To do so, we let $d_{t+1} = d_t = \bar{d}, \forall t \geq 0,$ in (2) and solving for s_t, we obtain the following result:

PROPOSITION 3. *Suppose that the debt-GDP ratio reaches the steady state. Then the following equation describes the long-run relationship between the primary surplus/deficit and government debt:*

$$\bar{s} = \frac{(r-g)}{(1+r)}\bar{d}, \tag{17}$$

which is the fraction of GDP that the government needs to save, i.e., the primary budget surplus, in order to service a debt in the steady state, which is equal to the discounted growth-adjusted interest payments on a debt that is sustainable.

Equation (17) reveals that the spread between r and g and the debt-GDP ratio determine the sustainable rate of the budget deficit or surplus. If $r > g$, the government needs to have a primary surplus to maintain a given level of the debt-GDP ratio. If $r < g$, however, the government can run a primary deficit every period and still keep the debt-income ratio constant. When $g = 0$, equation (17) describes a fraction of GDP for interest payments that the government is required to pay in order to service a given debt-GDP ratio. Figure 2 illustrates the relation between the primary-surplus ratio and the debt-GDP ratio in the steady state. It shows that to keep the debt-GDP ratio constant or at a steady state, $r > g$ requires a primary surplus, but $r < g$ demands a primary deficit. An implication of this result is that *frequent or chronic budget deficits are consistent with a sustainable debt if $r < g$.*

The interest rate and growth rate also determine the primary surplus required to keep the debt-GDP ratio constant. A higher interest rate causes more interest payments on a given debt level, which requires the government to have a higher primary surplus or a lower deficit. On the other hand, a higher growth rate can enable the government to afford reduced primary surpluses or higher deficit. However, as discussed before, changes in the interest rate and growth rate also affect the long-run debt-GDP ratio. To see the effect on the steady-state primary surplus from a change in the interest rate and growth rate, we substitute $\bar{d}$ in (9) into (17) to get

$$\bar{s} = \frac{-(r-g)\mu}{(1+g)-(1+r)(1-f(r,g))}, \qquad (18)$$

Figure 3 illustrates the effect of an increase in growth rate on the steady-state primary surplus ratio when the growth rate is higher than interest rate. A higher growth rate causes the debt-GDP ratio to increase from $\bar{d}_1$ to $\bar{d}_2$. It also causes the government to have a higher deficit in steady state, moving $\bar{s}_1$ to $\bar{s}_2$. While we discussed the effect of the growth rate, Figure 3 is also relevant for discussing the effects of the interest rate on the primary surplus ratio and the debt-GDP ratio in steady state. In this case, the effect on the debt-GDP ratio and the surplus-GDP ratio is the opposite of a change in growth rate. These results are worth summarizing:

PROPOSITION 4. *A high growth rate relative to the interest rate can allow a country to have a high budget deficit in the steady state. Moreover for sufficiently high/low derivatives of f(r,g), an increase in the growth rate or a decrease in the interest rate may allow a country to have a higher sustainable debt-GDP ratio and a higher steady state budget deficit.*

III. A State Space Model

The fiscal reaction function (3) plays a central role in analysis of fiscal sustainability. Hence a proper estimation of this function is essential in appropriately assessing fiscal sustainability. Existing studies estimate the fiscal reaction function by treating the reaction coefficient constant (see, e.g., Bohn, 1998; Canzoneri, Cumby, and Diba, 2001; Mendoza and Ostry, 2008; Collignon, 2012). The previous section demonstrated that the reaction coefficient is not constant but is determined endogenously. This section presents a cogent empirical framework for implementing the generalized fiscal reaction function composed of equations (3) and (4). The theoretical model in the previous section is formulated, for convenience, with constant interest rates and growth rates as well as the constant value of unmodeled determinants of the surplus-GDP ratio. These variables are taken to vary over time in empirical analysis. Then the time-varying nature of the reaction coefficient (4) becomes crucial in our analysis of fiscal sustainability. In order to capture the possibly time-varying behavior of fiscal policy, we propose to model the dynamics of the debt-GDP ratio in a state space framework. We also conjecture that unobserved factors, possibly capturing consumers' preferences and political tastes, may influence the reaction to debt stockpiles, which makes the state space model even more suited for analyzing fiscal sustainability. Moreover, the state space model makes explicit allowance for permanent and temporary changes in the growth rate as well as in the interest rate.

The fiscal reaction coefficient (4) measures the degree of the government's fiscal response to increasing debt-GDP ratio, which depends on the interest rate and growth rates. We argue that the government's response to higher debts is likely influenced by long-term changes in the interest rate and growth rate. That is, a change in the growth rate of potential GDP or the natural rate of interest is more likely to change how fiscal policy responds to the current debt level than would year to year fluctuations. To incorporate this idea with unobservable shocks, we specify the fiscal reaction coefficient as a time-varying function of the following form:

$$\rho_t = \rho_{t-1} + \alpha_1 r^*_{t-1} + \alpha_2 g^*_{t-1} + e_t, \forall t \geq 0, \tag{19}$$

where r^*_{t-1} is the natural rate of interest rate at period *t*-1, g^*_{t-1} is the natural rate of output or GDP growth at period *t*-1, and e_t is an error term that captures unobservable shocks. For example, it is easy to imagine that other factors, such as political and social dynamics that are difficult to measure, can also affect the way fiscal policy is set relative to the economy's debt. Thus, we may desire that the fiscal reaction coefficient is itself subject to shocks, which necessitates the inclusion of *e*. The natural rates of interest and of potential output growth are also unobservable.

For the natural rate of interest rate, we follow the work of Laubach and Williams (2003) and specify it as

$$r^*_t = \gamma_1 g^*_{t-1} + \phi_{t-1}, \forall t \geq 0, \tag{20}$$

where ϕ_t captures other unobservable determinants of r^* such as households' rate of time preference, which are not directly observable. We assume that it follows a random walk:

$$\phi_t = \phi_{t-1} + \upsilon^{\phi}_t, \forall t \geq 0, \tag{21}$$

where υ^z_t is a white noise error.[11] We also consider a transitory or cyclical component of the interest rate (r^c_t) and specify it as

$$r^c_t = \gamma_2 r^c_{t-1} + \gamma_3 g^*_{t-1} + \upsilon^c_t, \forall t \geq 0, \tag{22}$$

where υ_t^c is a white noise error. The observed real interest rate is specified as the sum of the natural rate of interest and the transitory or cyclical component:

$$r_t = r_t^* + r_t^c, \forall t \geq 0. \tag{23}$$

To specify the natural rate of output growth (g_t^*), we adopt the trend-cycle decomposition widely used in the time series literature (see Clark, 1987; Morley, Nelson, and Zivot, 2003; Kim and Murray, 2002) by expressing real GDP (y_t) in logarithm. In this decomposition, the observed output series (y_t) is the sum of unobservable trend (y_t^τ) and transitory or cyclical (y_t^c) components. The trend follows a random walk with a drift, which is considered the natural rate of output growth. The cyclical part is an AR(1) with the cyclical interest rate (Laubach and Williams, 2003) Formally,

$$y_t = y_t^\tau + y_t^c, \forall t \geq 0, \tag{24}$$

$$y_t^\tau = y_{t-1}^\tau + g_{t-1}^* + \eta_t^\tau, \forall t \geq 0, \tag{25}$$

$$g_t^* = g_{t-1}^* + \eta_t^g, \forall t \geq 0, \tag{26}$$

$$y_t^c = \gamma_4 y_{t-1}^c + \gamma_5 r_{t-1}^c + \eta_t^c, \forall t \geq 0, \tag{27}$$

where η_t^τ, η_t^g, and η_t^c are independent white noise processes.

To estimate the fiscal reaction function (3), there is a set of unmodelled other determinants influencing the surplus-GDP ratio that may cause temporary deviations from the fiscal reaction rule. Following Bohn (1996), we consider cyclical fluctuations in output and in military spending, so the fiscal reaction function is specified as

$$s_t = \delta_0 + \rho_t d_t + \delta_1 y_t^c + \delta_2 m_t^c + \delta_3 s_{t-1} + \theta_t, \forall t \geq 0, \tag{28}$$

where θ_t is an error term. The lagged surplus-GDP ratio is also included to account for possible serial correlation in the series. To estimate cyclical real military spending m_t^c, which is not directly observable, we follow the trend-cycle decomposition and specify the observed military spending series m_t in a logarithm as consisting of the trend and cyclical components:[12]

$$m_t = m_t^\tau + m_t^c, \forall t \geq 0, \tag{29}$$

$$m_t^\tau = m_{t-1}^\tau + g_{t-1}^m + \omega_t^\tau, \forall t \geq 0, \tag{30}$$

$$g_t^m = g_{t-1}^m + \omega_t^g, \forall t \geq 0, \tag{31}$$

$$m_t^c = \gamma_5 m_{t-1}^c + \omega_t^c, \forall t \geq 0, \tag{32}$$

where ω_t^τ, ω_t^g, and ω_t^c are independent white noise processes. The transitory component of military spending is interpreted as build-ups in spending surrounding military conflicts such as World War II and the wars in Korea and Vietnam.

All of the equations described above can be written and analyzed in a state space framework (see Kim and Nelson, 1999). The transition equation, which describes the evolution of unobservable variables, can be written as

$$\beta_t = F_t \, \beta_{t-1} + \mathrm{E}_t, \forall t \geq 0, \tag{33}$$

where

$$\beta_t = \begin{bmatrix} \rho_t \\ r_t^* \\ \phi_t \\ r_t^c \\ y_t^\tau \\ g_t^* \\ y_t^c \\ m_t^\tau \\ g_t^m \\ m_t^c \end{bmatrix} \qquad F = \begin{bmatrix} 1 & \alpha_1 & 0 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \gamma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_2 & 0 & \gamma_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_5 & 0 & 0 & \gamma_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_5 \end{bmatrix} \qquad \mathrm{E}_t = \begin{bmatrix} e_t \\ 0 \\ \upsilon_t^\phi \\ \upsilon_t^c \\ \eta_t^\tau \\ \eta_t^g \\ \eta_t^c \\ \omega_t^\tau \\ \omega_t^m \\ \omega_t^c \end{bmatrix}$$

The measurement equation relates observed data to unobservable variables:

$$X_t = H\beta_t + JZ_t + \Xi_t, \forall t \geq 0, \tag{34}$$

where

$$X_t = \begin{bmatrix} r_t \\ y_t \\ s_t \\ m_t \end{bmatrix} \quad H = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ d_t & 0 & 0 & 0 & 0 & 0 & \delta_1 & 0 & 0 & \delta_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad J = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \delta_0 & \delta_3 \\ 0 & 0 \end{bmatrix} \quad \mathrm{Z} = \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \quad \Xi_t = \begin{bmatrix} 0 \\ 0 \\ \theta_t \\ 0 \end{bmatrix}$$

We assume that $\mathrm{E}_t \sim N(0,Q)$ and $\Xi_t \sim N(0,R)$. To maintain the parsimony of the model, we assume that Q is a diagonal matrix and that the shocks to the various state variables are independent of each other at all leads and lags. Since there is no error associated with the natural rate of interest rate (see equation (22)), its variance is zero; thus there are nine variances that make up for Q: [$\sigma^2(\rho_t)$, $\sigma^2(\phi_t)$, $\sigma^2(r_t^c)$ $\sigma^2(y_t^\tau)$, $\sigma^2(g_t^*)$, $\sigma^2(y_t^c)$, $\sigma^2(m_t^\tau)$, $\sigma^2(g_t^m)$, $\sigma^2(m_t^c)$]'. There is only one variance that makes up for R: the variance of innovations to the surplus-GDP ratio itself, $\sigma^2(s_t)$. The state space model is estimated by maximum likelihood using the Kalman filter (see Kim and Nelson, 1999). A detailed discussion of estimation methods is provided in the next section.

IV. Estimation and Results

A. Data

We estimate the model over the years from 1916 to 2016. Most of our data comes from standard sources (i.e., information on real GDP and real military spending is obtained from the Bureau of Economic Analysis, while the debt-GDP and surplus-GDP ratios are taken from the U.S. Office of Management and Budget). To extend the data set to 1916 (the first year analyzed in Bohn (1998)), we obtain information on most of the key variables for our model from the online database associated with Bohn (2008). The one exception to that rule is that, to extend the data set for the yield on the 10-year Treasury note, we must appeal to the database published alongside Shiller (1989) and Shiller (2015).

Figure 4 reports the time series of the debt- GDP ratio over the course of our sample period. Debt is generally rising between 1916 and the end of World War II, peaking above 100 percent of GDP before falling dramatically over a span of decades. It begins to rise in the 1980s, falls in the late 1990s, and has generally been increasing since the turn of the new millennium. Figure 5 offers an analogous depiction of the primary budget surplus as a share of GDP. The time series is characterized by large negative values coinciding with World War I, World War II, and the Great Recession of 2007 to 2009. Outside of these episodes, the surplus-GDP ratio tends to hover around zero, although there has a very subtle negative trend in the series since the start of the 1950s. A negative trend in the surplus combined with rising debt would seem to support the idea that the relationship between the two series has been changing over time.

B. Estimation Methods

In order to estimate the state space model described by equations (33) and (34) for the time-varying fiscal reaction function, we adopt Bayesian techniques. A primary reason for the use of these techniques is that our model is fairly heavily parameterized and the sample period is somewhat short, so Bayesian methods incorporating prior knowledge on the behavior of some of the dynamic relationships may help to produce more efficient estimates. Specifically, we employ Gibbs sampling Markov Chain Monte Carlo (MCMC) posterior simulation methods for state space models, as introduced in Carter and Kohn (1994) and described in Koop and Korobilis (2009).[13]

For the purposes of the Gibbs sampling, we divide each of the parameters of the model and the unobserved state variables into blocks. The first block is the set of state variables, β_t = [ρ_t r_t^* ϕ_t r_t^c y_t^τ g_t^* y_t^c m_t^τ g_t^m m_t^c]'. The second is the set of parameters in the matrix F that governs the evolution of β_t, namely, $\Pi = [\alpha_1 \;\; \alpha_2 \;\; \gamma_1 \; \gamma_2 \; \gamma_3 \; \gamma_4 \; \gamma_5]$'. The third is the set of variance terms that make up the variance-covariance matrix Q associated with the error terms for the state vector E_t, namely, [$\sigma^2(\rho_t)$, $\sigma^2(\phi_t)$, $\sigma^2(r_t^c)$ $\sigma^2(y_t^\tau)$, $\sigma^2(g_t^*)$, $\sigma^2(y_t^c)$, $\sigma^2(m_t^\tau)$, $\sigma^2(g_t^m)$, $\sigma^2(m_t^c)$]'. The fourth is the set of parameters found in the measurement equation, namely, $\Delta =$ [δ_0 δ_1 δ_2 δ_3]'. Finally, there is the set of variances that make up the variance-covariance matrix R, i.e., $\sigma^2(s_t)$.

The Gibbs sampling algorithm proceeds as follows. We start with an initial guess for Π_t, Q, Δ, and $\sigma^2(s_t)$. Conditional on these initial guesses, we draw $\beta^T \equiv [\beta_0,...,\beta_T]'$, running this draw through the Kalman Filter. The distribution of β_t is assumed to be multivariate normal for every period. Then, conditional on the draw of β^T, we draw values for Π from a multivariate normal, though it should be noted that we need to impose that the cyclical components of real GDP, the real interest rate, and real military spending be stationary, so γ_i for i = 1; 4; 5 are actually drawn from a truncated normal distribution with their absolute values constrained to be less than one. Conditional on β^T and Π, we then draw the elements of Q from an inverse Wishart distribution (again imposing that the second diagonal element be zero). Following that, we draw the elements of Δ from a multivariate normal, and conditional on the realization of β^T, Π, and Δ, we draw $\sigma^2(s_t)$ from an inverse Wishart. Once this is done, we again draw a new set of β^T and repeat the entire process again. We allow 20,000 "burn-in" replications to purge any influence of the initial guesses for the parameters, and then repeat the process an additional 50,000 times. Our simulated posterior distributions comprise only every fifth replication after the burn-in period, so as to remove any serial correlation in the sequence of draws.

Table 1 contains the prior distributions for each of the parameters. In many cases, the priors are quite uninformative, although a few draw on previous analyses. For example, priors on the variances of the permanent and transitory components of GDP, as well as its drift are taken from the maximum likelihood estimates reported in Kim and Nelson (1999). The priors for the variances of the analogous terms for military spending take the same values. For the interest rate, we take as our priors the variance estimates in Laubach and Williams (2003). For $\delta_0, \delta_1, \delta_2$, and $\sigma^2(s_t)$, the prior values are those estimated in Bohn (1998), while the prior for δ_3 derives from simple OLS estimation of the observation equation, estimating cyclical components for real GDP and real military spending via the regression technique introduced in Hamilton (2017).

C. Results

In this section, we report the results from the estimation of the state space model, starting by examining the estimates of the model's coefficients.

C1. Model Parameter Estimates. Figure 6 depicts the posterior distributions of each of the eleven model coefficients with histograms. For comparison, the figures offer the kernel densities of the prior distributions of each coefficient as well. Table 2 contains the same results in tabular form, reporting the median, as well as the 5th and 95th percentiles, of the posterior distribution, alongside a convergence diagnostic statistic which tests for the equality of means of the parameter in the first 10 percent of draws and the last 40 percent of draws. This statistic assesses the extent to which the influence of the initial parameter guess has been purged. Table 2 shows that, for most parameters, we cannot reject the null of no difference in means in the two different parts of the distribution. While we do reject the null hypothesis of equality for δ_2 and δ_0, the differences in means from the early set of draws and the late set of draws are quantitatively quite small.

Turning to the parameter estimates themselves, we first note that neither α_1 nor α_2 are statistically significant and both are very close to zero, indicating that changes in the growth rate of potential output (illustrated in Figure 7) or the natural rate of interest (depicted in Figure 8) do not have a strong influence on the way fiscal policy makers react to debt stocks. The coefficients also do not have the expected signs. We do observe that the natural rate of interest is significantly negatively related to changes in the growth rate of potential GDP (posterior median of γ_1 = -0.0032). As expected, the surplus is serially correlated (with an AR(1) parameter of 0.3383) and it is positively associated with the cyclical component of GDP and negatively associated with the transitory component of military spending (implying that the government is willing to run large deficits in wartime). These results accord well with our intuition.

C2. Evolution of the Fiscal Reaction Coefficient. Figure 9 contains one of the main results of the paper, namely the estimated variation over time in the reaction coefficient ρ that relates the surplus-GDP ratio to the debt-GDP ratio. Since the reaction coefficient does materially depend on the interest rate of growth rate, it is likely characterized by a random walk process with unobservable factors clearly affecting the government's response to changes in debt (see equation (19)). A higher value of ρ indicates that the government is more aggressive about controlling the rise of debt. The figure also, for comparison, shows the estimated value of ρ = 0.054 in Bohn (1998) (for a further discussion, see Section V). Our results indicate that, in many periods in the last 100 years, the U.S. government was actually more proactive about constraining increases in the debt-GDP ratio than even the rather robust results of Bohn (1998) might have implied. In the years before the onset of the Great Depression, ρ generally stayed well above 0.1, reaching as high as about 0.3 in 1920. During the Depression years, ρ tended to hover around the benchmark value of 0.054, estimated by Bohn (1998).

It is interesting to note that ρ turns steeply negative during the war years. This implies that the large run-up in debt during World War II was not solely due to a temporary surge in military expenditures, but it also reflected a weaker willingness on the part of policymakers to rein in the debt. While this may seem obvious, it is comforting that our model is able to distinguish between increases in debt resulting from rising war spending and those increases that result from a reduced willingness to respond to debt.

Following the war, ρ rises steadily, hitting a local peak around 0.24 in 1969, and then begins to trend downward. In the years surrounding the turn of the millennium, the coefficient drops below the benchmark value of 0.054, and it turns negative in the lead-up to the Great Recession. In the last year of the sample, 2016, the value of ρ is -0.0134, and it has been negative every year since 2008. The fact that the reaction coefficient has declined steadily since 2000 and been negative in a number of fiscal years since 2008 implies that the federal government has become less concerned with its debt stockpile. This result is also consistent with the findings of D'Erasmo, Mendoza, and Zhang (2016), who note that the large deficits experienced by the government years after the end of the Great Recession do not have precedent in U.S. history.

The blue bars in Figure 10 indicate periods in which the stability condition, $\kappa = \frac{1+r}{1+g}(1-\rho)$ < 1, fails to hold. In these periods, the debt-GDP ratio is following a process that can be described as explosive and hence unsustainable. This occurs briefly during the Great Depression (1932), World War II (1943), and after the Great Recession (from 2009 to 2011). When the debt-GDP ratio is on an explosive trajectory, it can be restored to a stable or sustainable trajectory if potential output growth rises, the natural rate of interest falls, or ρ rises. After the Great Recession, the debt-GDP ratio returned to a stable path due to continued declines in the natural rate of interest, and, more importantly, a recovery of the ρ coefficient.

C3. Steady-State Debt-GDP Ratios. Figure 10 illustrates the steady state debt-GDP ratio calculated as $\bar{d} = \frac{-(1+r)\mu}{(1+g)-(1+r)(1-\rho)}$ for each time period in our sample. This ratio is a long-run value of the debt-GDP ratio that will be reached at each period if we let the existing debt-GDP ratio continues to grow over time with the prevailing economic conditions described by the interest rate and growth rate as well as the implied reaction coefficient and given value of μ at each period. If the stability condition is satisfied, the debt-GDP ratio will converge to its stationary or steady-state value; hence it will be sustainable. If the stability condition is not satisfied, the debt-GDP ratio will continue to grow with no bound; hence it will be unsustainable. (we discuss more about the stability condition in the next section.) For comparison, the plot includes the actual debt-GDP ratio as well. In the periods when the stability condition does not hold and debt is explosive, we constrain the steady state debt ratio to zero for convenience. It is clear from the figure that there are periods when the steady-state debt-GDP ratio is extremely high, i.e., the debt is multiple times GDP. In general, the most extreme data points are in periods surrounding those when the stability condition does not hold (suggesting that these are periods in which the stability condition is only just satisfied). Thus, during the Depression, World War II, and in the years surrounding the Great Recession, steady-state debt-GDP ratios soar to well over 100 percent. It is also notable that the steady-state debt-GDP ratio creased 100 percent of GDP in the early 1980s, when the natural rate of interest was relatively high, and in 2003, the first year in the postwar period when the reaction coefficient ρ turned negative. Our model indicates that, as of the end of 2016, the steady-state debt-GDP ratio was approximately 58 percent of GDP, below the actual ratio of around 72 percent. This implies that, assuming no changes in fiscal behavior, debt should decline as a share of GDP going forward, a result of a slightly higher reaction coefficient (relative to the years of the Great Recession) and a lower natural rate of interest in this period. If either were to change, then it is possible that the debt-GDP ratio will resume an unsustainable trajectory.

C4. Steady-State Surplus-GDP Ratios. We next turn to the surplus-GDP ratio in steady state, which is plotted in Figure 12. The steady state surplus-GDP ratio is calculated as $\bar{s} = \frac{(r-g)}{(1+r)}\bar{d}$, so it rises with the natural rate of interest, declines in the growth rate of potential output, and rises in steady state debt as a share of GDP. An actual value of the surplus-GDP ratio higher (lower) than the steady state value is consistent with a declining (rising) debt-GDP ratio. As in the case of Figure 10, shaded bars indicate periods when the stability condition for debt does not hold, so we set the steady state surplus to zero to ease the interpretation of the rest of the figure. Of particular note in Figure 11 is that actual surpluses were well in excess of steady state surpluses in the years immediately after World War II, setting the debt-GDP ratio on its declining mid-century trend. In the early-to-mid-1980s, the opposite scenario obtained, indicating that debt was about to start rising persistently, which it did until the late 1990s, when surpluses again exceeded their steady state value. Since the turn of the millennium, the surplus-GDP ratio has largely tracked its steady state value, although this does not account for the three years that the stability condition is not holding. Our model suggests that the debt-GDP ratio has nearly doubled in that time primarily because of the failure of the stability condition between 2009 and 2011.

C5. Speed of Convergence. We estimated the steady-state debt-GDP ratio and surplus-GDP ratio for each period. An issue of interest is how long it takes to reach the steady state. This can be investigated using the speed of convergence given by $\phi = 1 - \kappa$. For illustration, we use Bohn's (1995) estimation results for 1916-1995. During this period, the average real return on government debt was about 0.1 percent, while the average real GDP-growth rate g was about 3.3 percent; hence, average δ = -0.0332 with the steady-state debt-GDP ratio of $\bar{d} = 0.3396$. With ρ = 0.054, this gives us κ = 0.9146, which is less than 1. Thus the stability condition guarantees that the debt-GDP ratio will converge to its steady state of $\bar{d} = 0.3396$. The rate of convergence ϕ toward the steady state is 0.0854. This suggests that, absent further shock, about 9% of the debt gap (the difference between a given year's debt-GDP ratio and its steady state level) will disappear each year. With this rate, it will take some time to reach the steady state. To get a rough idea, the rule of 70 (or 69) indicates that the half-life of the debt gap is about 9 years (70/8%), meaning that it will take every 9 years to eliminate the half of the debt gap. With the initial value of $d = 0.05$ in 1916, to eliminate the half of initial debt gap of about 0.292% (= 0.34 -0.05), it will take about 9 years. It will take another 9 years to eliminate the half of the remaining gap, and so on. With this rate, it will take about 40 for initial debt ratio to reach its steady state.

C6**. Decomposition of Primary Surplus/Deficit.** We can use the results from our estimated model to assess the contribution of each element in the fiscal reaction function (28) to the observed surplus-GDP ratio. Specifically, we identify that part of the observed surplus that is attributable to the government's reaction to debt, the transitory part of GDP, the transitory part of military spending, other components (such as the effects of the constant term and serial correlation in the surplus), and that part which is unexplained by our model. The decomposition is reported in Figure 12.

Most notably, we observe that the contribution of the desire of policy makers to respond to debt fits a similar pattern as the reaction coefficient itself. The contribution is strongly positive in the early 1920s, as the federal government attempted to pay off World War I era debts, and remains positive up until World War II. After the Second World War, the reaction to debt produces a large share of the observed surplus, more than 10 percentage points worth of GDP, and it remains a substantial contributor for much of the rest of the 20th century. This implies that, were it not for large negative contributions from the surplus-GDP ratio having a negative constant value and from transitory military spending, the observed surplus-GDP ratio for much of the postwar period would have been even larger than it was.

This narrative does not hold after the turn of the new millennium. In this period, the coefficient on debt in the fiscal reaction function turns downward, and at most, fiscal policy makers' desires to constrain debt buildups does not add to the budget deficit. In other years, the less aggressive approach to debt actually makes the deficit larger than it would otherwise have been. This can be observed by noting that the red dashed-dotted line representing the contribution of the reaction to debt venturing below 0 in Figure 12. The only positive contributor to the surplus-GDP ratio is military spending that is somewhat below trend.

The transitory component of output makes very little contribution to the surplus-GDP ratio throughout our sample period. This is because our model assigns most of the variation in output to its permanent component, with the cyclical component having a very small variance. In principle, cyclical variations in output could be meaningful contributors to the surplus-GDP ratio, but, in practice, our model suggests that these variations are rather minimal.

V. Robustness Checks and Comparison with Other Specifications

It is possible that some of our results are, in part, an artifact of the sample period over which we estimate our model. One specific way in which this is the case is that the large increase in military spending around World War II strongly influences our model's estimate of the permanent and transitory components of military spending. This is especially possible given the large degree of persistence observed in this transitory component. In this case, the large magnitude of the fiscal reaction coefficient may only be a function of the transitory component of military spending being estimated as too positive, improperly assigning any excess deficit to the positive transitory component of military spending.

Figure 13 reports the estimates of the time-varying reaction coefficient when the model is only estimated on data starting in 1960. Indeed, it is clear that for much of the sample period, the reaction coefficient tends to hover around the value estimated by Bohn (1998), although there is a persistent decline in the 1980s and a sharp increase in the late 1990s. Importantly, however, we continue to find a large decline in the reaction coefficient in the 2000s, and the trend seems to be negative. Thus, our finding that policy makers have become less aggressive about countering increases in debt continues to hold.

We also estimate the model on a longer time series, using the historical data set of Bohn (2008), which includes information back to 1792. These results are provided in Figure 14. While there are large fluctuations in the reaction coefficient, especially before 1900, we continue to observe strong responses with respect to debt throughout much of the 20th century and a persistent decline in the 2000s. We conclude, therefore, that our main qualitative results are robust to changes in the sample period.

In order to capture the possibly time-varying behavior of fiscal policy makers, we formulated and estimated the generalized fiscal reaction function in a state space framework. Of course, time variation in the fiscal reaction coefficient might also be studied using a simpler model of the form in equation (5) with interaction terms by using the same variables for unmodelled determinants of the surplus-GDP ratio:

$$s_t = \rho_0 + \alpha_0 d_t + \alpha_1 (r_t x d_t) + \alpha_2 (g_t x d_t) + \alpha_3 y_t^c + \alpha_4 m_t^c + \alpha_5 s_{t-1} + \zeta_t, \forall t \geq 0, \qquad (35)$$

where ζ is an error term.[14] This model can be estimated via OLS. While this model has the virtue of simplicity, it has at least two important drawbacks relative to the state space technique that we have developed. For one, this model does not allow the fiscal reaction coefficient to be subject to shocks. The second reason is that it does not distinguish between long-term and short-term interest rates. These drawbacks are likely to lead to a biased reaction coefficient and to biased inference with respect to fiscal sustainability.

In this section, we compare the implications of our state space model with those of other plausible specifications for the behavior of the debt-GDP ratio in the United States. In particular, we compare our results estimated with the state space model with those that derive from a model in which we assume, as Bohn (1998) does, the fiscal reaction coefficient is constant over time, using both the original Bohn (1998) coefficient (0.054) and one from an updated estimation (0.0665).[15] We also compare our model results with the interaction term model specified in equation (35).

Figure 15 is an extension of Figure 9 in that it includes a line indicating the updated ρ coefficient and the implied ρ from a model that includes interaction terms. The most notable feature that the figure illustrates is that the ρ estimated in our state space model implies much greater variability in fiscal policy behavior than we would infer from a model relying on interaction terms. The interaction term model tends to suggest that the reaction coefficient has generally been rather stable over time, hovering around a level of 0.035, except for the periods immediately after the two world wars, when policy makers made a more concerted effort to bring down debt ratios. Another important difference in the interaction term model is that it implies no lesser willingness on the part of policy makers to respond to debt in recent years, in stark contrast to the implications of our state space model.

There are a few of possible reasons for the different conclusions from the interaction term model relative to that of our state space model. The first is that the interaction term model interacts current levels of growth and the interest rate with the debt-GDP ratio, whereas the state space model assumes that policy makers change behavior only with respect to permanent changes in the growth rate or the natural rate of interest. We find this latter assumption more plausible. The second reason is that the interaction term model imposes that only changes in growth or in the interest rate would influence fiscal policy makers' reaction function, while our state space model allows for innovations in the reaction coefficient unrelated to either, such as, for example, changing political preferences.

We can assess the implications of each value of ρ for the satisfaction of the stability condition governing the dynamics of debt. The stability condition is satisfied when $\kappa \equiv \frac{1+r}{1+g}(1-\rho) < 1$. In Figure 16, we plot the values of κ for each of the model specifications that we consider, paying close attention to whether the parameters is above or below unity. Clearly, κ is quite volatile for all of the models during the turbulent years around World War I, the Great Depression, and World War II. Interestingly, for much of the postwar period, our state space model provides stronger evidence for the stability of the debt-GDP ratio, as the stability parameter is much closer to zero than for any of the other models. This observation holds up until about the year 2000, after which the implications are much different. In the last 15 years or so, the state space model has been the most likely to suggest that the stability condition is failing, relative to the other models under consideration. For 2016, the implied κ of the state space model was about 0.985, the highest value since 2012. This implies that going forward, without a substantial increase in potential output growth or a change in fiscal behavior, the stability condition may fail and debt may become explosive in the near future.

Figures 17 and 18 illustrate the implied steady state debt-GDP and surplus-GDP ratios, respectively, for each of the four model specifications. One thing that stands out is that the stability condition fails much more frequently (especially early in the sample) for the other specifications than for the state space model. When the fiscal reaction coefficient is held constant, further, the implied steady state debt ratio rises very substantially in the years surrounding the Great Recession. For the interaction term model, the results suggest that the dynamics of the debt-GDP ratio in the 1980s were highly unstable, relative to the other periods of history that we study.

With respect to the surplus-GDP ratio, our state space model seems to fit the data much better than do the other models, always predicting a steady state surplus-GDP ratio with a magnitude reasonably close to actual surpluses. At times, the models using the original benchmark ρ =

0:054 and the interaction terms predict steady state surplus-GDP ratios several times GDP, an implausible result.[16]

VI. Summary and Conclusion

This paper has proposed a generalized fiscal reaction function to analyze the dynamic behavior of U.S. government debt and deficits for 1916-2016. We revisit the theoretical framework initially established by Bohn (1998), and we extend it to allow for a fiscal reaction coefficient that may vary over time, especially with in response to changes in the natural rates of interest and output growth. We estimate our model in a state space framework by treating the reaction coefficient as an unobservable stochastic process.

The estimated reaction coefficient exhibits significant time variation. Although it does not significantly change with interest rates or growth rates, as we had speculated, it does apparently respond to other, unobservable forces which may plausibly influence the willingness of policy makers to respond to debt. We find that, for much of the 20th century, the reaction coefficient was well above the value estimated in Bohn (1998), implying a more proactive fiscal policy with respect to debt than previously understood. On the other hand, since 2000, the reaction coefficient has steadily declined, and these declines are not merely a mechanical result of the Great Recession or the wars in Afghanistan and Iraq. As of 2016, the reaction coefficient was negative, though close to zero.

Our findings have implications for inference about the sustainability of debt in the United States. According to the stability condition that we derive, debt-GDP ratios were generally sustainable for most of the sample period, with exceptions being the years around the Great Depression and Great Recession, as well as World War II. In recent years, however, as the reaction coefficient has declined, the stability condition parameter has come much closer to a value of 1, above which the stability condition would be violated. This would suggest that, if current trends persist, debt-ratios may soon become very difficult to bring back under control.

Footnotes

[1] Throughout this paper, the term "debt" refers to "federal debt held by the public." According to the chapter on "Federal Borrowing and Debt" in the *Analytical Perspectives of the U.S. Government*, published by the Office of Management and Budget to accompany the President's budget request, this includes debt held by persons and institutions outside of the U.S. federal government, including private agents, state and local governments, and foreign governments. It does not include debt held by Federal government accounts, such as trust funds, which would be included in figures capturing Gross Federal Debt.

[2] See e.g., Hamilton and Flavin (1986), Hakkio and Rush (1991) Haug (1991), Smith and Zin (1991), Ahmed and Rogers (1995), Trehan and Walsh (1998, 2001), Uctum and Wickens (2000). However, Bohn (2007) has shown that time series tests are incapable of rejecting the assumption of sustainability, because the intertemporal budget constraint is satisfied if the time series of the relevant debt variable is stationary after any number of differencing operations.

[3] "Secular Stagnation' Even Truer Today, Larry Summers Says," *Wall Street Journal*, May 25, 2017.

[4] "Cheap Money Talks," *New York Times*, July 11, 2016.

[5] The fiscal reaction function has been widely adopted in theoretical dynamic general equilibrium models, such as, for example, Davig and Leeper (2007) (though in a slightly different form). Another approach to assessing fiscal sustainability is the model-based simulation exercises of Bi and Leeper (2013).

[6] This variable is an amalgam of a constant term and other variables (which we consider as cyclical output and military spending in empirical analysis) and an error term.

[7] There are studies employing a quadratic or cubic function that allows the reaction coefficient to vary over time (see, e.g., Bohn, 1998; Mendoza and Ostry, 2008; Ghosh et al.,2011; D'Erasmo, Mendoza, and Zhang, 2016). We provide an empirical analysis in section V.

[8] In empirical analysis, we allow the reaction coefficient to vary with other unobservable variables.

[9] Collignon (2012) examines the stability condition and steady state for fiscal sustainability in the context of Europe's fiscal policy rules. However, there are some fundamental conceptual problems in his model specification.

[10] What we refer to as a "steady state" debt-GDP ratio should not be confused with the fiscal limit studied by Bi and Leeper (2013) or Ghosh, et al. (2013). The former paper defines the fiscal limit as being that value of debt that may be expected to be paid down if the government raised the maximum tax revenue possible into the infinite future. The latter paper considers an

endogenous interest rate with risk-neutral investors who incorporate the probability of default. Their model includes an object that might be analogous to our steady state debt as well as a distinct affordable debt limit. The steady state debt-GDP ratio in our framework is the value of debt relative to GDP that the economy will tend towards assuming no future changes in the natural rate of interest, the growth rate of potential output, or the fiscal reaction coefficient.

[11] Laubach and Williams (2003) consider both random walk and stationary specifications of the zt term in the natural rate of interest. For simplicity, we consider only the random walk specification in this study.

[12] Bohn (1998) constructed fluctuations in output and military spending, but we treat them as unobservable variables.

[13] The MATLAB computer programs used to execute the estimation are also heavily based on programs available on the web pages associated with Koop and Korobilis (2009).

[14] In estimating this model, we follow the guidance of Balli and Sorensen (2013) in standardizing the interest rate and the growth rate. This means that ρ_0 in Equation (35) can be interpreted as the reaction coefficient when both the interest rate and growth rate are at their average values over the sample, and α_1 and α_2 can be interpreted as the effect on the reaction coefficient when either variable rises above its sample average.

[15] It may be surprising that the fiscal reaction coefficient in the updated sample (the original Bohn (1998) sample ran to 1995) would be greater than the original coefficient. This is likely a result of the fact that the estimates of the transitory components of output and military spending are fundamentally different in our updated sample than they were in the original Bohn (1998) paper. Our estimates are taken from the state space model, while those in Bohn (1998) are borrowed from Barro (1986). We are not able to extend these figures.

[16] In Bohn's (1998) fiscal reaction function, the reaction coefficient is constant; hence it does not depend on the debt level. There are several studies employing a nonlinear or cubic specification for the fiscal reaction function (see, e.g., Bohn, 1998; Mendoza and Ostry, 2008; Ghosh et al.,2011; D'Erasmo, Mendoza, 2016). This specification allows the reaction coefficient to depends on, and vary over time with, the debt level. We also estimated a cubic specification (along the lines of Bohn, 1998), although we do not report the results here. We find that the reaction coefficient exhibits substantial variation over time (of a much greater magnitude than for our other specifications), and the stability condition is violated much more often. For example, the stability condition with the cubic function is violated every year from the late 1960s to the early 1980s.

References

Ahmed, Shaghil and John H. Rogers. 1995. “Government Budget Deficits and Trade Deficits: Are Present Value Constraints Satisfied in Long-Term Data?” J*ournal of Monetary Economics* 36 (November), 351-374.

Alesina, Alberto and Guido Tabellini. 1990. “A Positive Theory of Fiscal Deficits and Government Debt,” *Review of Economic Studies,* 57(3), July, 403-414.

Balli, H. O., and B. E. Sorensen (2013): “Interaction Effects in Econometrics," *Empirical Economics*, 45(1), 583{603.

Barro, Robert J. 1979. “On the Determination of the Public Debt,” *Journal of Political Economy,* 87(5), 940-971.

Barro, Robert J. 1986. “U.S. Deficits since World War I," *Scandinavian Journal of Economics,* 88(1), 195-222.

Bi, Huixin and Eric M. Leeper. 2013. “Analyzing Fiscal Sustainability,” Working Paper, Bank of Canada and Indiana University and Monash University.

Bohn, Henning. 1995. “The Sustainability of Budget Deficits in a Stochastic Economy.” *Journal of Money, Credit, and Banking* 27 (February), 257-271.

Bohn, Henning. 1998. “The Behavior of U.S. Public Debt and Deficits.” *Quarterly Journal of Economics* 113 (2), 949-963.

Bohn, Henning. 2007. Are Stationarity and Cointegration Restrictions Really Necessary for the Intertemporal Budget Constraint? *Journal of Monetary Economics* 54:1837–1847

Bohn, Henning. 2008: “The Sustainability of Fiscal Policy in the United States," in *Sustainability of Public Debt,* ed. by R. Neck, and J.-E. Sturm, pp. 15{49. The MIT Press.

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba. 2001. “Is the Price Level Determined by the Needs of Fiscal Solvency?.” *American Economic Review* 91 (5), 1221-1238.

Carter, C., and R. Kohn (1994): “On Gibbs Sampling for State Space Models," *Biometrika*, 81(3), 541{553.

Checherita-Westphal, Cristina and Philipp Rother. 2012. “The Impact of High Government Debt on Economic Growth and its Channels: An Empirical Investigation for the Euro Area.” *European Economic Review* 56, 1392–1405.

Clark, Peter K. 1987. “The Cyclical Component of U.S. Economic Activity," *Quarterly Journal of Economics*, 102(4), 797{814.

Collignon, Stefan. 2012. “Fiscal Policy Rules and the Sustainability of Public Debt in Europe.” *International Economic Review* 53 (2), 539-567.

Davig, Troy and Eric M. Leeper. 2011. “Monetary-Fiscal Policy Interactions and Fiscal Stimulus,” *European Economic Review,* 55, 211-227.

D’Erasmo, P., Mendoza, E., and Zhang, J., 2016. “What is a Sustainable Public Debt?” in *Handbook of Macroeconomics*, Vol.II. pp. 2493-2507. North-Holland.

Elmendorf, Douglas, and Gregory Mankiw, 1999. “Government Debt”, in Calvo and Vegh (eds), *Handbook of Macroeconomics*, Vol.1I, Elsevier, 1615-1669.

Ghosh, A., Kim, J., Mendoza, E., Ostry, J., Qureshi, M., 2013a. “Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies,” *Economic Journal* vol. 123, pages F4-F30, 02.

Goff, Brian L. and Robert D. Tollison. 2002. “Explaining U.S. Federal Deficits: 1889-1998.” *Economic Inquiry* 40 (July), 457-469.

Hakkio, Craig S. and Mark Rush. 1991. “Is the Budget Deficit ‘Too Large?” *Economic Inquiry* 29 (July), 429-445.

Hamilton, James D. 2017. “Why You Should Never Use the Hodrick-Prescott Filter," NBER Working Paper 23429.

Hamilton, Jonathan and Marjorie Flavin. 1986. “On the Limitations of Government Borrowing: A Framework for Empirical Testing.” *American Economic Review* 76 (September), 808-819.

Haug, A. A. 1991. “Cointegration and Government Borrowing Constraints: Evidence for the United States.” *Journal of Business & Economic Statistics* 9 (January), 98-101.

Herndon, Thomas, Michael Ash, and Robert Pollin. 2014. “Does High Public Debt Consistently Stifle Economic Growth? A Critique of Reinhart and Rogo.” *Cambridge Journal of Economics* 38 (2), 257-279.

Kim, Chang-Jin, and Christian J. Murray. (2002. “Permanent and Transitory Components of Recessions," *Empirical Economics*, 27(2), 163{183.

Kim, Chang-Jin and Charles R. Nelson. 1999. *State-Space Models with Regime Switching*. The MIT Press.

Koop, Gary. 2003. *Bayesian Econometrics*. John Wiley & Sons, Inc.

Koop, Gary and D. Korobilis. 2009. “Bayesian Multivariate Time Series Methods for Empirical Macroeconomics," *Foundations and Trends in Econometrics*, 3(4), 267{358.

Kremer, Jeroen J.M. 1989. “U.S. Federal Indebtedness and the Conduct of Fiscal Policy.” *Journal of Monetary Economics* 23 (March), 219-238.

Laubach, T., and J. C. Williams. 2003. “Measuring the Natural Rate of Interest," *Review of Economics and Statistics*, 85(4), 1063-1070.

Mendoza, Enrique G. and Jonathan D. Ostry. 2008. “International Evidence on Fiscal Solvency: Is Fiscal Policy ‘‘Responsible’’?.” *Journal of Monetary Economics* 55 1081– 1093.

Morley, J. C., C. R. Nelson, and E. Zivot. 2003. “Why are the Beveridge-Nelson and Unobserved-Components Decompositions of GDP so Different?," *Review of Economics and Statistics*, 85(2), 235{243.

Panizza, Ugo and Andrea F. Presbitero. 2014. “Public Debt and Economic Growth: Is There a Causal Effect?” *Journal of Macroeconomics* 41, 21–41.

Reinhart, Carmen M. and Rogo_, Kenneth S. 2010. “Growth in a Time of Debt.” *American Economic Review*: Papers & Proceedings, 100 (2), 573-578.

Shiller, Robert J. 1989. *Market Volatility*. The MIT Press.

Shiller, Robert J. 2015. *Irrational Exuberance*. Princeton University Press.

Smith, Gregor W. and Stanley E. Zin. 1991. “Persistent Deficits and the Market Value of Government Debt.” *Journal of Applied Econometrics* 6 (Jan.-March), 31-44.

Trehan, B. and Carl E. Walsh. 1998. “Common Trends, the Government Budget Constraint, and Revenue Smoothing.” *Journal of Economic Dynamics and Control* 12 (June/Sept.), 425-444.

Trehan, B. and Carl E. Walsh. 1991. “Testing Intertemporal Budget Constraints: Theory and Applications to U.S. Federal Budget and Current Account Deficits.” *Journal of Money, Credit, and Banking* 21(May), 206-223.

Uctum, Merich and Michael Wickens. 2000. Debt and Deficit Ceilings, and Sustainability of Fiscal Policies: An Intertemporal Analysis.” *Oxford Bulletin of Economics and Statistics* 62 (May), 197-221.

Wilcox, David W. 1995. “The Sustainability of Government Deficits: Implications of the Present-Value Borrowing Constraints.” *Journal of Money, Credit, and Banking* 21 (August), 291-306.

Table 1: Prior Distributions

Parameter	Prior
β_t	N(0,$I_{10} \times 4$)
α_1	N(0,4)
α_2	N(0,4)
δ_0	N(-1.9,4)
δ_1	N(1.286,4)
δ_2	N(0.721,4)
δ_3	N(0.77,4)
γ_1	N(0,4)
γ_2	N(0.8002,4)
γ_3	N(0,4)
γ_4	N(0.8002,4)
γ_5	N(0.8002,4)
$\sigma^2(\rho)$	IW(0.01,T)
$\sigma^2(\phi)$	IW($4 \times (0.323^2)$,T)
$\sigma^2(r^c)$	IW($4 \times (0.340^2)$,T)
$\sigma^2(y^\tau)$	IW($4 \times (0.0056^2)$,T)
$\sigma^2(g^*)$	IW($4 \times (0.0002^2)$,T)
$\sigma^2(y^c)$	IW($4 \times (0.0061^2)$,T)
$\sigma^2(m^\tau)$	IW($4 \times (0.0056^2)$,T)
$\sigma^2(g^m)$	IW($4 \times (0.0002^2)$,T)
$\sigma^2(m^c)$	IW($4 \times (0.0061^2)$,T)
$\sigma^2(s)$	IW(1.96,5)

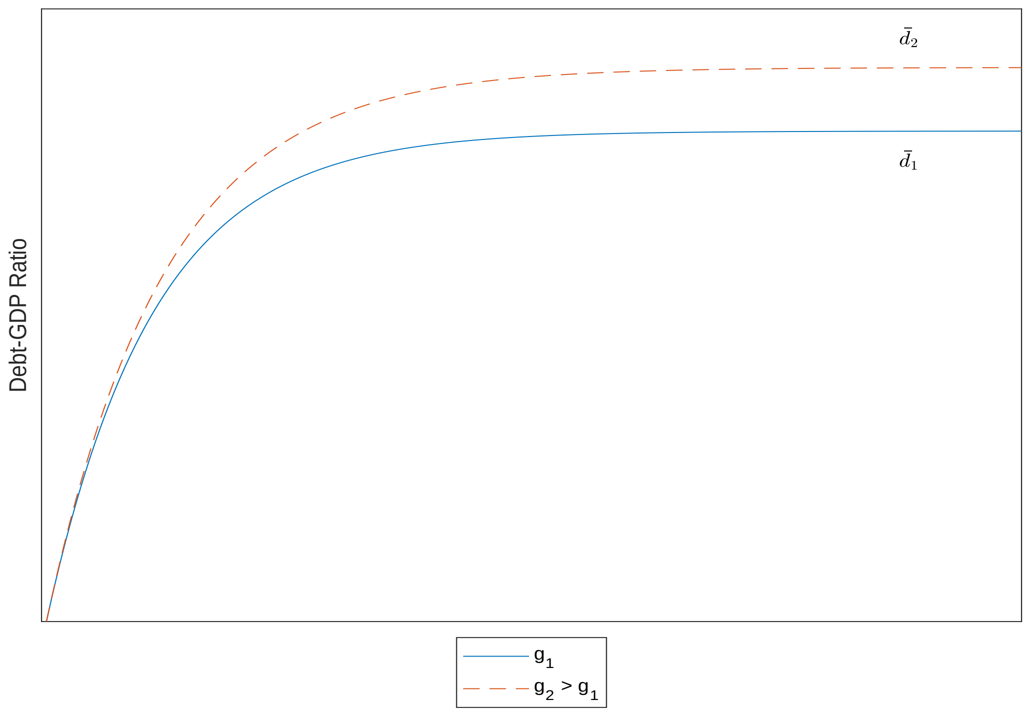
Notes: The table reports the prior distributions specified for each parameter in the state space model. In the table "N" indicates a normal distribution, and "IW" indicates an inverse Wishart distribution.

Table 2: Posterior Distributions

Parameter	Median	90% Confidence Interval	Convergence Diagnostic
α_1	-0.0024	$[-0.0372, 0.0325]$	0.7397
α_2	0.0166	$[-0.8009, 0.8108]$	0.7243
δ_0	-2.3088	$[-3.1972, -1.4678]$	2.3998
δ_1	1.1491	$[0.5472, 1.7670]$	-0.8498
δ_2	-3.0617	$[-3.6985, -2.4357]$	-2.4189
δ_3	0.3375	$[0.1883, 0.4879]$	-2.0174
γ_1	-0.0032	$[-0.8152, 0.8059]$	1.2439
γ_2	0.6847	$[0.0722, 0.9728]$	0.3974
γ_3	0.0010	$[-0.6063, 0.6172]$	0.9680
γ_4	0.7164	$[0.1023, 0.9797]$	-0.0767
γ_5	0.9290	$[0.7829, 0.9926]$	-0.7404
$\sigma^2(\rho)$	0.0055	$[0.0015, 0.0238]$	-0.5184
$\sigma^2(\phi)$	5.3608	$[4.5474, 6.3931]$	-0.4333
$\sigma^2(r^c)$	0.0078	$[0.0047, 0.0199]$	1.1217
$\sigma^2(y^\tau)$	1.3797×10^{-6}	$[1.0869 \times 10^{-6}, 1.7934 \times 10^{-6}]$	-0.9210
$\sigma^2(g^*)$	1.7554×10^{-9}	$[1.3886 \times 10^{-9}, 2.2670 \times 10^{-9}]$	0.1969
$\sigma^2(y^c)$	0.0043	$[0.0011, 0.0287]$	0.1067
$\sigma^2(m^\tau)$	1.3734×10^{-6}	$[1.0838 \times 10^{-6}, 1.7612 \times 10^{-6}]$	-0.5966
$\sigma^2(g^m)$	1.7525×10^{-9}	$[1.3847 \times 10^{-9}, 2.2557 \times 10^{-9}]$	0.0344
$\sigma^2(m^c)$	0.0721	$[0.0599, 0.0919]$	0.5283
$\sigma^2(s)$	2.2699	$[1.2231, 3.8784]$	-0.3012

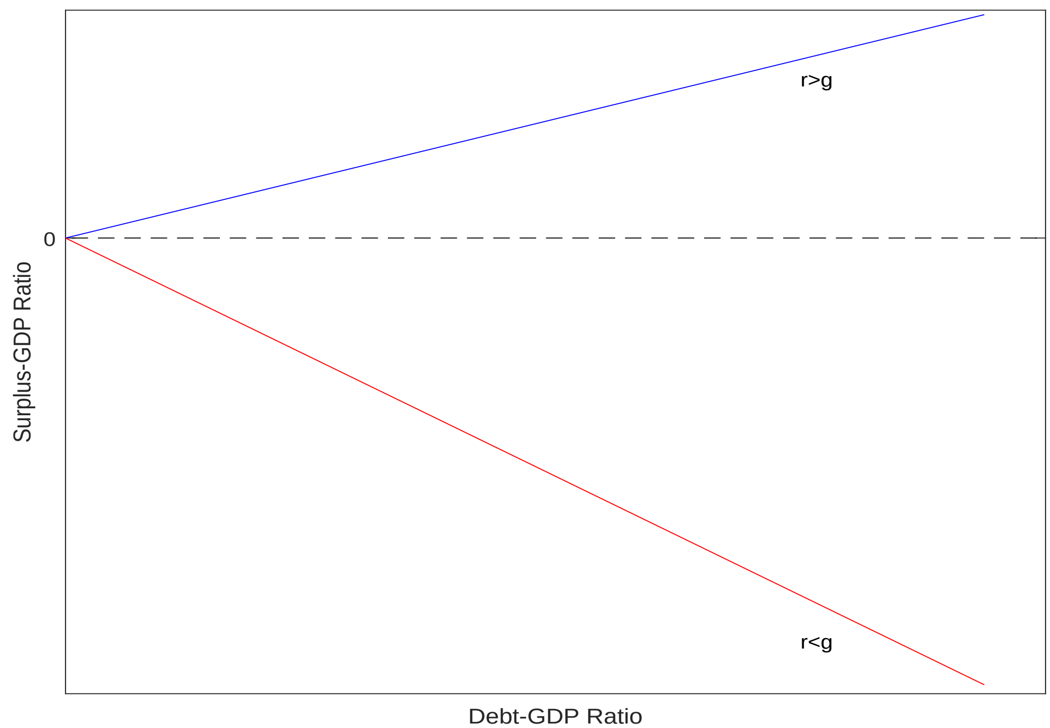
Notes: The table reports the main statistics associated with the simulated posterior distribution for each parameter in the model. The convergence diagnostic is defined as $CD = \frac{\hat{g}_{S_A} - \hat{g}_{S_C}}{\frac{\hat{\sigma}_A}{\sqrt{S_A}} + \frac{\hat{\sigma}_C}{\sqrt{S_C}}}$; see **?**. The convergence diagnostic statistics are compared with standard normal critical values. $\hat{g}$ is the average value of each parameter over the first 10% of draws after the burn-in period (S_A) or the last 40% of draws after the burn-in period (S_C).

Figure 1: Time Path of Government Debt: An Increase in the Growth Rate



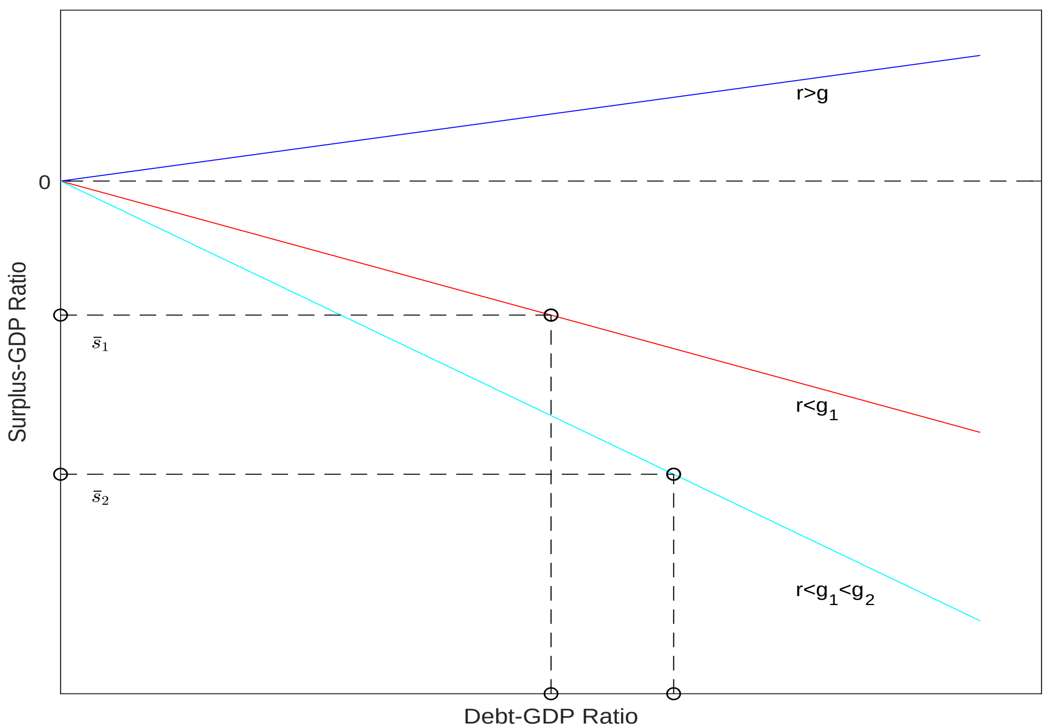
The figure shows how the debt-GDP ratio would transition to its new steady state value following a permanent increase in the growth rate of potential output, assuming that the fiscal reaction coefficient declines sufficiently as a result.

Figure 2: Long-run Relationship Between the Primary Surplus/Deficit and Debt



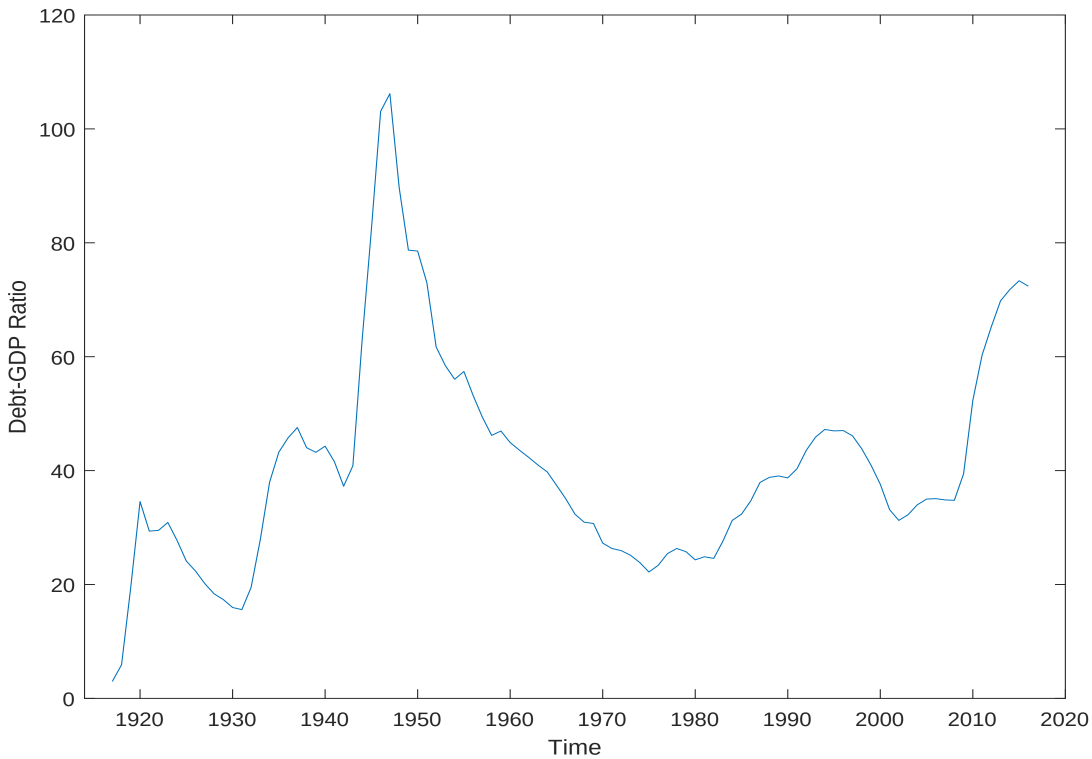
The figure plots the relationship between the steady state debt-GDP ratio and the steady state surplus-GDP ratio, depending on whether the natural rate of interest is greater than or less than the growth rate of potential output.

Figure 3: Effects of an Increase in the Growth Rate



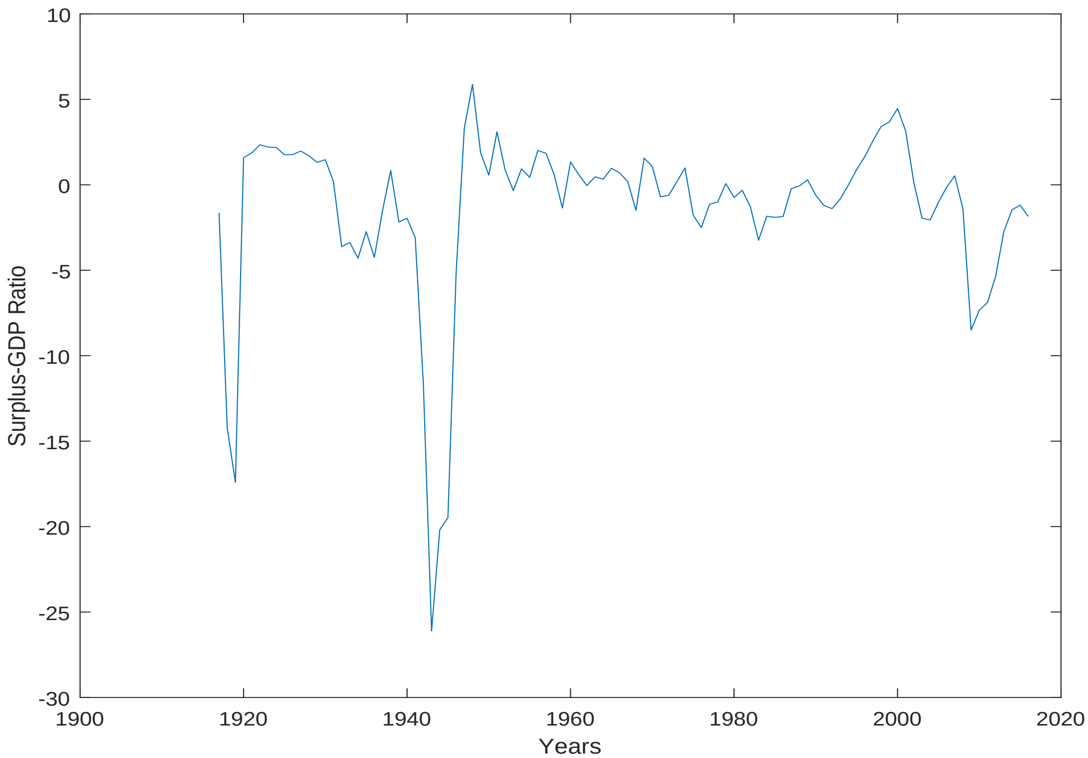
The figure plots how the steady state debt-GDP ratio and the steady state surplus-GDP ratio would respond to a permanent increase in the growth rate of potential output.

Figure 4: U.S. Debt Dynamics



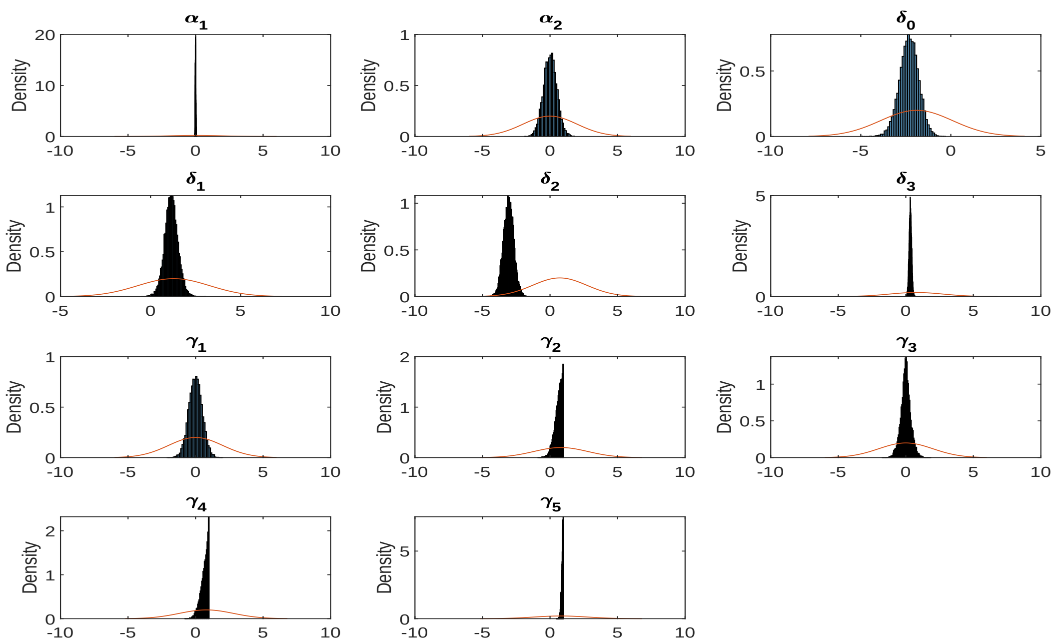
The figure plots the time series of U.S. government debt held by the public as a share of Gross Domestic Product.

Figure 5: U.S. Surplus Dynamics



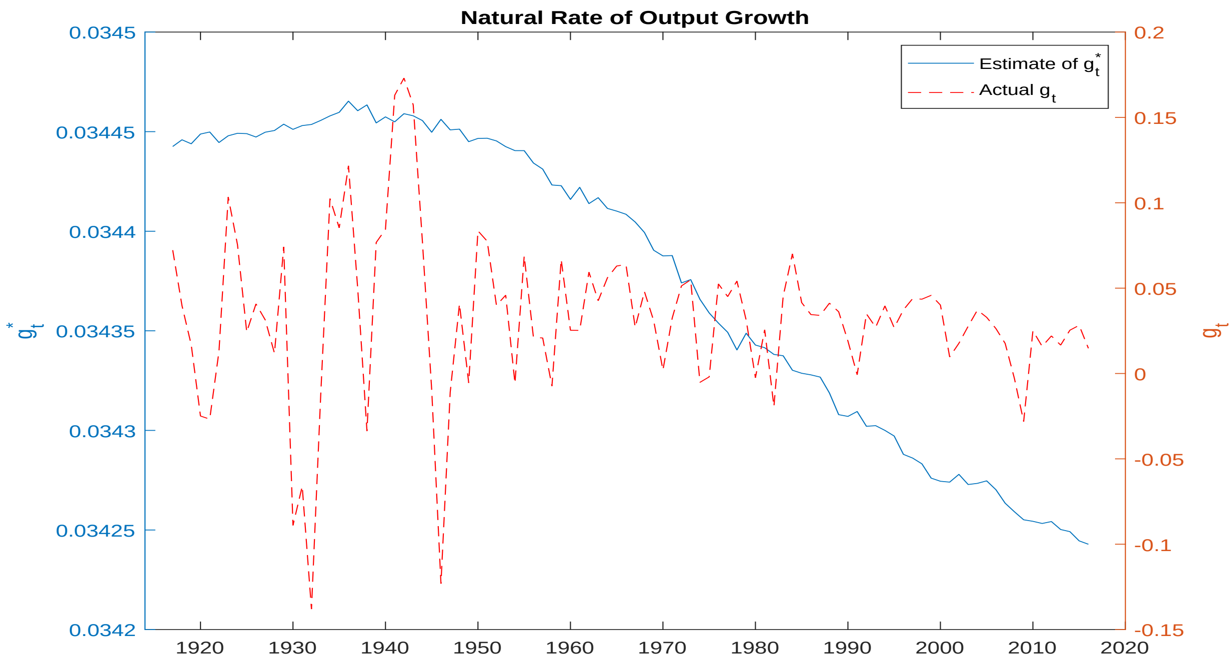
The figure plots the time series of U.S. government primary surplus as a share of Gross Domestic Product.

Figure 6: Parameter Posterior Distributions



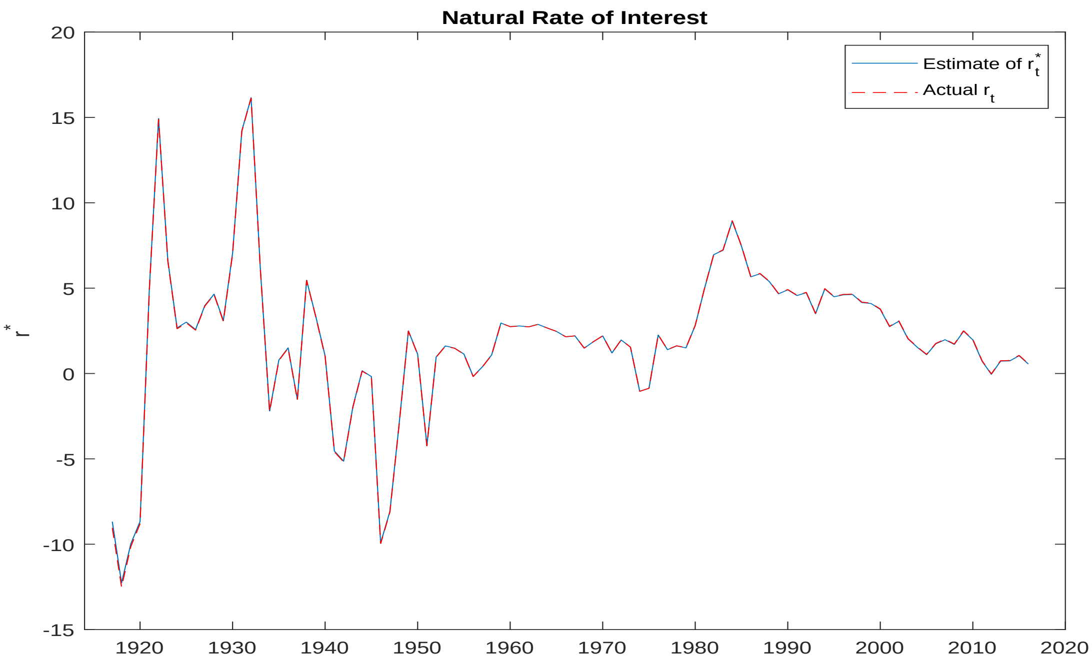
The figure reports plots of the prior and posterior distributions of each of the coefficients in the state space model for the dynamics of debt. The orange line in each subfigure provides the kernel density of the prior distribution, and the histograms illustrate the posterior distribution of draws for each parameter.

Figure 7: State Space Estimates



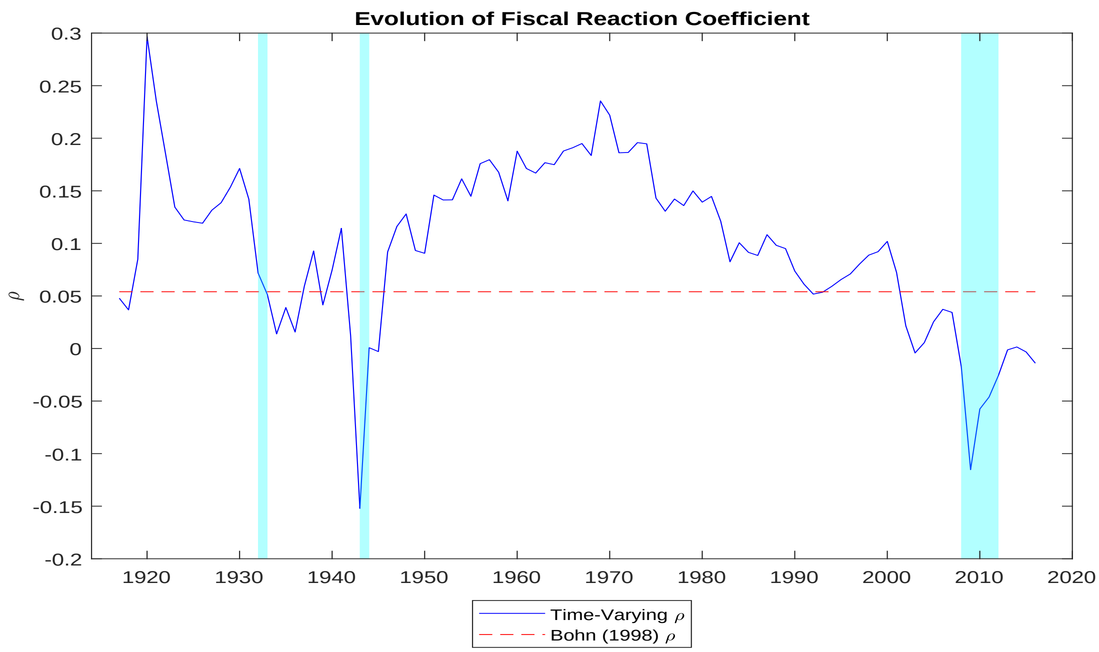
The figure reports the estimated time series of the growth rate of potential output (δ_y) from the state space model discussed in Section **??**.

Figure 8: State Space Estimates



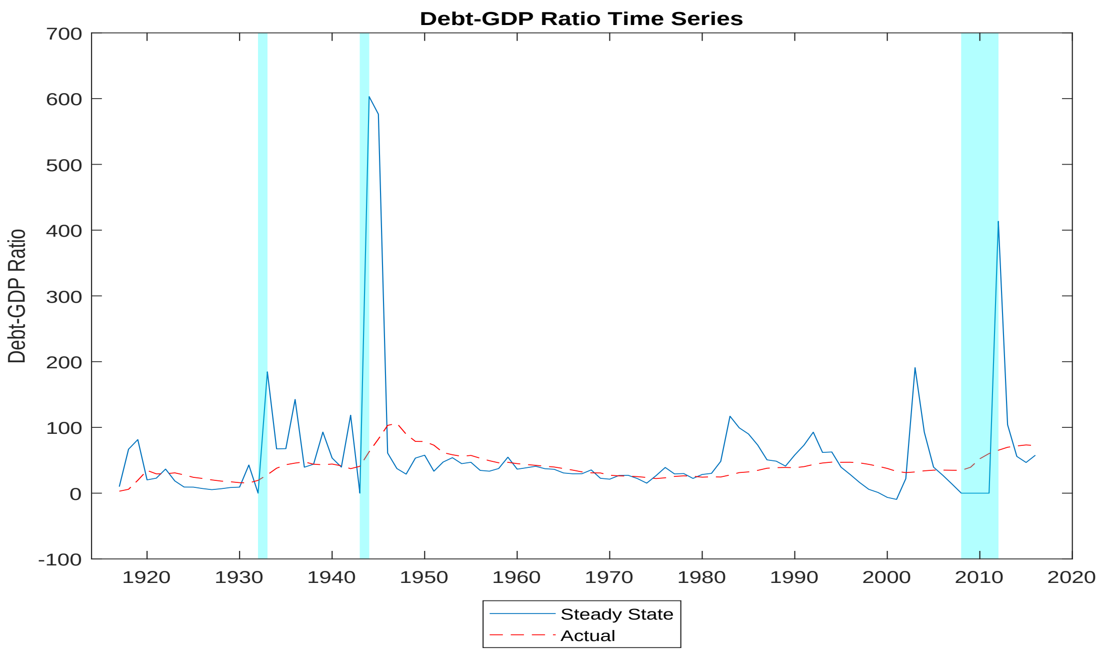
The figure reports the estimated time series of the natural rate of output (r^*) from the state space model discussed in Section **??**.

Figure 9: State Space Estimates



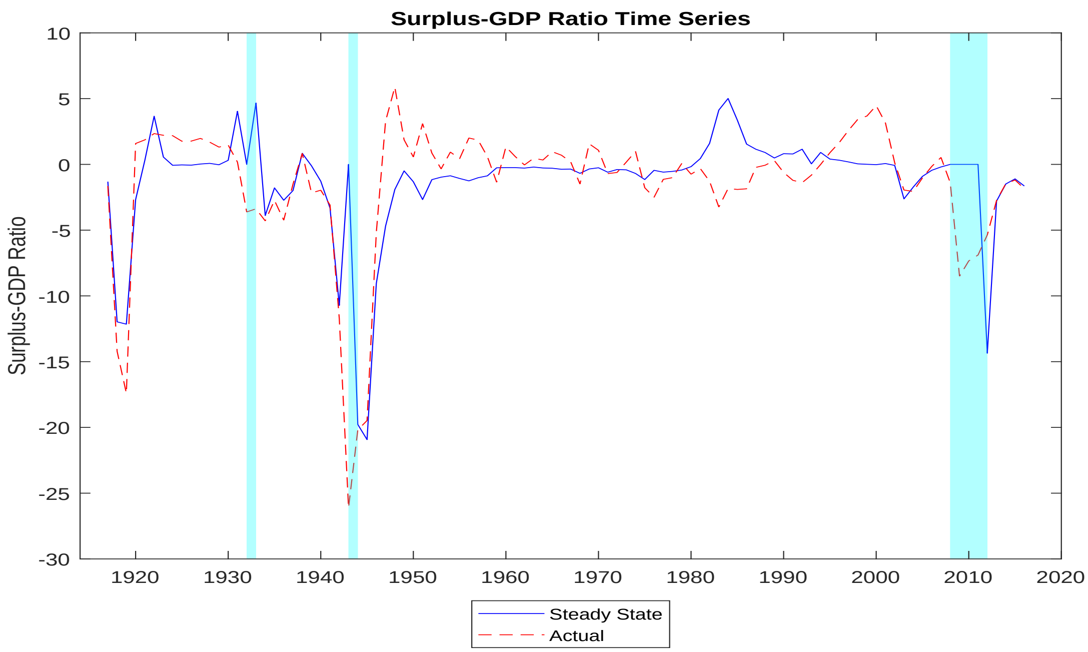
The figure reports the estimated time series of the coefficient on the debt-GDP ratio (ρ in the model discussed in Section **??**) from the state space model. The light blue shaded bars indicate periods in which ρ was not positive enough to ensure that the stability condition for the debt-GDP ratio was met.

Figure 10: State Space Estimates



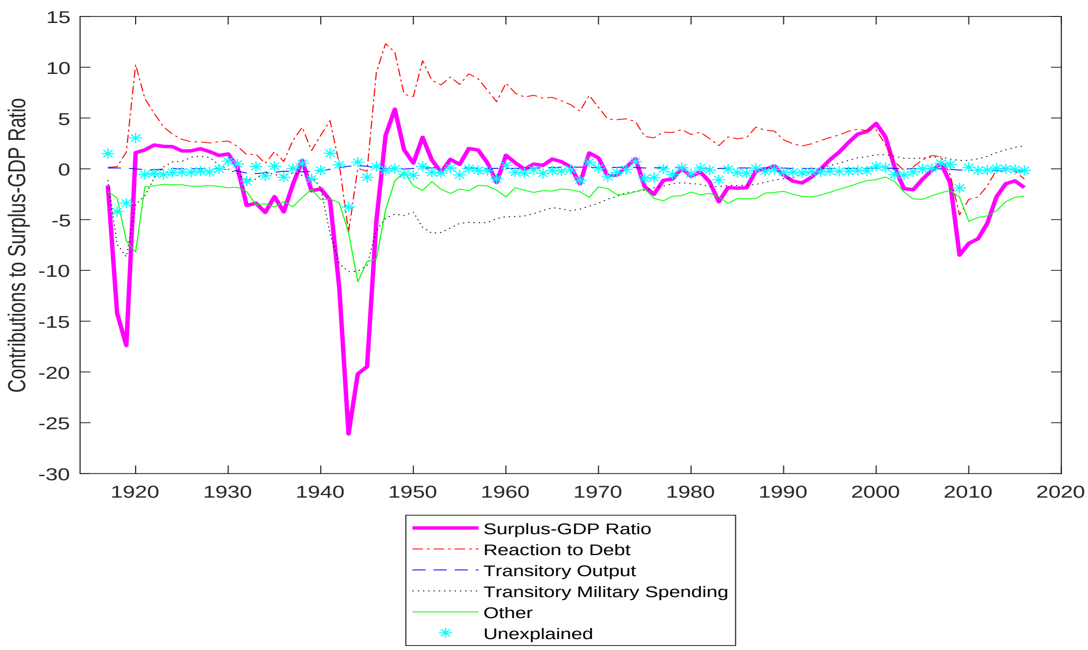
The figure reports the time series of the debt-GDP ratio alongside the estimated steady state debt-GDP ratio implied by the estimated time series of the natural rate of interest (r^*), the growth rate of potential output (δ_y), and the fiscal reaction coefficient (ρ). The light blue shaded bars indicate periods in which ρ was not positive enough to ensure that the stability condition for the debt-GDP ratio was met.

Figure 11: State Space Estimates



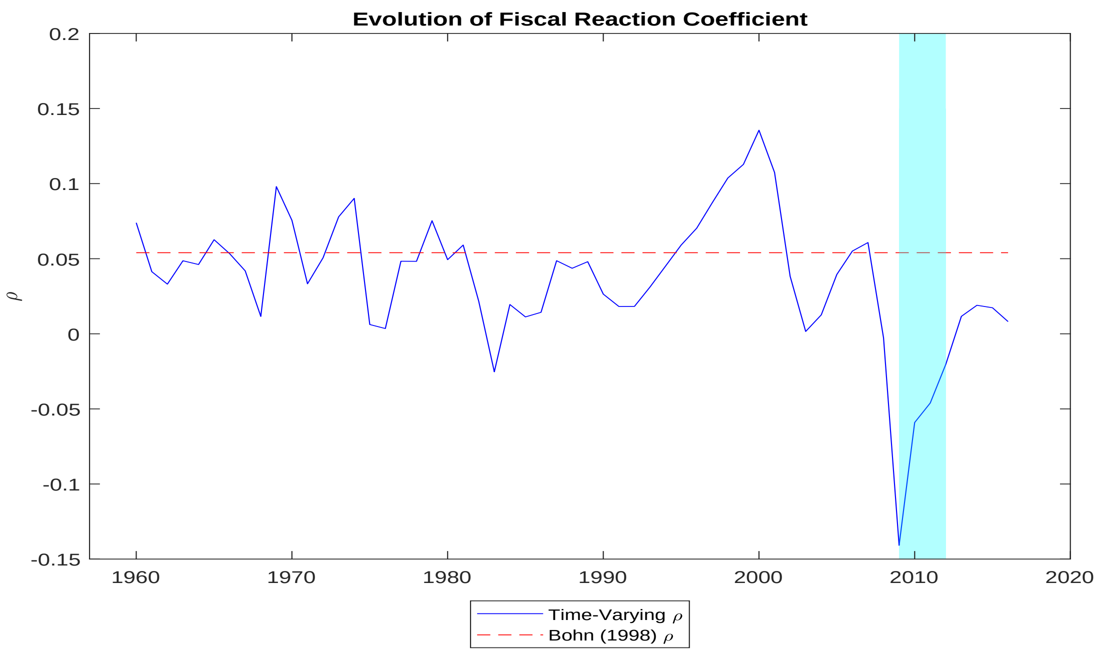
The figure reports the time series of the surplus-GDP ratio alongside the estimated steady state surplus-GDP ratio implied by the estimated time series of the natural rate of interest (r^*), the growth rate of potential output (δ_y), and the fiscal reaction coefficient (ρ). The light blue shaded bars indicate periods in which ρ was not positive enough to ensure that the stability condition for the debt-GDP ratio was met.

Figure 12: Contributions to Surplus-GDP Ratio



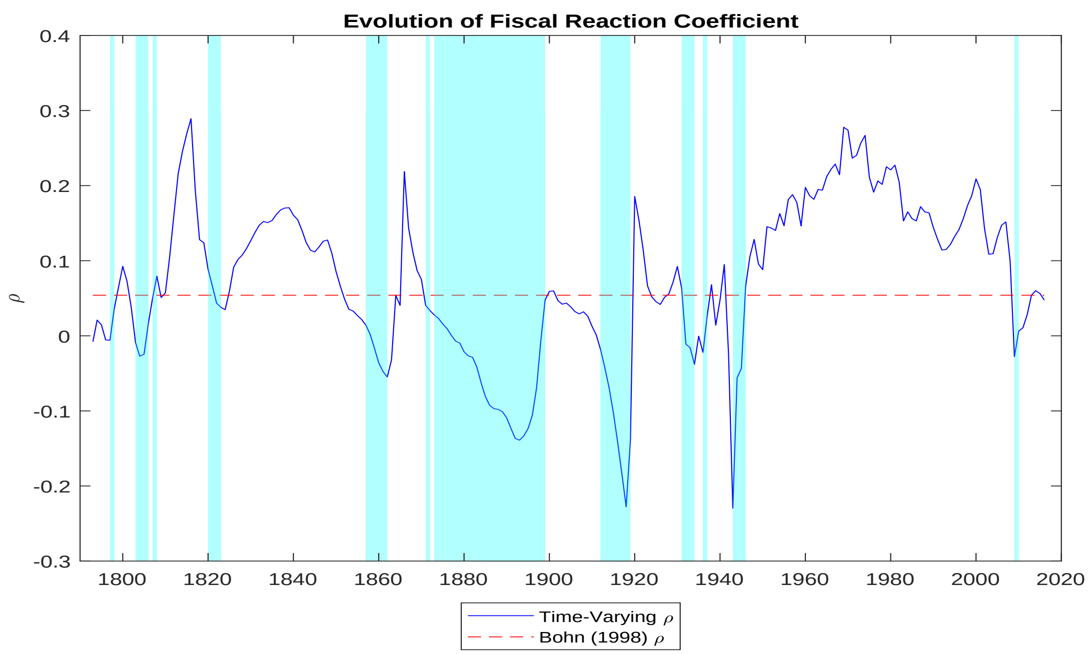
The figure reports the estimated contributions of each of the elements in the fiscal reaction function to the actual surplus-GDP ratio over time. The thick pink line, representing the actual surplus-GDP ratio, is the sum of the other five lines in the figure.

Figure 13: State Space Estimates: 1960-2016



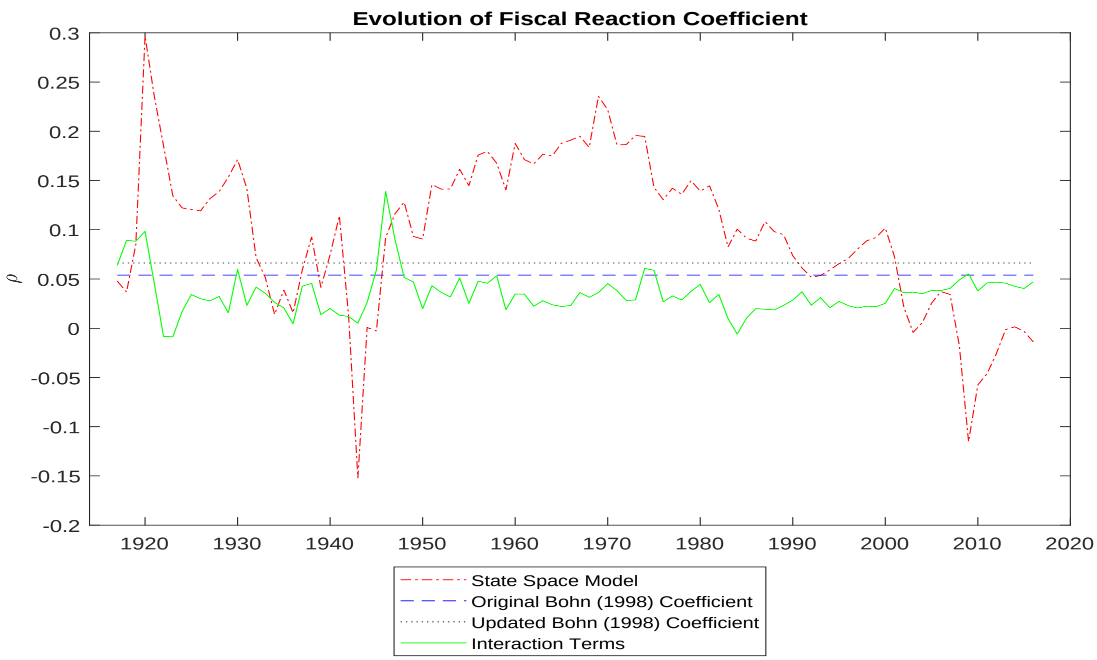
The figure reports the estimated time series of the coefficient on the debt-GDP ratio (ρ in the model discussed in Section **??**) from the state space model. The estimation sample is the preriod from 1960 to 2016. The light blue shaded bars indicate periods in which ρ was not positive enough to ensure that the stability condition for the debt-GDP ratio was met.

Figure 14: State Space Estimates: 1792-2016



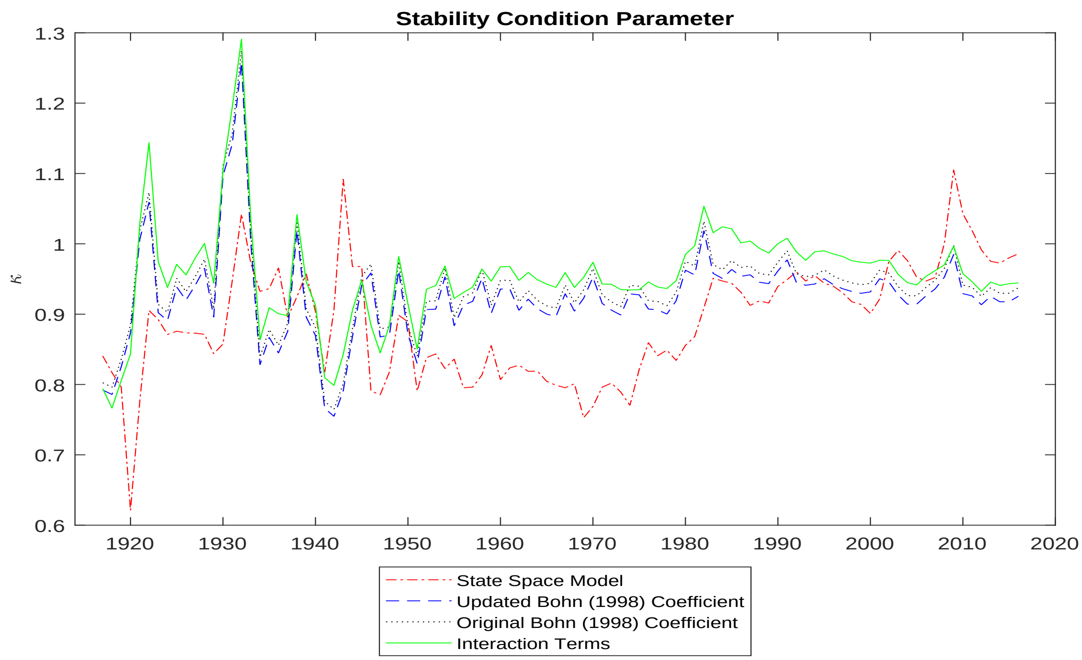
The figure reports the estimated time series of the coefficient on the debt-GDP ratio (ρ in the model discussed in Section **??**) from the state space model. The estimation sample is the period from 1792 to 2016. The light blue shaded bars indicate periods in which ρ was not positive enough to ensure that the stability condition for the debt-GDP ratio was met.

Figure 15: Model Comparisons



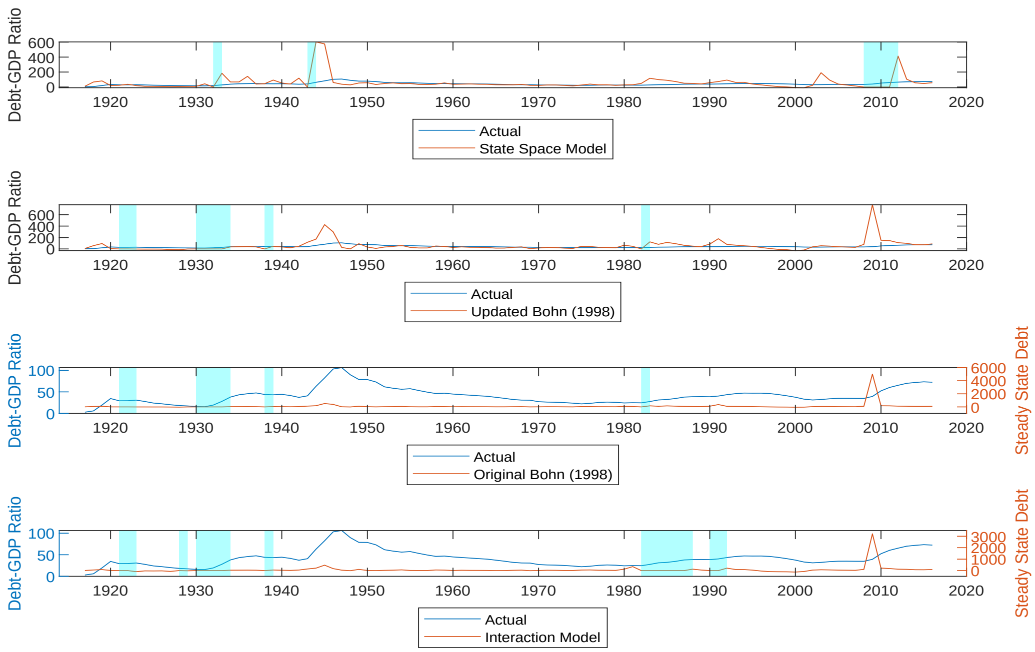
The figure reports the estimated time series of the coefficient on the debt-GDP ratio (ρ in the model discussed in Section **??**) from the state space model, alongside the original coefficient estimated in **?**, an updated coefficient based on a similar methodology and extended sample, and a coefficient derived from a model interacting the growth rate of GDP and the real yield on the 10-year Treasury note with the debt-GDP ratio, as in Equation (**??**).

Figure 16: Model Comparisons



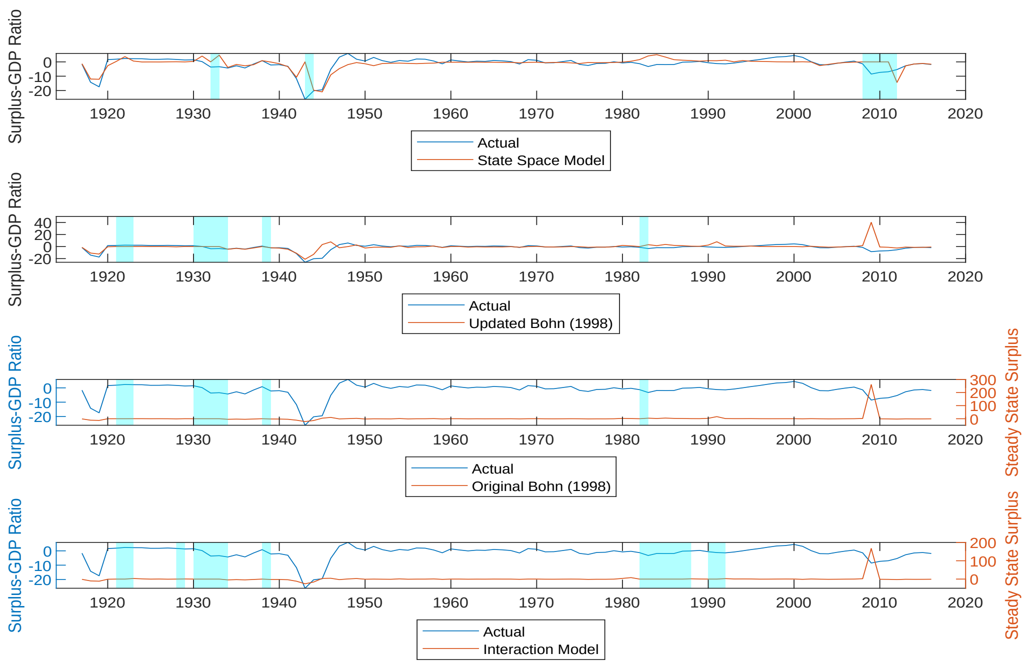
The figure reports the estimated time series of the stability condition parameter ($\kappa = \frac{1+r}{1+\delta_y}(1-\rho)$) from the state space model, assuming the original coefficient estimated in **?**, assuming an updated coefficient based on a similar methodology and extended sample, and assuming a coefficient derived from a model interacting the growth rate of GDP and the real yield on the 10-year Treasury note with the debt-GDP ratio, as in Equation (**??**). $\kappa < 1$ implies that the stability condition is satisfied.

Figure 17: Model Comparisons



The figure reports the estimated time series of the actual debt-GDP ratio and the implied steady state debt-GDP ratio from the state space model (top), assuming an updated coefficient based on the methodology of **?** and extended sample (second from top), assuming the original coefficient estimated in **?** (third from top), and assuming a coefficient derived from a model interacting the growth rate of GDP and the real yield on the 10-year Treasury note with the debt-GDP ratio, as in Equation (**??**) (bottom). Light blue bars imply that the stability condition is not satisfied.

Figure 18: Model Comparisons



The figure reports the estimated time series of the actual surplus-GDP ratio and the implied steady state surplus-GDP ratio from the state space model (top), assuming the original coefficient estimated in **?** (lower left), assuming an updated coefficient based on a similar methodology and extended sample (upper right), and assuming a coefficient derived from a model interacting the growth rate of GDP and the real yield on the 10-year Treasury note with the debt-GDP ratio, as in Equation (**??**) (lowe right). Light blue bars imply that the stability condition is not satisfied.