# Disclosure Volume

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**Abstract**

We study a shareholder voting game in which a manager chooses how many signals about an upcoming proposal to make publicly available, and must choose ex ante, without private information. Our main result is that this choice, called the *disclosure volume*, suffices to manipulate the vote. The manager does not need to bias the signals or hide unfavorable results. Instead, she exploits the fact that the shareholders know their information cannot be conditionally independent, as their signals come from the same finite population. This makes their inferences a function of the population size, which the manager controls.

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# 1 Introduction

We study a manager who chooses how much information to disclose in order to persuade shareholders to vote in favor of a proposal. As in information design, the manager makes her decision ex ante, with no private information, and commits to make available every signal she receives. But there is an important difference: her disclosures are unbiased signals of whether the proposal is in shareholder interest, and she cannot alter their distribution. The sole tool at her disposal is her choice of *disclosure volume*, i.e., the number of signals to generate. We show that her choice can increase the probability that an ex ante unfavorable proposal passes; the amount of available unbiased information can bias voters.

The shareholders are fully rational voters; their objective is to maximize the proposal's expected net present value (ENPV). They are ex ante identical, and we focus throughout on symmetric, pure-strategy equilibria (though if they acquire different information, they may behave differently). We envision them as institutional investors who cannot opportunistically sell their shares, such as those who offer unmanaged funds. But nothing stops the shareholders from affecting ENPV through voting, and by focusing on symmetric equilibria with ex ante identical shareholders, we avoid the incentive to abstain arising from the swing voter's curse (Feddersen and Pesendorfer, 1996). Thus, we assume throughout that all shareholders vote.

Despite their common goal of ENPV maximization, the shareholders are barred from explicitly pooling their information.[1] This distinguishes them from other voters with a common purpose, such as jurors, for whom deliberation is encouraged rather than restricted. Nevertheless, if the amount of available information is finite, a shareholder can infer from his own sample some information about what other shareholders could have seen. These inferences depend on the number of available signals, making the manager's disclosure volume choice a strategic tool.

To make these ideas precise, we model each signal the manager generates as an independent Bernoulli draw with an unknown success probability. Nature uses a uniform distribution to draw the true probability that the proposal has positive ENPV, revealing it to no one, and then uses the same success probability for each signal; in this sense,

[1] In the United States, the Securities and Exchange Commission requires Schedule 13D for an investor whose communications about voting intentions lead to classification as a group acting in concert. Malenko, Malenko, and Spatt (2025) cite the Delaware Court of Chancery, which has found that even innocuous communications such as exchanging information can trigger acting-in-concert rules, and note that Vanguard, BlackRock, and State Street maintain explicit policies against pre-vote communication to avoid these rules. In the absence of such constraints, Coughlan (2000) shows that pre-vote communication would enable the voters to pool their information and vote sincerely, and Hellwig and Veldkamp (2009) have similar observations in a trading setting, in which investors would try to coordinate on information acquisition if their actions are complementary.

the signals are unbiased.[2] At the start of play, the manager chooses the number $N$ of Bernoulli draws to request. Nature provides the $N$ realizations, which the shareholders can sample. We focus on a proposal that is ex ante unfavorable, so that if the manager were to choose $N = 0$, the shareholders would optimally vote against the proposal.

The choice of $N$ has two effects on the shareholders' probability of voting for the proposal. It limits the amount of information shareholders can sample, and it affects the shareholders' reasoning about whether their votes are pivotal, an issue that Persico (2004) addresses (in a setting with conditionally independent signals, which is the limit of our setting as $N \to \infty$).[3] To separate these two effects, we begin by considering a setting in which information processing is costless. Given that information is free, a shareholder does not need to consider decision relevance in order to justify sampling.

In the most informative costless information processing equilibrium, the shareholders each observe every signal, effectively becoming clones of one another because of their common prior and shared objective. This equilibrium isolates the effect of limiting the information available to the shareholders. Clones vote identically given symmetry and pure strategies, so no shareholder is pivotal. The manager can focus on a representative shareholder, and optimally chooses the smallest signal population such that the representative shareholder supports the proposal if and only if all the signals are successes.

This result is similar to findings in the Bayesian persuasion literature (Kamenica and Gentzkow, 2011), including its application to voting (Alonso and Câmara, 2016). The manager faces a pass-fail test, where a pass is convincing the representative shareholder that the success probability is sufficiently high. The shareholder has a cutoff, and the manager aims to pool as much probability mass below this cutoff as possible with the probability mass above it. Any test that tolerates even one failure signal is wasteful, as it potentially dilutes the posterior success probability. Even though the game differs from Bayesian persuasion in that the manager cannot alter the signal structure, the result persists because the manager can use her first-mover advantage.

The game changes if information processing is costly (as in Blankespoor, deHaan, and Marinovic, 2020; Devdariani and Hirsch, 2023), becoming a constrained information design problem.[4] A nonzero processing cost rules out an equilibrium in which all the

[2]Generalization to any Beta prior is conceptually straightforward, as conjugacy and the hypergeometric sampling structure survive. However, the uniform prior is uniquely tractable, because it induces a uniform prior over the number of successes in the population. Therefore, the weights from the prior cancel from the shareholders' posteriors. A general Beta prior instead induces a beta-binomial prior over the number of successes, whose weights survive into every posterior.

[3]See also Austen-Smith and Banks (1996); Feddersen and Pesendorfer (1997, 1998). Strategic voting of this kind is well documented: Guarnaschelli, McKelvey, and Palfrey (2000) and Battaglini, Morton, and Palfrey (2010) find it in laboratory elections, and Maug and Rydqvist (2009) find evidence of strategic voting by shareholders specifically.

[4]For a general discussion of constrained information design, see Doval and Skreta (2024). An overview

shareholders observe the whole population. Given a shared objective and a common posterior, an individual shareholder would optimally deviate by learning nothing and free-riding. Thus in any informative equilibrium, each shareholder leaves some signals unexplored.

In fact, each shareholder must ignore at least two signals. For instance, suppose the manager makes $N = 2$ signals available. We can immediately rule out an equilibrium in which each shareholder samples both signals, because all the shareholders would have a common posterior. But there is also no equilibrium in which each shareholder samples one signal. A given shareholder's vote changes the outcome (he is pivotal) only if the two signals split, so a shareholder who draws a good signal would have to condition his vote on the unobserved signal being bad. In the only case in which learning matters, the shareholder would have nothing to learn.

We provide conditions for the existence of equilibria in which each shareholder samples $0 < n \leq N - 2$ signals. The manager's equilibrium choice of signal population $N$ leads to the proposal being approved with greater probability than the probability the proposal has positive ENPV. The shareholders choose their sample size $n$, observe their signals, and then make inferences about available signals that they have not observed.

To illustrate, consider a minimally informative equilibrium, in which the manager makes $N = 3$ signals available and the shareholders each sample $n = 1$ signal. The total number of successes in the population can be any value in $\{0, 1, 2, 3\}$. However, a shareholder cannot be pivotal if the signals are all successes or all failures; the other shareholders must have differential information for their votes to split evenly. Thus the shareholder updates in two steps. First, he estimates the success probability twice: once assuming a single success in the population, once assuming two. Second, he averages the two estimates, weighting by how likely each scenario is given his own draw.

From the Uniform-Binomial prior, it follows that, for each $k \in \{0, 1, 2, 3\}$, the prior probability of $k$ successes is $1/4$. This means in particular that the prior probabilities of $k = 1$ and of $k = 2$ are the same, and when the shareholder conditions on being pivotal, each of these turns out to have probability $1/2$. Given one success on three trials, the posterior success probability would be distributed Beta(2,3), with a mean of $2/5$, and given two successes on three trials, the posterior success probability would be distributed Beta(3,2), with a mean of $3/5$. If $k = 1$, the shareholder draws a success with probability $1/3$, and if $k = 2$, the shareholder draws a success with probability $2/3$. By Bayes' rule, a shareholder drawing a success infers that $k = 1$ with probability $1/3$ and $k = 2$ with probability $2/3$. Thus, given $N = 3$ signals are available, a shareholder drawing $n = 1$ and seeing that it is a success would infer a posterior success probability of

of the information design literature is in Bergemann and Morris (2019).

$(1/3) \cdot (2/5) + (2/3) \cdot (3/5) = 8/15$. Analogously, a shareholder observing a single failure would infer a posterior success probability of $7/15$. If an $8/15$ success probability suffices for the proposal to have positive ENPV, then the manager issuing $N = 3$ signals and the shareholders each drawing $n = 1$ can be part of an equilibrium. (It is automatic that a $7/15$ success probability implies negative ENPV, as the proposal is unfavorable ex ante, with a prior of $1/2$.)

These two examples, $N = 2$ and $N = 3$, though far from exhaustive, illustrate the consequences of pivotality. As the number of shareholders grows, pivotality can fully determine the number of successes in the population. In that case, the logic of the $N = 2$ example recurs: a shareholder has nothing to learn if pivotal and no value in learning if not pivotal. The only other possibility is that pivotality asymptotically leads to exactly two possible values for the total number of successes, each with probability $1/2$. These two possible values are necessarily consecutive integers in $\{1, \ldots, N - 1\}$. And it is only in this latter case that there can be an informative equilibrium, as in the $N = 3$ example.

Which case obtains depends on $N$ and on the sample size $n$ in a way that we characterize explicitly. For instance, if $n = 2$, there are asymptotically two possible total successes in the population given pivotality if and only if there is an integer $k \in \{2, \ldots, N - 1\}$ satisfying the Diophantine equation $N^2 - N = 2(k - 1)^2$. If there is, the total number of successes in the population is either $k$ or $k - 1$. For $n = 1$, the condition is simpler: $N$ must be odd and at least 3; if so, pivotality implies the number of successes is either $(N - 1)/2$ or $(N + 1)/2$.

These cases can intersect, but the shareholder's incentive compatibility constraint uniquely determines the sample size. If the manager chooses $N = 9$, then the $n = 1$ requirement is satisfied with total population successes in $\{4, 5\}$, and if $n = 2$, the requirement is satisfied with total population successes in $\{6, 7\}$. For $n = 1$ to be part of an equilibrium, the shareholder must, in particular, be willing to support the proposal given 5 successes among the 9 signals, and for $n = 2$ to be part of an equilibrium, the shareholder must be unwilling to support the proposal given 6 (or fewer) successes among the 9 signals. In sum, everything a shareholder conditions on, and the shareholder's choice of how many signals to process, depends on the signal population size $N$ that the manager chooses.

These results show that the finite number $N$ of available signals alters the inferences one can make about information and voting, on technical and conceptual levels. From a technical viewpoint, if the level of disclosure were thought of as precision, varying continuously, then conditioning on pivotality would concentrate beliefs at a single balance point as the number of voters (our shareholders) increases. With finitely many signals, the balance point can fall at the midpoint between two integers. It is this feature that enables our manager to choose where the integers on either side of the balance point lie.

Conceptually, the central effect of finiteness is that it implies that individual voters draw interdependent samples. In the limit, as $N \to \infty$, the shareholders draw conditionally independent samples, as in Persico (2004). For smaller $N$, the voters know their samples are interdependent, even though they cannot communicate. It is this interdependence that the manager can use to her advantage.

Although interdependent information is natural in voting and other collective decision problems, modeling it is a known obstacle. Kaniovski (2010) and Pivato (2017) show that the difficulty is structural: pairwise correlations do not pin down how interdependent information aggregates, so results built on conditional independence do not extend by perturbation. Our framework offers a way forward: the finite signal population provides a one-parameter family of voting problems, indexed by $N$ (which is endogenous in our case, but could be exogenous in others). The framework is tractable, and it specifies the joint distribution of the voters' information completely.

The mechanism also explains why a vote can be manipulated by a sender who could not manipulate a market. When information feeds into a price, aggregation is continuous: a sender with no informational advantage, releasing unbiased signals, changes what the price reflects but not whether it reflects it (Grossman, 1976; Hellwig, 1980). A vote aggregates through a threshold. The proposal passes or fails, and the manager's choice of $N$ shifts where the shareholders' inferences fall relative to that threshold.

If the manager were to choose $N$ off the equilibrium path, or if the cost of sampling were prohibitive, our shareholders might be rationally ignorant, as in Downs (1957), but the mechanism is different. A Downsian voter weighs the cost of information against an exogenous probability of being pivotal. For our shareholders, the more important factor is what the information could reveal, rather than what it costs to observe it.

The structure of the rest of this paper is as follows. Section 2 introduces the model. Section 3 analyzes the case of no information processing costs. Section 4 addresses costly information processing and establishes our main result. A final section concludes. Appendix A has all the proofs.

## 2 The Model

We model a one-shot game of imperfect information. We adopt the following notation from the combinatorics literature:

For a positive integer $\alpha$, $[\alpha] := \{1, \ldots, \alpha\}$

For a real number $x$, $\lceil x \rceil := \min\{\alpha \in \mathbb{Z} | \alpha \geq x\}$ (the *ceiling* of $x$)

The players are a manager (Player 0) and an odd number, $M = 2m+1$ for some $m \in \mathbb{Z}_{++}$, of homogeneous shareholders. All players are risk neutral, rational, and have rational expectations. There is no ex ante information asymmetry. Denote a generic shareholder by $i \in [M]$.

There is an exogenously chosen proposal, on which the shareholders will eventually vote. Voting is by majority rule, without abstentions: if the proposal receives $m+1$ or more yes votes, we say the proposal passed or that the shareholders approved it. Otherwise, we say the proposal failed or that the shareholders rejected it. Let $y_i \in \{0, 1\}$ equal 1 if shareholder $i$ votes for the proposal and 0 if $i$ votes against the proposal, and let $y = (y_1, \ldots, y_M)$. The payoff to the manager is

$$u_0(y) = \begin{cases} 1, & \text{if } \sum_{i=1}^{M} y_i > m \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

i.e., the manager's utility is 0 if the shareholders reject the proposal and 1 if they approve it.

If the shareholders reject the proposal, they also get 0. If they approve it, each shareholder receives an identical random payoff $\tilde{v}$:

$$\tilde{v} = \begin{cases} U, & \text{with probability } \tilde{\theta} \\ D, & \text{with probability } 1 - \tilde{\theta} \end{cases} \tag{2}$$

where $D < 0 < U$. The success probability $\tilde{\theta}$ is unknown to all and has a uniform common prior, i.e., $\tilde{\theta} \sim U[0, 1]$. (Throughout, tildes denote random variables; plain symbols denote their realizations.) Thus, the payoff to shareholder $i$, given the vote profile $y$, is

$$u_i(y) = u_0(y)\tilde{v} \tag{3}$$

The manager's only decision is to choose the number of signals $N \in \mathbb{Z}_+$ that she commits to release publicly, denoted $\tilde{s}^N := (\tilde{s}_1, \ldots, \tilde{s}_N)$. She does not know the value of any of the signals when making this decision, and she cannot renege or draw more than $N$ signals and choose which to release.

After the manager chooses $N$, Nature privately draws the value of $\theta$ and the signal realizations $s_1, \ldots, s_N$. No one learns $\theta$, but for any $j \in [N]$, any shareholder can privately learn $s_j$ at signal processing cost $c \geq 0$.[5]

Conditional on the success probability $\theta$, the $\tilde{s}_j$ are mutually independent Bernoulli($\theta$) draws; i.e., they have the same success probability as the proposal:

$$\tilde{s}_1, \ldots, \tilde{s}_N | \theta \overset{iid}{\sim} \text{Bernoulli}(\theta) \tag{4}$$

The assumptions of Bernoulli signals and a uniform prior give convenient and tractable updating rules. The assumption that the signals have the same success probability as $\tilde{v}$ and are conditionally independent prevents the manager from engaging directly in information design, as the signals are required to be unbiased.

Once the signals are released, the shareholders decide which and how many signals to process. Shareholder $i$ chooses a number of signals $n_i$ to process, so that $i$ draws $n_i$ of the $N$ signals randomly and without replacement. The total information processing cost to $i$ is therefore $cn_i$.[6]

The shareholders then simultaneously and privately submit their votes. Nature determines the proposal outcome $v \in \{U, D\}$. The shareholders and the manager receive their payoffs, and the game ends.

Figure 1 summarizes the sequence of events.

| | | | | |
|---|---|---|---|---|
| Manager chooses the number of signals to promise | Nature draws the success probability and signal values | Shareholder $i$ processes $n_i$ public signals at cost $cn_i$ | Shareholders vote | Payoffs realized |

Figure 1: Timeline.

We restrict attention to pure strategy, subgame perfect, symmetric equilibria. Thus, for each shareholder $i \in [M]$, $n_i = n$, and shareholders with identical posteriors vote the same way. To break ties, we assume that shareholders who are indifferent vote for the proposal. In equilibria in which the shareholders process at least one signal, we consider

[5] The constant variable cost rules out learning-by-doing or knowledge prerequisites. Neklyudov (2019) and Nan, Song, and Zhang (2025) address these issues, though in different disclosure settings.

[6] We assume $i$ collects all $n_i$ signals before processing any of them, rather than looking at signals sequentially and then deciding whether to continue sampling. For example, an institutional investment firm might assign different research tasks to different individuals. This assumption avoids the technical subtleties in sequential analysis that Wald (1950) raises.

only threshold strategies. Finally, we focus on cases in which the prior expectation of $\tilde{v}$ is negative; this avoids trivialities, as Proposition 1 shows:

**Proposition 1.** *Suppose $U + D \geq 0$. Then there is an equilibrium in which*

1. *The manager chooses $N = 0$, i.e., she discloses nothing, and*
2. *Each shareholder $i$ votes $y_i = 1$, i.e., in favor of the proposal*

*Furthermore, in every pure strategy, subgame perfect equilibrium, the shareholders gather no information and all vote for the proposal.*

On account of Proposition 1, we assume $U + D < 0$.

*Remark* 1 (Known-probability benchmark). If the information processing cost $c = 0$ and if the number of public signals $N \to \infty$, the shareholders could learn $\theta$ arbitrarily well, and would approve the proposal if and only if

$$\theta U + (1 - \theta) D \geq 0 \quad \Leftrightarrow \quad \theta \geq \frac{-D}{U - D}$$

This policy would be optimal if $\theta$ were known. Evaluated ex ante, this policy passes the proposal with probability

$$\frac{U}{U - D}$$

# 3 Costless Information Processing

We begin by considering the case in which $c = 0$, so that there is no cost of processing information. There are many equilibria in this setting. We focus on one in which the shareholders observe every released signal. Costless information processing is necessary for such an equilibrium to exist, as no voter is pivotal.

**Lemma 1.** *Suppose the information processing cost $c = 0$. Given the manager's choice of $N$, there is an equilibrium in the voting subgame in which all the shareholders observe all the signals $\tilde{s}^N$ and vote for the proposal if and only if $E[\tilde{v}|s^N] \geq 0$.*

Given Lemma 1, we proceed to find the manager's optimal choice of $N$. This turns out to be the smallest possible number that could make the shareholder decide to approve the proposal, conditional on all the released signals being positive.

The manager first considers how a representative shareholder updates for an arbitrary sample size $n$. Let $\sigma$ be the number of successes the shareholder observes out of $n$

processed signals. The shareholder's posterior distribution of $\tilde{\theta}$ is

$$\tilde{\theta}\left|\sum_{i=1}^{n} s_i = \sigma \right. \sim Beta(\sigma+1, n-\sigma+1)$$

and the posterior expectation of $\tilde{\theta}$ is

$$E\left[\tilde{\theta}\left|\sum_{i=1}^{n} s_i = \sigma\right.\right] = \frac{\sigma+1}{n+2}$$

Therefore, the shareholder votes in favor of the proposal if and only if

$$\begin{aligned} E[\tilde{v}|s^N] &= \frac{(\sigma+1)U + (n-\sigma+1)D}{n+2} \geq 0 \\ \Rightarrow \qquad \sigma &\geq -\frac{U+(n+1)D}{U-D} \end{aligned} \tag{5}$$

Denote by $\sigma^*(n)$ the minimal number of successes satisfying (5) conditional on the shareholder processing $n$ signals. Then

$$\sigma^*(n) = \left\lceil -\frac{U+(n+1)D}{U-D} \right\rceil \tag{6}$$

If $\sigma^*(n) > n$, then the shareholder would vote to reject regardless of the outcomes of the $n$ observed signals. This is equivalent to choosing $n = 0$, in which case it is easily verified that $\sigma^*(0) = 1$. Lemma 2 establishes that $\sigma^*(\cdot)$ is weakly increasing and grows by at most one when $n$ increases by one:

**Lemma 2.** *For any $n \in \mathbb{Z}_+$, $\sigma^*(n+1) \in \{\sigma^*(n), \sigma^*(n)+1\}$, and if $\sigma^*(n+1) = \sigma^*(n)$ then $\sigma^*(n+2) = \sigma^*(n)+1$.*

Using Lemma 1 and Lemma 2, we verify the equilibrium in which the manager releases the fewest signals under which the proposal could pass:

**Theorem 1.** *Let $n_0 \in \mathbb{Z}_{++}$ satisfy*

$$\sigma^*(n_0) \leq n_0$$

*and*

$$(\forall \hat{n} < n_0) \quad \sigma^*(\hat{n}) > \hat{n}$$

*Then $\sigma^*(n_0) = n_0$, and there is a subgame perfect, symmetric, pure strategy equilibrium satisfying*

1. *The manager chooses $N = n_0$;*

*2. the representative shareholder processes all $n = n_0$ signals; and*

*3. the representative shareholder approves the proposal if and only if all $n_0$ signals are successes.*

*The expected payoff to the shareholder is*

$$\frac{(n_0+1)U + D}{(n_0+1)(n_0+2)},$$

*and the expected payoff to the manager is*

$$\frac{1}{n_0+1}$$

Theorem 1 implies that there is an equilibrium with nondegenerate but limited disclosure given $c = 0$. Corollary 1 establishes that in this equilibrium, the probability that the proposal is approved is higher than the probability it is in shareholder interest.

**Corollary 1.** *In the equilibrium of Theorem 1, the shareholder approves the proposal with strictly greater probability than the known-probability benchmark.*

Thus, under costless information processing, an uninformed sender who cannot design the structure of her signals can nevertheless increase the probability that the receivers take her preferred action, purely through her choice of disclosure volume.

The intuition is as follows: the proposal is ex ante unfavorable, so each signal is likely to work against the manager's goal. Therefore, she wishes to release as few signals as she can, subject to the constraint that she needs to release enough to potentially change the shareholders' minds.

# 4 Costly Information Processing

We now move to the case in which information processing is costly. In contrast to the costless processing case, there cannot be an equilibrium in which the manager releases $N \geq 1$ signals and in which the shareholders observe and process all the public information. If all the shareholders observe the same information, their common prior implies a common posterior. In that case, the shareholders would all vote the same way, so none of them would be pivotal. An individual shareholder optimally deviates by not observing any signals, saving the processing costs without changing the outcome.

More generally, for any $n \in [N]$, there cannot be an equilibrium in which the shareholders coordinate on processing the same $n$ signals, as this results in a common posterior. We summarize this in Proposition 2.

**Proposition 2.** *In equilibrium, the number of signals each shareholder processes must be at least two fewer than the total number available, i.e.,* $n \leq N - 2$. *Moreover, the shareholders cannot credibly commit to pool their observed signals unless* $n = 0$.

From Proposition 2, we see that a nonzero information processing cost makes it impossible for shareholders to coordinate their sampling (in contrast to the setting of Section 3). As long as they process at least one signal, the shareholders must have some independent sampling and cannot pool their information. For the rest of this paper, we assume this sampling is completely independent.[7]

## 4.1 Last stage: shareholder updating and voting

In the last stage of the game, the manager's choice of $N$ is sunk, as is each shareholder's sample of $n$ signals (where, by Proposition 2, $n \leq N - 2$). Given that shareholder $i$ observes $\sigma_i$ successes and $n - \sigma_i$ failures in his sample, the difference in $i$'s expected utility between voting yes and voting no on the proposal is

$$E[\tilde{v}|i \text{ pivotal}, \sigma_i, n, N] \cdot Pr(i \text{ pivotal}|\sigma_i, n, N),$$

because the shareholders are instrumental voters, not deriving any utility from voting with or against the manager for its own sake. Thus, shareholder $i$ chooses his vote $y_i$ as follows:

$$y_i = \begin{cases} 1, & \text{if } E[\tilde{v}|i \text{ pivotal}, \sigma_i, n, N] \geq 0 \\ 0, & \text{if } E[\tilde{v}|i \text{ pivotal}, \sigma_i, n, N] < 0 \end{cases} \tag{7}$$

As (7) indicates, the shareholders condition their beliefs on their observed successes $\sigma_i$ and failures $n - \sigma_i$, and also on being pivotal, which requires them to make inferences about the signals they have not observed. We first provide an illustration, then move to the general case.

### 4.1.1 An illustration

Suppose, as in the introduction, the manager makes $N = 3$ signals available and the shareholders each independently sample $n = 1$. By Proposition 2, these are the minimal possible values of $(N, n)$ in any informative equilibrium. If shareholder $i$ is pivotal, then

[7] It is natural to expect shareholders to reduce their information processing costs by assigning different signals to different shareholders. For instance, we could imagine Shareholder 1 observing the first $N/(2m + 1)$ signals, Shareholder 2 the next $N/(2m + 1)$, and so on, and then broadcasting their information. This would reveal all $N$ signals to all the shareholders, with the costs being split. However, Proposition 2 rules this out: pooled signals would give the shareholders a common posterior and hence a common vote, so no shareholder would be pivotal and each would prefer to free-ride. An alternative approach is imperfectly correlated sampling. For instance, a third party such as a proxy advisory firm could sample a subset of the signals and sell a summary report to the shareholders, enabling the shareholders to split the costs of processing some of the signals, but not all of them. This case is taken up in Feng (2025).

$m$ of the other shareholders must have drawn a signal of 0 and the other $m$ must have drawn a signal of 1. Although the number of successes $k$ in the population of 3 signals can be 0, 1, 2, or 3 (each with equal probability by the Uniform-Binomial prior), each shareholder updates conditional on $k$ being either 1 or 2, as these are the only scenarios in which the shareholder's vote matters.

Table 1 shows shareholder $i$'s posterior probability distribution of there being $k$ successes in the population, given that his own signal $s_i = 1$.

| $k$ | Prob($k$) | Prob($s_i = 1$ & pivotal$\|k$) | Prob($k\|$pivotal & $s_i = 1$) |
|---|---|---|---|
| 0 | $\frac{1}{4}$ | 0 | 0 |
| 1 | $\frac{1}{4}$ | $\binom{2m}{m}\left(\frac{1}{3}\right)^{m+1}\left(\frac{2}{3}\right)^{m} = \binom{2m}{m}\frac{2^m}{3^{2m+1}}$ | $\frac{2^m}{2^m+2^{m+1}} = \frac{1}{3}$ |
| 2 | $\frac{1}{4}$ | $\binom{2m}{m}\left(\frac{2}{3}\right)^{m+1}\left(\frac{1}{3}\right)^{m} = \binom{2m}{m}\frac{2^{m+1}}{3^{2m+1}}$ | $\frac{2^{m+1}}{2^m+2^{m+1}} = \frac{2}{3}$ |
| 3 | $\frac{1}{4}$ | 0 | 0 |

Table 1: Shareholder $i$'s posterior probabilities of $k$ successes among the $N = 3$ signals the manager releases, given that each shareholder observes $n = 1$ signal and that shareholder $i$ observes a success, i.e., $s_i = 1$

Given an observed success, the shareholder's posterior distribution of $\tilde{\theta}$ is

$$\tilde{\theta} \sim \begin{cases} \text{Beta}(2,3), & \text{with probability } \frac{1}{3} \\ \text{Beta}(3,2), & \text{with probability } \frac{2}{3} \end{cases}$$

and the posterior expectation of $\tilde{v}$ is

$$\left(\frac{1}{3}\right)\left(\frac{2U+3D}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{3U+2D}{5}\right) = \frac{8U+7D}{15}$$

Therefore, the shareholder will vote $y_i = 1$ given $s_i = 1$ if and only if $U/|D| \geq 7/8$. If, on the other hand, $s_i = 0$, a similar calculation shows that $i$'s posterior given pivotality is $k = 1$ with probability $2/3$ and $k = 2$ with probability $1/3$. In this case, the expectation of $\tilde{v}|i$ pivotal, $s_i = 0$ is $(7U + 8D)/15 < 0$ because $U < |D|$. Thus, $i$ optimally votes $y_i = 0$ if $s_i = 0$.

We end this illustration by noting the resulting pass probability. Provided $7/8 \leq U/|D|$ ($U/|D| < 1$ holds by assumption), each shareholder votes for the proposal if and only if he draws a success. By the symmetry of the Uniform-Binomial prior, $k$ and $3 - k$ are equally likely. Therefore, the probability that a majority of the shareholders draw successes is $1/2$, so the proposal passes with probability $1/2$, which is strictly above the known-probability benchmark of $U/(U - D)$.

### 4.1.2 General setting: conditioning on pivotality

In the general case, shareholder $i$'s probability of getting $\sigma$ successes is the hypergeometric probability mass, given sample size $n$, the population's total successes $k$, and the total number of signals $N$ in the population:

$$Pr(\sigma \text{ successes}|n,k,N) = \frac{\binom{k}{\sigma}\binom{N-k}{n-\sigma}}{\binom{N}{n}}$$

Let $F_{n,k,N}(\sigma)$ be the hypergeometric cumulative distribution of $\sigma$ given the sample size, population successes, and population size $(n,k,N)$. For $h \in \{0,\ldots,n-1\}$, let $n-h$ be the cutoff, i.e., the minimum number of successes a shareholder must observe in order to vote for the proposal. Then, given $k$, a shareholder is pivotal with probability

$$\begin{aligned} Pr(\text{pivotal}|n,k,N) &= \binom{2m}{m}\left(\sum_{\sigma=0}^{n-h-1}\frac{\binom{k}{\sigma}\binom{N-k}{n-\sigma}}{\binom{N}{n}}\right)^m\left(1-\sum_{\sigma=0}^{n-h-1}\frac{\binom{k}{\sigma}\binom{N-k}{n-\sigma}}{\binom{N}{n}}\right)^m \\ &= \binom{2m}{m}\left(F_{n,k,N}(n-h-1)\left[1-F_{n,k,N}(n-h-1)\right]\right)^m, \end{aligned} \tag{8}$$

where the binomial coefficient arises because it does not matter which $m$ of the $2m$ other shareholders vote yes and which vote no.

From the Uniform-Binomial prior, the prior probability of $k$ total population successes is constant. Therefore, the posterior probability of $k$ successes in the population, given pivotality, is

$$Pr(k|\text{pivotal},n,N) = \frac{\left(F_{n,k,N}(n-h-1)\left[1-F_{n,k,N}(n-h-1)\right]\right)^m}{\sum_{k'=n-h}^{N-h-1}\left(F_{n,k',N}(n-h-1)\left[1-F_{n,k',N}(n-h-1)\right]\right)^m} \tag{9}$$

Proposition 3 characterizes (9) as the number of shareholders gets large.

**Proposition 3.** *As $m \to \infty$, the probability (9) of $k$ successes in the population given pivotality, $n$, and $N$, is zero everywhere except those values of $k$ for which*

$$k \in \underset{k'\in\{n-h,\ldots,N-h-1\}}{\operatorname{argmin}}\left|F_{n,k',N}(n-h-1)-\frac{1}{2}\right|, \tag{10}$$

*i.e., except for values of $k$ for which $n-h-1$ is closest to the median.*

A hypergeometric random variable may not have a median. For instance, as in the illustration of 4.1.1 with $N=3, h=0$, and $n=1$, the probability that a shareholder observes $n-h-1=0$ successes given $k$ is $(3-k)/3$. There is no $k$ with $F_{1,3,k}(0)=1/2$, and the values of $k$ in (10) are $\{1, 2\}$.

In general, there are no closed-form expressions for the quantiles of a hypergeometrically distributed random variable, although there are algorithms for calculating them (see Alvo and Cabilio, 2000). The special case of $h = 0$ is of particular interest for two reasons. First, the lower bound $n - h$ and the upper bound $N - h - 1$ on the population successes $k$ are decreasing in $h$. Secondly, if $h = 0$, Corollary 2 simplifies (10).

**Corollary 2.** *For $h = 0$, as $m \to \infty$, the support of* (9) *has either one element $k^*$ or two elements $\{k^* - 1, k^*\}$. The latter case holds if and only if*

$$\binom{N}{n} = \binom{k^*}{n} + \binom{k^* - 1}{n}, \tag{11}$$

*and the conditional probabilities of $k^*$ and $k^* - 1$ are asymptotically* $1/2$.

Figure 2 illustrates Proposition 3 and Corollary 2. As $m$ increases, the continuous approximation to $F_{n,k,N}$ becomes increasingly concentrated. The maximum with respect to $k$ may not be at an integer, and Equation (11) holds if and only if the maximum is equidistant between $k^* - 1$ and $k^*$.

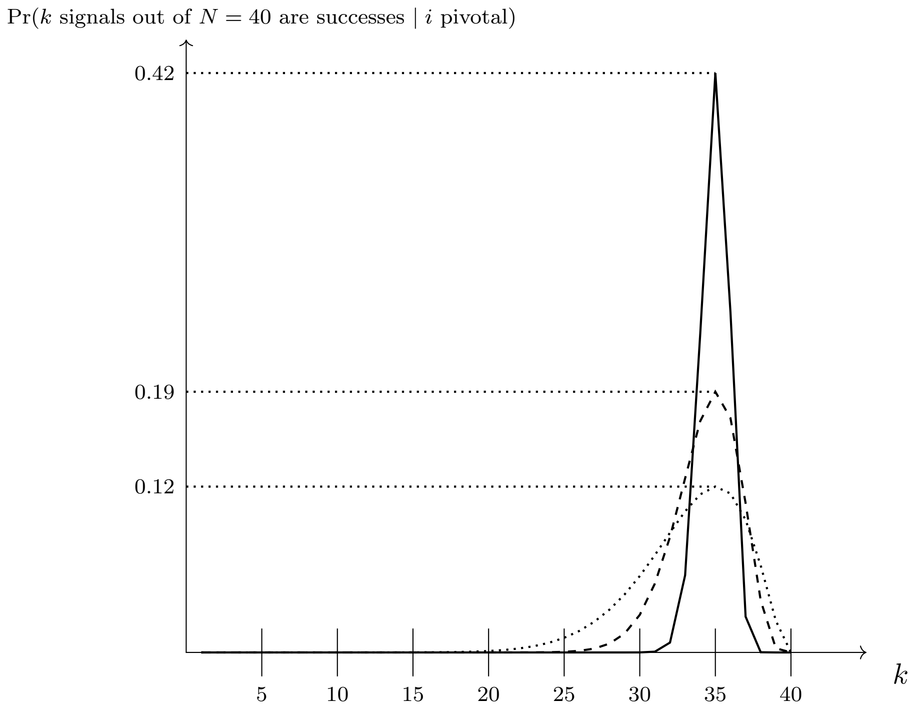


Figure 2: The probability that $k$ out of $N = 40$ signals are successes, given that shareholder $i$ is pivotal, the shareholders each process $n = 5$ signals, and that each shareholder votes if and only if observing $h = 0$ failures. Dotted line: $m = 2$ shareholders; dashed line: $m = 5$; solid line: $m = 25$.

*Remark* 2. Equation (11) is a necessary condition for shareholders to be willing to gather information as $m$ gets large. If (11) does not hold, then for large enough $m$, pivotality fully determines the number of successes in the population of signals. Each shareholder would thus conclude that either his vote is irrelevant or that he has nothing to learn from sampling. On the other hand, if (11) does hold, then asymptotically in $m$, a pivotal shareholder's signals determine whether $k^*$ or $k^* - 1$ successes are more likely.

**Proposition 4.** *For $n = 1$, a solution to* (11) *exists if and only if $N$ is odd and $k^* \geq 2$. There are also solutions to* (11) *if either $n = 2$ or $k^* = N - 1$, but only in special cases.*

*Remark* 3. Although we cannot solve (11) explicitly for $n > 2$ and $k^* < N - 1$, we searched numerically for solutions with $N \leq 1000$ and $n \in \{3, N-2\}$ and found none.

As shown in the proof of Proposition 4, there are many solutions to (11) if each shareholder draws a sample size of $n = 1$; the only requirement is that $N$ is odd and at least 3. In these cases, $k^* = (N+1)/2$. Thus, the ratio $k^*/N$ is strictly decreasing to $1/2$ and maximized at $(k^*, N) = (2, 3)$ (with a value of $2/3$). The ratio $(k^* - 1)/N$ is strictly increasing to $1/2$, and is minimized at $(k^*, N) = (2, 3)$, with a value of $1/3$.

Solutions to (11) with $n = 2$ are spaced much further apart. Substituting $n = 2$ into (11) yields

$$N^2 - N - 2(k^* - 1)^2 = 0,$$

which is the square-triangular problem, and is equivalent to the Pell equation

$$(2N - 1)^2 - 8(k^* - 1)^2 = 1 \tag{12}$$

The first several examples are

$$(k^*, N)|n = 2 \in \{(7, 9), (36, 50), (205, 289), (1190, 1682), (6931, 9801), \ldots\}$$

**Corollary 3.** *Solutions are generated by powers of the fundamental unit $3 + 2\sqrt{2}$ of the Pell equation* (12)*. Consequently, successive values of $(k^*, N)$ follow the recursion*

$$\begin{aligned} (k_0^*, N_0) &= (1, 1) \\ (k_1^*, N_1) &= (2, 2) \\ k_{t+2}^* &= 6k_{t+1}^* - k_t^* - 4 \quad N_{t+2} = 6N_{t+1} - N_t - 2 \end{aligned}$$

*so that $k_t^* - 1$ is sequence A001109 at the On-Line Encyclopedia of Integer Sequences (OEIS Foundation Inc., 2026) and $N_t$ is sequence A055997. The ratio $k^*/N$ for $N \geq 3$ and $n = 2$ strictly decreases, with a limit at $\sqrt{2}/2$, and the ratio $(k^* - 1)/N$ strictly increases to $\sqrt{2}/2$.*

*The initial conditions $k_0^* = 1$ and $k_1^* = 2$ correspond to the degenerate cases of $N \leq 2$; these are excluded by the requirement that $n \leq N - 2$ from Proposition 2, for which $n = 2$*

*requires* $N \geq 4$.

Although the solution is classical (due to Euler), we provide a self-contained proof in Appendix A to illustrate the underlying mechanism in which a fundamental unit generates an entire family.

Because the first non-degenerate case occurs at $(k^*, N) = (7, 9)$, Corollary 3 implies that the lower bound $(k^* - 1)/N$ is minimized (at a value of 2/3) and the upper bound $k^*/N$ is maximized (at a value of 7/9) at $(k^*, N) = (7, 9)$.

Solutions to (11) with $k^* = N - 1$ are also few and far between, and likewise have long been known. The first several examples with $n > 2$ are

$$(n, N)|k^* = N - 1 \in \{(6, 16), (40, 105), (273, 715), (1870, 4896), \ldots\}$$

We have the following corollary:

**Corollary 4** (Anning and Underwood, 1931)**.** *Solutions to* (11) *with* $k^* = N - 1$ *satisfy* $n(N - 1) = (N - n)(N - n - 1)$*, the equation of Anning and Underwood (1931). They are generated by*

$$\begin{aligned} n_t &= 7n_{t-1} - n_{t-2} - 1, \quad n_0 = 1,\ n_1 = 6 \\ N_t &= 7N_{t-1} - N_{t-2} - 4, \quad N_0 = 3,\ N_1 = 16 \end{aligned}$$

*equivalently* $n_t =$ *Fibonacci*$(2t + 1) \cdot$ *Fibonacci*$(2t + 2)$ *and* $N_t =$ *Fibonacci*$(2t + 2) \cdot$ *Fibonacci*$(2t + 3) + 1$*, sequences A081016 and (from its second term) A206351 at the OEIS (OEIS Foundation Inc., 2026).*

*Remark* 4. The rewriting in terms of Fibonacci numbers is due to Shirshov (1999).

The ratio of successive terms in the $n_t$ sequence is asymptotically $(1 + \sqrt{5})^4/16$, so solutions arise at an exponentially slow rate. The ratios $(k^* - 1)/N$ and $k^*/N$ in these cases can be rewritten as $(N - 2)/N$ and $(N - 1)/N$, so both asymptotically approach 1 from below. The lower bound in the smallest instance with $n > 2$ is 7/8, and the upper bound is 15/16. For the second-smallest instance, the lower bound is 103/105, and for subsequent instances the lower bound quickly approaches 1.

Given the simplification to (11) in Corollary 2, we assume from here on that $h = 0$ and that $m$ is sufficiently large for us to focus on the asymptotic distribution.

Proposition 5 characterizes the likelihood that a pivotal shareholder observes $n$ signals in this setting:

**Proposition 5.** *For large enough* $m$ *with* $h = 0$*, the probability that a pivotal shareholder* *draws all $n$ successes is* $1/2$.

### 4.1.3 Voting decision

Given $(N, n)$, it must be incentive compatible for shareholder $i$ to vote yes on the proposal if the $n$ signals are all successes and to vote no otherwise:

$$E[\tilde{v}|n \text{ successes, pivotal}] \geq 0$$
$$E[\tilde{v}|(n-1) \text{ successes, pivotal}] < 0$$

To calculate these expectations, $i$ first observes that

$$\tilde{\theta}|\text{pivotal} \sim \begin{cases} \text{Beta}(k^*, N - k^* + 2), & \text{if } k = k^* - 1 \\ \text{Beta}(k^* + 1, N - k^* + 1) & \text{if } k = k^* \end{cases}$$

If $i$ draws all $n$ successes, then

$$E[\tilde{v}|n \text{ successes, pivotal}] = Pr(k^* - 1|n \text{ successes, pivotal}) \left[\frac{k^* U + (N - k^* + 2)D}{N+2}\right] + Pr(k^*|n \text{ successes, pivotal}) \left[\frac{(k^* + 1)U + (N - k^* + 1)D}{N+2}\right] \quad (13)$$

Proposition 6 characterizes the condition that $i$ votes yes if observing successes on all $n$ draws and votes no if observing at least one failure:

**Proposition 6.** *Conditional on a population of $N$ signals, on a shareholder observing all successes on $n$ signals, and on the shareholder being pivotal, and assuming $\{k^* - 1, k^*\}$ are the solutions to* (11)*, the shareholder votes yes if and only if*

$$[-2k^{*2} + (2N + n + 3)k^* - (N + 2)n]D + [k^*(2k^* - n + 1)]U \geq 0 \quad (14)$$

*If instead the shareholder observes at least one failure, he votes no if and only if*

$$[k^*(-2k^{*2} + (2N + n - 1)k^* + 2N - Nn - n + 1)]U + [2k^{*3} - (4N + n + 3)k^{*2} + (2N(N + n + 1) + 3n - 1)k^* - (N + 2)(N + 1)(n - 1)]D < 0 \quad (15)$$

In the case with $n = 1, k^* = 2, N = 3$ inequality (14) becomes $7D + 8U \geq 0$, as we find above in Subsection 4.1.1, and inequality (15) becomes $16D + 14U < 0$, which does not bind because $U < |D|$.

## 4.2 Second-last stage: shareholder sample choice

We now seek conditions under which the shareholders optimally choose a sample of size $n$ that solves (11), given that the manager has released $N$ signals and that the other shareholders independently sample $n$ signals. The shareholder's probability of being pivotal must be large enough to justify the cost of gathering $n$ signals. Also, the cost $c$ of processing the signals must not be so low that the shareholder deviates and processes one or more additional signals.

The intuition is similar to that of Stigler (1961). The shareholder's problem is to find the optimal number of costly searches to perform for a decision maker who conducts all the searches before evaluating the results (such as an institutional investor that might assign different employees to research tasks, and later receives the summary reports from each employee). Stigler's (1961) solution is to choose the number of $n$ searches so that the expected incremental benefit of the $n^{\text{th}}$ search weakly exceeds the search cost $c$, and the expected incremental benefit from an $(n+1)^{\text{st}}$ search does not.

To apply Stigler's (1961) argument, we need to establish that the benefit of drawing $n$ signals is decreasing in $n$. We can decompose this benefit as

$$Pr(\text{pivotal})Pr(n \text{ successes}|\text{pivotal})E[\tilde{v}|n \text{ successes, pivotal}]$$

The first term, the probability that shareholder $i$ is pivotal, does not depend on whether $i$ draws $n$ signals or deviates. Therefore, we focus on showing that the product of the last two terms decreases in $n$.

For the last term, we introduce some notation. Let

$$\underline{v} := \frac{k^*U + (N - k^* + 2)D}{N + 2}$$
$$\delta := \frac{U - D}{N + 2}$$

Using this notation, we can rearrange (13) as

$$E[\tilde{v}|n' \text{ successes, pivotal}] = \underline{v} + Pr(k^*|n' \text{ successes, pivotal})\delta$$

Thus, the net benefit of additional information is bounded above because, conditional on pivotality, there must be either $k^* - 1$ or $k^*$ successes in the population.

For the middle term, recall that (11) holds provided a shareholder who observes $n$ signals votes $y_i = 1$ if all the observed signals are successes and votes $y_i = 0$ otherwise. Thus, we can restrict attention to a shareholder who deviates by drawing more than $n$ signals. We limit discussion here to a deviation to observe an additional, $(n+1)^{\text{st}}$ signal.

We note that the shareholder would deviate from choosing $n$ to $n+1$ signals only if the additional signal must be a success in order for the posterior of $\tilde{v}$ to be positive (given pivotality). Otherwise, the additional signal is superfluous and would only generate a cost to the shareholder. Therefore, the benefit to the shareholder of drawing $n+1$ signals is

$$\begin{aligned}
&Pr(\text{pivotal})\left(\frac{\binom{k^*}{n+1}+\binom{k^*-1}{n+1}}{2\binom{N}{n+1}}\right)\left[\underline{v}+\frac{\binom{k^*}{n+1}}{\binom{k^*}{n+1}+\binom{k^*-1}{n+1}}\delta\right]\\
=&Pr(\text{pivotal})\left[Pr(n+1 \text{ successes}|\text{pivotal})\underline{v}+\frac{\binom{k^*}{n+1}}{2\binom{N}{n+1}}\delta\right] \qquad (16)
\end{aligned}$$

The coefficient in (16) on $\underline{v}$ is easily seen to be decreasing. The last term is also decreasing, as it reduces to

$$\frac{k^*!}{2N!}\frac{(N-n)!}{(k^*-n)!}\delta,$$

which decreases in $n$ because $N > k^*$. Lastly, we need to establish that the cost $c$ is justified at the value of $n$ satisfying (11). Rewrite (13) as $\psi(n,N)$; expanding and using Proposition 6, we obtain:

$$\psi(n,N) := E[\tilde{v}|\text{all } n \text{ successes, pivotal}] = \frac{[-2k^{*2}+(2N+n+3)k^*-(N+2)n]D+[k^*(2k^*-n+1)]U}{(N+1)(2k^*-n)}$$

Proposition 7 makes this idea precise:

**Proposition 7.** *A shareholder is willing to process $n$ signals provided the search cost $c$ satisfies*

$$c \le \frac{\binom{2m}{m}\psi(n,N)}{2(N+1)n}\sum_{k=n}^{N-1}\left[\left(1-\frac{\binom{k}{n}}{\binom{N}{n}}\right)\left(\frac{\binom{k}{n}}{\binom{N}{n}}\right)\right]^m \qquad (17)$$

*If the shareholder will not vote yes given $n-1$ successes on $n-1$ tries, then he will not deviate to choose fewer than $n$ signals provided* (17) *holds.*

## 4.3 Initial stage: manager choice of $N$

We can now state the following existence result:

**Theorem 2.** *There exist $N^* \in \mathbb{Z}_{++}$ and $n^* \in [N^*-2]$ such that*

- *The manager optimally commits to releasing $N^*$ signals;*
- *Each shareholder optimally observes $n^*$ signals at total individual cost $cn^*$;*
- *Each shareholder optimally votes in favor of the proposal if and only if all $n^*$ of the*

*shareholder's observed signals are successes; and*

- *The probability that the proposal is approved strictly exceeds the known-probability benchmark $U/(U-D)$*

In the case of $n^* = 1$, we have seen that any odd value of $N \geq 3$ satisfies (11). A straightforward substitution into (16) shows that the expectation of $\tilde{v}$, given pivotality and a signal that is a success, is

$$\frac{(N+1)^2U + (N^2+2N-1)D}{2N(N+2)} = \frac{(N+1)^2(U+D) - 2D}{2N(N+2)}$$

If this benefit is positive for any given $N$, it is also positive for any $N' \in \{3, 5, \ldots, N\}$. Thus, if the cost of additional sampling is too small to implement a choice of a given $N$, there is some chance that the manager can satisfy (17) with $N+2$ and have the benefit of additional signals drop sufficiently fast in this case.

We note that in every equilibrium with $n^* = 1$, the proposal passes with probability asymptotically approaching 1/2. Each value of $k \in \{0, \ldots, N^*\}$ is ex ante equally likely, and each shareholder observes a success with probability $k/N^*$. Therefore, the probability that an individual shareholder votes yes is greater than 1/2 for every $k \in \{((N^*+1)/2, (N^*+3)/2, \ldots, N^*\}$, that is, in half of the possible realizations of $k$. As $m$ increases, the proposal is approved in exactly these cases. On the other hand, as $U < |D|$, the known-probability benchmark is strictly below 1/2.

# 5 Discussion and Conclusion

To summarize, there are two reasons a sender can bias voters' beliefs, and thereby their votes, through choosing the amount of unbiased information to make available. The first is statistical: limiting information limits updating. The second arises from strategic voting. Conditioning on pivotality is itself informative, because the amount of available information bounds what any voter could have learned. The statistical effect is likely to be more robust to different information environments; the strategic voting effect is deeper, and is closely connected to the fact that, in practice, the amount of available information is always finite.

The key to the statistical effect is that noise is not bias, but control over the amount of noise can introduce bias. The amount of information available to voters determines the likelihood that their beliefs cross a threshold, above which a proposal is in their interests. This effect is asymmetric. The sender gains from moving beliefs in her favored direction on a good draw, but not from moving them only when the state truly warrants it. As in Bayesian persuasion, her aim is to pool unfavorable states with favorable ones, up to the point at which further pooling would leave even a favorable draw short of the threshold. Noise is a blunter instrument than signal design, but it achieves some of the same effect, and the sender's first-mover advantage lets her choose how much.

The strategic voting effect is about more than noise; finite amounts of information play a central role. With infinitely many signals, voters' samples would be conditionally independent given the state. Finiteness opens the possibility of reasoning about signals that a voter has not seen, beyond what the voter's own sample conveys, because it destroys conditional independence.

It is natural to ask whether mandating disclosure can mitigate the bias a sender induces, and we conclude with some remarks on that question. In the case of an ex ante favorable proposal, the answer is clearly yes: Proposition 1 establishes that a sender in this case prefers to disclose nothing. The voters are all ready to back her preferred decision, giving her nothing to gain from bringing out a magnifying glass. Forcing disclosure when none would otherwise be forthcoming trivially leads to more disclosure, which weakly increases the probability an ex ante favorable policy gets rejected.

The less trivial case is the one we focus on here, in which the proposal is ex ante unfavorable. A sender who chooses how much information to make available manipulates voters by motivating them to become informed. She wants them to make mistakes that go in her favor, and therefore needs them to sample. Mandating a particular amount of disclosure, or a floor, can remove the incentive to sample entirely (see Remark 2). Changing the number of available signals may make pivotality determine the population's aggregate information uniquely, leaving a voter nothing to learn from his own draw. The same thing happens, less abruptly, as the amount of disclosure grows: the difference between what a favorable and an unfavorable sample tell a pivotal voter shrinks until it is too small to change how he votes. Thus, for a proposal that is ex ante unfavorable, the mandate removes the information along with the manipulation: the sender's influence worked by inducing information acquisition that would not otherwise exist, and overriding her choice eliminates both. A floor or a requirement can effectively become a ban. Either way, no voter has reason to become informed, and the unfavorable prior then dooms the proposal.

# A Proofs

*Proof of Proposition 1.* The common prior on $\tilde{\theta}$ is uniform over [0,1], so with no additional information, shareholder $i$'s expectation if the proposal passes is

$$\frac{U+D}{2} \geq 0$$

so everyone voting for the proposal is optimal. Given no disclosure, as the shareholders all have the same information, the only other candidate for a pure strategy symmetric equilibrium has all shareholders voting against the proposal. If $U + D > 0$, then every shareholder is choosing a weakly dominated strategy, so this is subgame imperfect, and if $U + D = 0$, this equilibrium violates the assumed tie-breaking rule.

As there are at least three shareholders, no shareholder is pivotal, and therefore no individual can benefit from deviating and voting against the proposal or from deviating and gathering information. Thus, the manager's choice of $N$ is irrelevant, and in particular choosing $N = 0$ is optimal (as no one observes any of the signals). □

*Proof of Lemma 1.* The shareholders all have the same prior and objective, and by hypothesis they all have the same posterior. Therefore, they will vote the same way, so no shareholder benefits from deviating. The reason for the cutoff is the same as in Remark 1. □

*Proof of Lemma 2.* From (6),

$$\begin{aligned}\sigma^*(n+1) &= \left\lceil -\frac{U+(n+2)D}{U-D} \right\rceil \\ &= \left\lceil -\frac{U+(n+1)D}{U-D} + \frac{-D}{U-D} \right\rceil\end{aligned}$$

The second term is positive, so $\sigma^*(n+1) \geq \sigma^*(n)$, i.e., $\sigma^*(\cdot)$ is weakly monotone. Moreover,

$$\frac{-D}{U-D} = \frac{|D|}{U+|D|} \in \left(\frac{1}{2}, 1\right)$$

because $-D < 0 < U < |D|$. Therefore,

$$\begin{aligned}\sigma^*(n+1) &= \left\lceil -\frac{U+(n+1)D}{U-D} + \frac{-D}{U-D} \right\rceil \\ &\leq \underbrace{\left\lceil -\frac{U+(n+1)D}{U-D} \right\rceil}_{\sigma^*(n)} + \underbrace{\left\lceil \frac{-D}{U-D} \right\rceil}_{1}\end{aligned}$$

and

$$\sigma^*(n+2) = \left\lceil -\frac{U+(n+1)D}{U-D} + \frac{-2D}{U-D} \right\rceil,$$

where the last term is strictly greater than 1. □

*Proof of Theorem 1.* Lemma 1 implies we can consider a representative shareholder, and that the shareholder is willing to sample all the signals that the manager releases.

Note that $\sigma^*(n_0 - 1) > n_0 - 1$. Therefore, by Lemma 2, $\sigma^*(n_0) \geq \sigma^*(n_0 - 1) \geq n_0$, and by hypothesis $\sigma^*(n_0) \leq n_0$.

The expected payoff to the shareholder from processing $n \in \mathbb{Z}_+$ signals is

$$\begin{aligned} &Pr(\sigma \geq \sigma^*(n)|n \text{ signals drawn}) \cdot E[\max\{0, E[\tilde{v}]\}|n \text{ signals drawn}] \\ = &\sum_{\sigma=\sigma^*(n)}^{n} \underbrace{Pr(\sigma|n \text{ signals drawn})}_{1/(n+1)} \frac{(\sigma+1)U + D}{n+2} \end{aligned} \tag{18}$$

Conjecture that the manager chooses $N = n_0$ signals. If the shareholders choose to observe $n \in \{0, \ldots, n_0 - 1\}$ signals, they always gets a payoff of 0. If the shareholders choose $n = n_0$, their expected payoff is, from (18),

$$\left(\frac{1}{n_0+1}\right)\left(\frac{(n_0+1)U + D}{n_0+2}\right)$$

For the manager, choosing $N < n_0$ gives zero probability of the proposal being approved, so the manager prefers to choose at least $n_0$, and having the proposal approved with probability (from the Uniform-Binomial prior)

$$\frac{1}{n_0+1} \tag{19}$$

To show that the manager cannot improve on this, we first note that, for any $n \geq n_0$, if $\sigma^*(n+1) = \sigma^*(n) + 1$, then having the shareholder process $n+1$ signals cannot be optimal. This follows because

$$Pr(\sigma \geq \sigma^*(n)|n \text{ signals drawn}) = 1 - \frac{\sigma^*(n)}{n+1}$$

Therefore, if $\sigma^*(n+1) = \sigma^*(n) + 1$,

$$\begin{aligned} Pr(\sigma \geq \sigma^*(n)|n \text{ signals drawn}) - Pr(\sigma \geq \sigma^*(n+1)|n+1 \text{ signals drawn}) \\ = 1 - \frac{\sigma^*(n)}{n+1} - \left(1 - \frac{\sigma^*(n)+1}{n+2}\right) \\ = \frac{n+1-\sigma^*(n)}{(n+1)(n+2)} > 0 \end{aligned}$$

where the last inequality follows from Lemma 2.

Also from Lemma 2, the sequence $\sigma^*(n)$ is weakly increasing in $n$, with each term equal to either the previous term or one plus the previous term. Let $k_0$ be the first $k \in \mathbb{Z}_{++}$ such that $\sigma^*(n_0 + k_0) = \sigma^*(n_0 + k_0 + 1)$. Then the probability of the shareholder approving the proposal is strictly decreasing from $n_0$ to $n_0 + k_0 - 1$, so the manager would prefer choosing $N = n_0$ to choosing a higher value of $N$ if the shareholder would choose $n \in \{n_0 + 1, \ldots, n_0 + k_0 - 1\}$.

Lemma 2 implies that $\sigma^*(n_0 + k_0) = n_0 + k_0 - 1$, so the probability that the shareholder would approve the proposal after having processed $n_0 + k_0$ signals is

$$1 - \frac{\sigma^*(n_0 + k_0)}{n_0 + k_0 + 1} = 1 - \frac{n_0 + k_0 - 1}{n_0 + k_0 + 1} = \frac{2}{n_0 + k_0 + 1} \tag{20}$$

The manager weakly prefers the shareholder to process $n_0$ signals rather than $n_0 + k_0$ signals if and only if (19) is larger than (20). The difference in probability of accepting the proposal is

$$\frac{1}{n_0 + 1} - \frac{2}{n_0 + k_0 + 1}$$

which is nonnegative if and only if $k_0 \geq n_0 + 1$. We therefore want to establish that $k_0$ must be greater than $n_0$.

First note that if the shareholder were to observe $n_0 - 1$ signals and all were successes, the posterior of $\tilde{\theta}$ would be Beta$(n_0, 1)$ with mean $n_0/(n_0 + 1)$ and the expectation of $\tilde{v}$ would be negative (by the definition of $n_0$). Therefore, it must be the case that

$$\frac{n_0}{n_0 + 1} U + \frac{1}{n_0 + 1} D < 0$$

Next, suppose that $k_0 \leq n_0$. The posterior mean of $\tilde{\theta}$ after observing $n_0 + k_0$ signals with one failure and the rest successes would be Beta$(n_0 + k_0, 2)$ with mean $(n_0 + k_0)/(n_0 + k_0 + 2)$. The posterior mean is strictly increasing in $k_0$, so it is at most

$$\frac{2n_0}{2n_0 + 2} = \frac{n_0}{n_0 + 1},$$

and as we have just seen, the expectation of $\tilde{v}$ at this posterior mean is negative. Therefore, $k_0$ must be at least $n_0 + 1$.

To finish the proof, we use induction on the values of $n$ where $\sigma^*(n) = \sigma^*(n - 1)$. For any $t \in \mathbb{Z}_{++}$, let

$$n_t = n_{t-1} + k_{t-1}$$

and let $k_t$ be the first $k \in \mathbb{Z}_{++}$ with $\sigma^*(n_t + k_t) = \sigma^*(n_t + k_t - 1)$. For each $t$, the manager is better off if the shareholder chooses $n_t$ than any $n \in \{n_t + 1, \ldots, n_t + k_t - 1\}$. Observe that $\sigma^*(n_t) = n_t - t$ and $\sigma^*(n_t + k + t) = n_t - t - 1$. By an analogous argument to the case for $t = 0$ above, it follows that the manager weakly prefers having the shareholder draw $n_t$ to $n_t + k_t$ signals if and only if

$$\frac{t+1}{n_t+1} \geq \frac{t+2}{n_t+k_t+1}$$

which holds if and only if

$$k_t \geq \frac{n_t+1}{t+1} = \frac{n_{t-1}+k_{t-1}+1}{t+1}$$

As we show above, for $t = 0$, $k_0 \geq n_0 + 1$. For $t = 1$, we have

$$k_1 \geq \frac{n_0+k_0+1}{2} \geq \frac{n_0+(n_0+1)+1}{2} = n_0 + 1$$

and for larger values of $t$, the argument is similar, and the rest follows the argument above for the $k_0$ case. □

*Proof of Corollary 1.* The probability that a shareholder approves the proposal after observing $n_0$ signals is $1/(n_0 + 1)$. Consider an shareholder who draws $n_0 - 1$ signals. By the definition of $n_0$, the expected net present value of $\tilde{v}$ would be strictly negative even if all were successes. The posterior of $\tilde{\theta}$ in this case is $\text{Beta}(n_0, 1)$, so

$$\begin{aligned}
&\frac{n_0 U + D}{n_0+1} < 0 \\
&\quad \Rightarrow\ n_0 < \frac{-D}{U}, \text{ so } n_0 + 1 < \frac{U-D}{U} \\
\Rightarrow\ &\frac{1}{n_0+1} > \frac{U}{U-D}
\end{aligned}$$

By comparison, the known-probability benchmark would be to approve the proposal if and only if

$$\tilde{\theta}U + (1-\tilde{\theta})D \geq 0 \quad \Leftrightarrow \quad \tilde{\theta} \geq \frac{-D}{U-D}$$

which occurs, by the uniform prior, with probability

$$1 - \frac{-D}{U-D} = \frac{U}{U-D}$$

Therefore, the approval probability given $N = n = n_0$ is strictly greater than the known-probability benchmark. □

*Proof of Proposition 2.* First, note that the posterior of $\tilde{\theta}$ is increasing in the number of successes a shareholder observes, so we can restrict attention to threshold strategies. Let $\underline{n}$ be the threshold, so that each shareholder votes for the proposal if and only if $\underline{n}$ of the shareholder's signals are successes.

For an individual shareholder $i$ to be pivotal, $m$ of the other shareholders must observe at least $\underline{n}$ successes, so at least $\underline{n}$ of the $N$ signals in the population must be successes. Similarly, the remaining $m$ of the other shareholders must observe at most $\underline{n}-1$ successes, so at least $n - \underline{n} + 1$ of the signals in the population must be failures.

Thus, pivotality restricts the possible values of $\underline{n} + n - \underline{n} + 1 = n + 1$ of the signals, leaving $N - n - 1$ degrees of freedom in the population. Therefore, if $N = n$, $i$ cannot be pivotal, and if $n = N - 1$, there are zero degrees of freedom left in the population, so $i$ cannot learn anything from sampling. In both of these cases, $i$ optimally deviates by not sampling, saving the processing cost without changing the outcome.

Next, observe that if the shareholders coordinate on observing the same set of $n$ signals, then they will have a common posterior and will vote the same way. In this case, $i$ cannot be pivotal, so $i$ optimally deviates by not sampling. Thus, if $n > 0$, each shareholder must ignore at least two of the signals, and the shareholders cannot pool their information. □

*Proof of Proposition 3.* Consider the numerator of (9) first. For each value of $k$, this value is strictly below 1, so as $m$ increases, the numerator gets closer to zero exponentially fast. The denominator rescales, so that the sum of (9) over all $k$ remains 1. In the limit as $m \to \infty$, the probability of $k$ given pivotality becomes concentrated on

$$\operatorname*{argmax}_k F_{n,k,N}(n-h-1)[1 - F_{n,k,N}(n-h-1)]$$

For any $x \in (0,1)$, the product of $x(1-x)$ has a unique global maximum at $x = 1/2$, and decreases symmetrically as $x$ moves away from $1/2$. □

*Proof of Corollary 2.* From Proposition 3, as $m \to \infty$, the posterior probability of $k$ is zero except where

$$\left| F_{n,k,N}(n-1) - \frac{1}{2} \right|$$

is minimized.

Given that each shareholder draws a sample of size $n$, the cumulative distribution of $n-1$ successes simplifies to $1 - Pr(n \text{ successes})$, so rewrite $F_{n,k,N}(n-1)$ as

$$1 - \frac{\binom{k}{n}\binom{N-k}{0}}{\binom{N}{n}} = \frac{\binom{N}{n} - \binom{k}{n}}{\binom{N}{n}}$$

and $1 - F_{n,k,N}(n-1)$ as $\binom{k}{n}/\binom{N}{n}$. Then (8) becomes

$$\frac{\binom{2m}{m}}{\binom{N}{n}^{2m}} \left[ \left( \binom{N}{n} - \binom{k}{n} \right) \binom{k}{n} \right]^m,$$

which is maximized where $\binom{N}{n}$ is as close as possible to $2\binom{k}{n}$. Note that

$$\binom{N}{n} - 2\binom{k}{n} \tag{21}$$

is strictly decreasing in $k$. At $k = n$, this difference is strictly positive, and at $k = N$, this difference is strictly negative. Therefore, there is a unique largest value $\underline{k}$ for which (21) is positive and a unique smallest $\bar{k}$ for which (21) is negative, and it must be the case that $\bar{k} - \underline{k} = 1$.

If

$$\binom{N}{n} - 2\binom{\underline{k}}{n} \neq 2\binom{\bar{k}}{n} - \binom{N}{n},$$

then there is a unique maximizer $k^*$ of $\left|\binom{N}{n} - 2\binom{k}{n}\right|$, either $\underline{k}$ or $\bar{k}$, whichever is closer to $\binom{N}{n}/2$. By Proposition 3, as $m \to \infty$, $Pr(k = k^*|i \text{ pivotal}) \to 1$.

Otherwise, denote $\bar{k}$ by $k^*$, so that $\underline{k} = k^* - 1$. Then

$$\begin{aligned}
\binom{N}{n} - 2\binom{k^*-1}{n} &= 2\binom{k}{n} - \binom{N}{n} \\
\Rightarrow 2\binom{N}{n} &= 2\binom{k^*}{n} + 2\binom{k^*-1}{n} \\
\Rightarrow \binom{N}{n} &= \binom{k^*}{n} + \binom{k^*-1}{n}
\end{aligned}$$

For each $m$, the value of (8) is equal for $k^*$ and $k^* - 1$, so in the limit they must both have probability 1/2. $\square$

*Proof of Proposition 4.* Rewrite (11) as follows:

$$\frac{N!}{\cancel{n}!(N-n)!} = \frac{k^*!}{\cancel{n}!(k^*-n)!} + \frac{(k^*-1)!}{\cancel{n}!(k^*-n-1)!},$$

so that

$$\begin{aligned}
&N(N-1)\cdots(N-n+1) \\
&\qquad = k^*(k^*-1)\cdots(k^*-n+1) + (k^*-1)\cdots(k^*-n+1)(k^*-n) \\
&\qquad\qquad = (2k^*-n)(k^*-1)\cdots(k^*-n+1), \qquad (22)
\end{aligned}$$

which is a $n^{\text{th}}$-order polynomial in $k$ (or alternatively in $N$). A solution to (11) is therefore an integer solution to (22); i.e., (22) is a Diophantine equation.

For $n = 1$, (22) simplifies to

$$N = 2k^* - 1 \quad \Rightarrow \quad k^* = \frac{N+1}{2}$$

Given that $N \geq 3$ in any informative equilibrium with costly processing, it follows that there is an integer solution if and only if $N$ is odd.

For $n = 2$, (22) requires a solution to the quadratic Diophantine equation

$$N(N-1) = k^*(k^*-1) + (k^*-1)(k^*-2) = 2(k^*-1)^2$$

The smallest solution for $N \geq 3$ is at $N = 9$ and $k^* = 7$. The next-smallest solution is at $N = 50$ and $k^* = 36$.

For $k^* = N - 1$, (22) requires a solution to the quadratic Diophantine equation

$$N(N-1) = (2N-2-n)(N-n) \quad \Leftrightarrow \quad N^2 - (1+3n)N + n(n+2) = 0$$

The smallest solution for $N \geq 3$ is at $N = 16$ and $n = 6$. The next-smallest solution is at $N = 105$ and $n = 40$. □

*Proof of Corollary 3.* We proceed in steps.

**Step 1: Pell equation and fundamental unit** Define $x := 2N - 1$ and $y := k^* - 1$. Then (12) is

$$x^2 - 8y^2 = 1,$$

which is a Pell equation. On the other hand, substituting $n = 2$ into (11) and rearranging gives us

$$N^2 - N - 2(k^* - 1)^2 = 0$$

To see that these are equivalent, multiply by 4 and add 1 to both sides:

$$\overbrace{(4N^2 - 4N + 1)}^{(2N-1)^2} - 8(k^* - 1)^2 = 1$$

which is (12).

Decompose the Pell equation as

$$(x + 2\sqrt{2}y)(x - 2\sqrt{2}y) = 1 \tag{23}$$

By a standard result in number theory (e.g., Theorem 7.26 of Niven, Zuckerman, and Montgomery, 1991, p. 354), if $(x_1, y_1)$ is the smallest strictly positive integer solution of the Pell equation, then the positive integer powers of the positive root $(x_1 + 2\sqrt{2}y_1)$ provide all the solutions. That is, for $t \in \mathbb{Z}_{++}$, $(x_1 + 2\sqrt{2}y_1)^t = (x_t + 2\sqrt{2}y_t)$.

The smallest possible positive value for $y_1$ is 1, which would give

$$x_1^2 - 8 = 1 \;\Rightarrow\; x_1 = 3$$

Thus, $(x_1, y_1) = (3, 1)$, and the expression

$$3 + 2\sqrt{2}$$

is called the *fundamental unit* of the Pell equation. We note that there is also a degenerate solution at $(x_0, y_0) = (1, 0)$.

**Step 2: Recursion** We recover $N_t$ as $(x_t + 1)/2$ and $k_t^*$ as $y_t + 1$. Then $k_0^* = 1$ and $k_1^* = 2$ give the initial conditions in the recursion that we now derive.

Write

$$\begin{aligned}(3 + 2\sqrt{2})^{t+1} &= (3 + 2\sqrt{2})(3 + 2\sqrt{2})^t \\ &= (3 + 2\sqrt{2})(x_t + 2\sqrt{2}y_t) \\ &= 3x_t + 8y_t + 2\sqrt{2}(x_t + 3y_t)\end{aligned}$$

Thus,

$$x_{t+1} = 3x_t + 8y_t \tag{24}$$

$$y_{t+1} = x_t + 3y_t \tag{25}$$

Rearranging,

$$x_t = y_{t+1} - 3y_t,$$

so (24) becomes

$$y_{t+2} - 3y_{t+1} = 3y_{t+1} - y_t \;\Rightarrow\; y_{t+2} = 6y_{t+1} - y_t$$

Rewriting in terms of $k_t^*$ gives

$$k_{t+2}^* - 1 = 6k_{t+1}^* - 6 - k_t^* + 1 \;\Rightarrow\; k_{t+2}^* = 6k_{t+1}^* - k_t^* - 4$$

Note that an analogous argument gives the same recursion in $x_t$:

$$x_{t+2} = 6x_{t+1} - x_t,$$

so we also have a recursion for $N_t$:

$$N_{t+2} = 6N_{t+1} - N_t - 2$$

**Step 3: Limiting ratio** We now show that $(k_t^* - 1)/N_t$ and $k_t^*/N_t$ converge to $\sqrt{2}/2$.

Let $u := 3 + 2\sqrt{2}$ be the fundamental unit, and let $\bar{u} := 3 - 2\sqrt{2}$ be its conjugate. Recall $u$ has the property that $u^t = x_t + 2\sqrt{2}y_t$. By the binomial theorem,

$$u^t = \sum_{s=0}^{t} \binom{t}{s} 3^{t-s}(2\sqrt{2})^s \text{ and } \bar{u}^t = \sum_{s=0}^{t} \binom{t}{s} 3^{t-s}(-2\sqrt{2})^s,$$

so the terms with even powers of $2\sqrt{2}$ are the same in both expressions (and are integers), and the terms with odd powers of $2\sqrt{2}$ are even integer multiples of $\sqrt{2}$ with opposite signs. Hence, $\bar{u}^t = x_t - 2\sqrt{2}y_t$. Collecting these results, we have

$$\begin{aligned}
u^t &= x_t + 2\sqrt{2}y_t \\
\bar{u}^t &= x_t - 2\sqrt{2}y_t \\
\Rightarrow & \\
x_t &= \frac{u^t + \bar{u}^t}{2} \\
y_t &= \frac{u^t - \bar{u}^t}{4\sqrt{2}}
\end{aligned}$$

Also, observe that $\bar{u} \approx 3 - 2.828 = 0.172 < 1$. The ratio $y_t/x_t$ is

$$\left(\frac{1}{2\sqrt{2}}\right)\left(\frac{(3+2\sqrt{2})^t - (3-2\sqrt{2})^t}{(3+2\sqrt{2})^t + (3-2\sqrt{2})^t}\right)$$

Dividing upstairs and downstairs by $(3 + 2\sqrt{2})^t$, the ratio becomes

$$\left(\frac{1}{2\sqrt{2}}\right)\frac{1 - \left(\dfrac{3-2\sqrt{2}}{3+2\sqrt{2}}\right)^t}{1 + \left(\dfrac{3-2\sqrt{2}}{3+2\sqrt{2}}\right)^t}$$

As $t \to \infty$, this ratio converges to $1/(2\sqrt{2})$. Substituting $k_t^* - 1$ for $y_t$ and $2N_t - 1$ for $x_t$, we obtain

$$\lim_{t\to\infty} \frac{k_t^* - 1}{2N_t - 1} = \frac{1}{2\sqrt{2}}$$

The limit depends only on the leading coefficients in the ratio, and not on the constant in the numerator or denominator. Thus, this is the same limit as for $(k_t^* - 1)/2N_t$ and for $k_t^*/2N_t$. Multiplying by 2, we get

$$\lim_{t\to\infty} \frac{k_t^* - 1}{N_t} = \lim_{t\to\infty} \frac{k_t^*}{N_t} = \frac{1}{\sqrt{2}}$$

as desired.

**Step 4: Direction of convergence** Finally, we need to establish that $k_t^*/N_t$ strictly decreases to this limit and that $(k_t^* - 1)/N_t$ strictly increases to this limit. We require that

$$\frac{k_{t+1}^*}{N_{t+1}} < \frac{k_t^*}{N_t}$$

Substituting, the requirement becomes

$$\frac{(y_{t+1} + 1)}{x_{t+1} + 1} < \frac{(y_t + 1)}{x_t + 1}$$

The denominators are both positive, so we can multiply by $(x_t + 1)(x_{t+1} + 1)$:

$$\begin{aligned}(y_{t+1} + 1)(x_t + 1) &< (y_t + 1)(x_{t+1} + 1)\\ x_t y_{t+1} + x_t + y_{t+1} + 1 &< x_{t+1} y_t + x_{t+1} + y_t + 1\end{aligned}$$

Substituting (24) and (25) and subtracting 1 from both sides,

$$\begin{aligned} x_t(x_t + 3y_t) + x_t + x_t + 3y_t &< (3x_t + 8y_t)y_t + 3x_t + 8y_t + y_t\\ \Rightarrow\ x_t^2 + 3x_t y_t + 2x_t + 3y_t &< 3x_t y_t + 3x_t + 9y_t + 8y_t^2\\ \Rightarrow\ x_t^2 - 8y_t^2 &< x_t + 6y_t\end{aligned}$$

The left-hand side is the Pell equation (12), so it equals 1. The right hand side is the sum of two strictly positive integers, so the inequality holds. The argument that $(k_t^* - 1)/N_t$ increases to the limit is analogous.

□

*Proof of Corollary 4.* In (11), let $k^* = N - 1$. The condition becomes

$$\binom{N}{n} = \binom{N-1}{n} + \binom{N-2}{n}$$

Multiply both sides by $n!$:

$$\frac{N!}{(N-n)!} = \frac{(N-1)!}{(N-n-1)!} + \frac{(N-2)!}{(N-n-2)!}$$

Multiply both sides by $(N-n)!$:

$$N! = (N-1)!(N-n) + (N-2)!(N-n)(N-n-1)$$

Divide both sides by $(N-2)!$:

$$N(N-1) = (N-1)(N-n)+(N-n)(N-n-1) = N(N-1)-n(N-1)+(N-n)(N-n-1)$$

Subtract $N(N-1)-n(N-1)$ from both sides:

$$n(N-1) = (N-n)(N-n-1),$$

which is the equation that Anning and Underwood (1931) solve. $\square$

*Proof of Proposition 5.* From Proposition 4, for large enough $m$ pivotality implies $Pr(k^*) = Pr(k^*-1) = 1/2$. Therefore,

$$\begin{aligned}
Pr(n \text{ successes}|\text{pivotal}) &= \frac{1}{2}\left[Pr(n \text{ successes}|\text{pivotal}, k^*)\right. \\
&\quad \left.+Pr(n \text{ successes}|\text{pivotal}, k^*-1)\right] \\
&= \left(\frac{1}{2}\right)\left(\frac{\binom{k^*}{n}+\binom{k^*-1}{n}}{\binom{N}{n}}\right) \\
&= \left(\frac{1}{2}\right)\left(\frac{\binom{N}{n}}{\binom{N}{n}}\right) \text{ by (11)} \\
&= \frac{1}{2}
\end{aligned}$$

$\square$

*Proof of Proposition 6.* Conditional on shareholder $i$ being pivotal and $k^*$ successes in the population of $N$ signals, the probability that $i$ draws $n$ successes and no failures is

$$Pr(n \text{ successes}|\text{pivotal}, k^*) = \frac{\binom{k^*}{n}\binom{N-k^*}{0}}{\binom{N}{n}} = \frac{k^*!(N-n)!}{N!(k^*-n)!}$$

Replacing $k^*$ with $k^*-1$,

$$Pr(n \text{ successes}|\text{pivotal}, k^*-1) = \frac{\binom{k^*-1}{n}\binom{N-k^*+1}{0}}{\binom{N}{n}} = \frac{(k^*-1)!(N-n)!}{N!(k^*-n-1)!}$$

The prior, conditional on pivotality, is that $Pr(k^*-1) = Pr(k^*) = 1/2$, so by Bayesian updating, the posterior probabilities are

$$\begin{aligned} Pr(k^* - 1 | n \text{ successes, pivotal}) &= \frac{k^* - n}{2k^* - n} \\ Pr(k^* | n \text{ successes, pivotal}) &= \frac{k^*}{2k^* - n} \end{aligned}$$

If there are $k^*-1$ successes in the population, then the posterior of $\tilde{\theta}$ is Beta$(k^*, N-k^*+2)$, in which case the expectation of $\tilde{v}$ is

$$\frac{k^*U + (N - k^* + 2)D}{N + 2}$$

If there are $k^*$ successes in the population, then the posterior of $\tilde{\theta}$ is Beta$(k^*+1, N-k^*+1)$, in which case the expectation of $\tilde{v}$ is

$$\frac{(k^* + 1)U + (N - k^* + 1)D}{N + 2}$$

Combining these, the expectation of $\tilde{v}$ given $n$ successes and no failures and assuming pivotality is

$$\left(\frac{k^* - n}{2k^* - n}\right)\left(\frac{k^*U + (N - k^* + 2)D}{N + 2}\right) \\ + \left(\frac{k^*}{2k^* - n}\right)\left(\frac{(k^* + 1)U + (N - k^* + 1)D}{N + 2}\right),$$

which simplifies to

$$E[\tilde{v}|\text{all } n \text{ successes, pivotal}] = \\ \frac{[-2k^{*2} + (2N + n + 3)k^* - (N + 2)n]D + [k^*(2k^* - n + 1)]U}{(N + 2)(2k^* - n)}$$

The posterior expectation of $\tilde{v}$ is nonnegative if and only if the numerator is nonnegative, which is inequality (14).

Next, suppose the shareholder observes $n - 1$ successes and 1 failure. Conditional on $k^*$ population successes, this occurs with probability

$$\frac{\binom{k^*}{n-1}(N - k^*)}{\binom{N}{n}},$$

and conditional on $k^* - 1$ population successes, this occurs with probability

$$\frac{\binom{k^*-1}{n-1}(N - k^* + 1)}{\binom{N}{n}}$$

The posterior probabilities are then

$$
\begin{aligned}
&Pr(k^*|n-1 \text{ successes, pivotal}) \\
&\qquad = \frac{-k^{*2} + Nk^*}{-2k^{*2} + (2N+n)k^* - (N+1)(n-1)} \\
&Pr(k^*-1|n-1 \text{ successes, pivotal}) \\
&\qquad = \frac{-k^{*2} + (N+n)k^* - (N+1)(n-1)}{-2k^{*2} + (2N+n)k^* - (N+1)(n-1)}
\end{aligned}
$$

Let $\eta = (N+2)[-2k^{*2} + (2N+n)k^* - (N+1)(n-1)]$. Then the posterior expectation of $\tilde{v}$ given $n-1$ successes and one failure is $1/\eta$ times

$$
\begin{aligned}
&(-k^{*2} + Nk^*)[(k^*+1)U + (N-k^*+1)D] \\
&\quad + [-k^{*2} + (N+n)k^* - (N+1)(n-1)][k^*U + (N-k^*+2)D] \\
&\quad = [k^*(-2k^{*2} + (2N+n-1)k^* + 2N - Nn - n + 1)]U + \\
&[2k^{*3} - (4N+n+3)k^{*2} + (2N(N+n+1) + 3n - 1)k^* - (N+2)(N+1)(n-1)]D
\end{aligned}
$$

so the posterior expectation is negative if and only if the term in square brackets is negative, corresponding to (15). □

*Proof of Proposition 7.* From the proof of Proposition 6, the expectation $\psi(n,N)$ of $\tilde{v}$, conditional on a given shareholder being pivotal and voting yes is

$$
\begin{aligned}
&\psi(n,N) := E[\tilde{v}|\text{all } n \text{ successes, pivotal}] = \\
&\qquad \frac{[-2k^{*2} + (2N+n+3)k^* - (N+2)n]D + [k^*(2k^* - n + 1)]U}{(N+2)(2k^* - n)}
\end{aligned}
$$

under our assumption that $m$ is large enough to allow us to treat the conditional probability of $k$ as differing negligibly from having support $\{k^*-1, k^*\}$ (each with probability mass $1/2$). The shareholder's benefit from $n$ signals is then

$$
\underbrace{1/2}_{Pr(n \text{ successes}|\text{pivotal})} \underbrace{\frac{\binom{2m}{m}}{N+1} \sum_{k=n}^{N-1} \left[ \left(1 - \frac{\binom{k}{n}}{\binom{N}{n}}\right) \left(\frac{\binom{k}{n}}{\binom{N}{n}}\right) \right]^m}_{Pr(\text{pivotal})} \psi(n,N) - nc
$$

(approximately, as the probability of $n$ successes given pivotality is $1/2$ asymptotically). The shareholder is willing to participate by drawing a sample of size $n$ if this benefit is nonnegative; rearranging, we see that this holds if and only if

$$c \leq \frac{\binom{2m}{m}\psi(n,N)}{2(N+1)n} \sum_{k=n}^{N-1} \left[ \left(1 - \frac{\binom{k}{n}}{\binom{N}{n}}\right) \left(\frac{\binom{k}{n}}{\binom{N}{n}}\right) \right]^m$$

Next, suppose the shareholder deviates by drawing an additional $j$ signals, and that he would vote yes only if all $n+j$ signals in his sample are successes. From the proof of Proposition 6, the expectation of $\tilde{v}$ conditional on all $n+j$ signals being successes and the shareholder being pivotal exceeds $\psi(n,N)$ by

$$\delta(n,N;j) := \frac{jk^*(U-D)}{(N+2)(2k^*-n)(2k^*-n-j)},$$

which is strictly increasing and strictly concave in $j$. Therefore, to rule out this deviation, it is enough for the increase in expected payoff from choosing $n+1$ signals over $n$ is be less than $c$, as the probability of obtaining all successes is strictly decreasing in $j$. In particular, from a direct calculation, we obtain the probability of getting $(n+1)$ successes on $n+1$ tries, given pivotality and given that all others draw a sample of size $n$, as

$$\left(\frac{1}{2}\right)\left(\frac{2k^*-n-2}{2k^*-n-1}\right)\left(\frac{k^*-n-1}{N-n-1}\right)$$

Thus the shareholder will not draw more than $n$ signals and require all to be a success provided

$$c \leq \left(\frac{2k^*-n-2}{2k^*-n-1}\right)\left(\frac{k^*-n-1}{N-n-1}\right)\frac{\binom{2m}{m}\left[\psi(n,N)+\delta(n,N;1)\right]}{2(N+1)(n+1)} \sum_{k=n}^{N-1} \left[ \left(1 - \frac{\binom{k}{n}}{\binom{N}{n}}\right) \left(\frac{\binom{k}{n}}{\binom{N}{n}}\right) \right]^m$$

Consider now a shareholder who deviates from sampling $n$ to sampling $n+1$ signals. If the shareholder will vote yes regardless of the additional signal's outcome, then the additional signal is not decision-relevant, and the cost $c$ of the additional signal is unjustified. Therefore, we can assume that the shareholder who draws an additional signal will vote yes only if all $n+1$ signals are successes. In this case, the posterior probability of $k^*$ given $n+1$ successes increases from $k^*/(2k^*-n)$ to $k^*/(2k^*-n-1)$. This increases the benefit conditional on voting yes and being pivotal from $\psi(n, N)$ by

$$\begin{aligned}\big([-4k^{*3} &+ 4(2N+n+3)k^{*2} \\ &- (5N(2n+1) + n(14+n) + 6)k^* + (4+3N)n(n+1)]D \\ &+k^*[2(n+3)k^{*2} - (N(n-2)+5n-2)k^* - (N-n+2)(n+1)]U\big) \\ &\qquad /[(N+1)(N+2)(2k^*-n)(2k^*-n-1)]\end{aligned}$$

$\square$

*Proof of Theorem 2.* By construction. Let $U = 15$ and $D = -16$. Then $U + D < 0$ and $8U + 7D = 8 > 0$. Let $N^* = 3$ and $n^* = 1$. Equation (11) holds by Proposition 4. The threshold rule is incentive compatible by Proposition 6. Expression $\psi(1, 3)$ in Inequality (17) equals $2/3 > 0$, and all the other terms in the inequality are strictly positive, so there is always a $c$ small enough to satisfy (17); Proposition 7 therefore guarantees that the shareholders are willing to sample. By Proposition 2, the shareholders do not deviate to draw more than $1 = N^* - 2$ signals. For the probability of the proposal passing, observe from Table 1 that, for each $k \in \{0, 1, 2, 3\}$, the prior probability of there being $k$ successes in the population is $1/4$. Given $k$ successes, a shareholder votes yes with probability $k/3$, so the probabilities of voting yes given $k$ and given $3 - k$ sum to one. As the number of shareholders is odd, the probability the proposal passes given $k$ and the probability it passes given $3 - k$ likewise sum to one, so pairing $k = 0$ with $k = 3$ and $k = 1$ with $k = 2$ gives a passing probability of exactly $1/2$, for any number of shareholders. This exceeds the prior probability $U/(U - D) = 15/31$ of the proposal being positive ENPV.

The manager cannot benefit from changing $N$. If $n^*$ remains at 1, then increasing $N$ to an even number or reducing $N$ below 3 results in no sampling, guaranteeing that the proposal fails. If the manager increases $N$ to an odd number (maintaining the assumption that $n = 1$), the posterior expectation of $\tilde{v}$ given a success decreases, as the posterior success probability declines toward $1/2$, reducing (after some tedious algebra) to $(N + 1)^2/[2N(N + 2)]$. On the other hand, the posterior failure probability increases to $1/2$. This gives the shareholder $U = 15$ with decreasing probability and $D = -16$ with increasing probability, so that $E[\tilde{v}|1 \text{ success}]$ falls in $N$. In the case of $N = 5$, this expectation is $-2/35$, so with $N > 3$, the shareholder would prefer not sampling to drawing $n = 1$ signals.

Finally, we show the manager does not benefit from inducing the shareholders to increase their samples to $n > 1$ signals each. From the Uniform-Binomial assumption, for any given $N$ and for any $k \in \{0, \ldots, N\}$, the prior probability of having exactly $k$ population successes is $1/(N+1)$. For sufficiently large $m$,

$$\Pr(\text{Proposal passes } |k \text{ population successes}) = \begin{cases} 0, & \text{if } \Pr(\text{all } n \text{ successes}) < 1/2 \\ 1/2, & \text{if } \Pr(\text{all } n \text{ successes}) = 1/2 \\ 1, & \text{if } \Pr(\text{all } n \text{ successes}) > 1/2 \end{cases}$$

As all $k$ are equally likely, it is enough to show that the fraction of states in which the probability of drawing all $n$ successes is at least $1/2$ is decreasing in $n$ for every $N$.

For a given $k$, the probability that an individual shareholder draws all $n$ successes is given by the hypergeometric distribution:

$$\frac{\binom{k}{n}}{\binom{N}{n}} = \frac{k \cdot (k-1) \cdots (k-n+1)}{N \cdot (N-1) \cdots (N-n+1)}$$

Observe that, for $k < N$, $k/N > (k-1)/(N-1)$: multiplying both sides by the denominators, the left-hand side becomes $k(N-1) = Nk - k$, and the right-hand side becomes $(k-1)N = Nk - N$. So the probability of the shareholder drawing all $n$ successes is bounded above by

$$\left(\frac{k}{N}\right)^n,$$

which also holds for $k = N$.

This bound is at least $1/2$ if and only if $k/N \geq 2^{-1/n}$, so the number of values of $k$ for which the proposal passes is at most

$$N - \lceil N \cdot 2^{-1/n} \rceil + 1 \leq N + 1 - N2^{-1/n}$$

Each $k$ has prior probability $1/(N+1)$, so the prior probability of the proposal passing at a given $(n, N)$ is bounded above at

$$1 - \left(\frac{N}{N+1}\right) 2^{-1/n}$$

At $(n, N) = (2, 4)$, this is approximately 0.43, and is strictly decreasing in $n$ and in $N$. Thus, for any $n > 1$ and $N \geq n + 2$, the manager achieves a lower probability of the proposal passing than under the candidate equilibrium. $\square$

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