

Federalism, fiscal savings, and information asymmetries

Matthew Wilson

Department of Economics

University of Michigan

mgwilson@umich.edu

12/27/2018

Abstract

This paper seeks to characterize aspects of U.S. state-level fiscal policy over the business cycle. I identify three stylized facts: U.S. state governments engage in precautionary savings, states are less insured by the federal government for local downturns than for national downturns, and states whose cycles are more independent tend to build higher balances. These facts imply that the federal government's transfer policy is an important factor for state-level finances over the cycle. I interpret the facts in the context of a framework which models U.S. states as economies in a fiscal union. Regional governments are subject to balanced budget rules, but the central government receives a noisy signal about economic shocks. I calibrate the model to U.S. data, and show that it fits important features of the data well. Implied information frictions are large, suggesting a significant obstacle to the ability of central government policy alone to conduct fiscal policy over the cycle; I conclude that space exists for U.S. states to engage in active countercyclical fiscal policy, contrary to some conventional wisdom.

Keywords: fiscal policy, fiscal federalism, information asymmetries, business cycles, precautionary savings.

1 Introduction

In the United States, fiscal policy at the state level is an important component of the economic environment. Spending by subnational governments totaled 2.85 trillion dollars in 2016, representing almost 15 percent of the size of GDP; state government spending alone made up 28 percent of total government spending.¹ In addition, the makeup of state government spending is fundamentally different than that of the federal government. While federal government spending primarily constitutes defense, social security, and interest payments, state governments tend to spend primarily on education and public welfare (this includes Medicaid). Constraints faced by state governments differ substantially from the federal government as well; state governments interact with different tax bases than the federal government, and most face deficit limits of varying strengths.

In light of these observations, policy analysts should not expect state-level fiscal policy to behave like federal policy. One major way in which state governments differ from the U.S. government is their use of Budget Stabilization Funds, or “Rainy Day Funds”, to improve their ability to fund programs in times of fiscal distress. All fifty state make use of rainy day funds; in 2017, Montana became the fiftieth state to establish such a fund.² The median balance of these funds in 2016 was 477 million dollars, having built up significantly since their depletion after the Great Recession. The presence and size of these funds indicates that fiscal savings may be an important way in which U.S. states can enact fiscal policy over the business cycle, even in the presence of (sometimes strict) deficit limits.

Motivated by the idea of the potential presence of a savings motive, this paper seeks to answer three main questions. First, how do U.S. state governments respond to business cycles, and what factors are important in determining these responses? Second, upon characterizing some patterns of fiscal behavior, what sort of a model might be able to reproduce that behavior and make meaningful predictions? Finally, what are the welfare

¹BEA, 2018.

²<https://www.pewtrusts.org/en/research-and-analysis/articles/2018/08/29/states-make-more-progress-rebuilding-rainy-day-funds>

implications of various policies and frictions that might exist in such a model?

1.1 Overview of results

The broad results are as follows. I present three stylized facts about state public finances over the business cycle: states engage in precautionary savings, transfers from the federal government respond less to local shocks than national shocks, and states whose cycles are less correlated with the national cycle tend to save more. I obtain these facts using data on state government finances from two different sources, under four possible definitions of state government savings; the results are robust to the choice of definition for state government savings.

I interpret these facts as evidence that state government savings behavior is driven both by balanced budget rules and by the transfer policies of the federal government. To illustrate, I propose a model of a small open endowment economy in a fiscal union; i.e., an economy under both a regional government and a central government, much like a U.S. state. The regional government faces a debt limit, but the central government observes the state of the world with a noisy signal. The central government, therefore, is not able to fully insure the regional household against adverse shocks, and the regional government must build up a stock of savings.

I calibrate the model to U.S. data, and find that it fits qualitative features of the data quite well. The implied noise shock in the central government's signal is almost three times as large as the actual economic shocks, indicating significant frictions to optimal policy making at the centralized level. The baseline calibration of the model implies a 3.2 percent welfare loss relative to the social planner solution, indicating a sizeable influence of the frictions in the model on household utility. I conclude that information (or other political) frictions at the centralized level of policy making may create large deviations from the socially optimal policy; this implies states should consider actively pursuing countercyclical fiscal policies.

1.2 Literature and outline

This paper connects to a number of distinct strands of literature in economics research. On the empirical side, Hines Jr (2010) characterizes the behavior of state-level spending over the business cycle, arguing that small and large states behave differently in response to macroeconomic conditions. Nakamura and Steinsson (2014) estimate government spending multipliers for U.S. states, using military spending shocks from the central government. Owyang and Zubairy (2008) find heterogeneous effects of fiscal stimulus in different states and regions of the union, depending on regional makeup. In addition to these, Chodorow-Reich (2017) has a nice summary of findings from empirical literature on the effects of fiscal policy at subnational levels.

In the public finance literature, economies with fiscal federalism have been much studied (Oates, 2008). This literature tends to compare public goods provision at the local level to that at the central level. These models are static, however, and do not say much about cyclical policy. Furthermore, they tend to compare two types of public spending rather than having both local and central governments spending at the same time. For the purposes of this model, I abstract from the choice of public goods and study the optimal taxation behavior of states subject to an exogenous stream of public goods. This is the approach taken by optimal fiscal policy papers such as Schmitt-Grohe and Uribe (2004), Chari and Kehoe (1999), and Bhandari et al. (2017).

This paper also draws on the precautionary savings under credit constraints literature. Aiyagari (1994) is a key early example of this literature. Additionally, Durdu, Mendoza, and Terrones (2009) analyze precautionary savings explicitly in a small open economy, which is the approach I take. Bhandari et al. (2017) consider explicitly fiscal policy for a credit constrained government with access to imperfect markets. I contribute to this literature by considering the problem of a government conducting fiscal policy under debt limits, but in a small open economy when there is a “higher” level of government which is also conducting policy, i.e., in a fiscal union or federation.

Models in which a central government might not have the same access to information as local governments have been explored in other contexts. Bordignon, Manasse, and Tabellini (2001), for example, consider optimal redistribution policy when information is asymmetric. Silva and Cornes (2000) examine information asymmetries in the context of interregional transfers and public goods provision. In his survey of the future of the fiscal federalism literature, (Oates, 2005) also mentions information asymmetries between the different levels of government as a feature of federal systems. This paper contributes to the information asymmetries literature in fiscal federalism by applying the idea to a dynamic model of fiscal policy.

The rest of the paper proceeds as follows. Section 2 presents the main stylized facts of the paper. Section 3 introduces the information model with which the stylized facts are interpreted. Section 4 calibrates, analyzes, and interprets the model. Section 5 concludes.

2 State government savings: three stylized facts

This section lays out three stylized facts apparent in the data on U.S. state government savings. First, state governments overwhelmingly engage in precautionary savings: savings are positive and procyclical. Second, transfer receipts from the federal government are countercyclical, but depend more on the aggregate U.S. economy than on a state's idiosyncratic business cycle. Finally, states whose business cycles are less correlated with the national business cycle tend to save more than states experiencing fluctuations more in step with the aggregate cycle. This section first describes the data sources and definitions; the second subsection presents the three facts.

2.1 Data and descriptions

2.1.1 Data sources

Several sources are used to assemble the data for this part of the paper. Data on rainy day funds and end-period balances for state governments are obtained from the National Association of State Budget Officers' - hereafter, NASBO - "Fiscal Survey of the States." I use the spring edition of this semiannual report from 1979 to 2017 to obtain data from previous years which is self-reported by states and collected by NASBO. Due to heterogeneity in the structure of BSFs, some state governments do not report BSF balances separately from end-year balances, rendering analysis of rainy day funds alone a bit hairy; I discuss this below when considering all possible definitions of 'savings.' A fuller explanation of this procedure is given in Appendix [A](#).

Data on state government revenues, spending, and debt holdings comes from the U.S. Census Bureau's "Census of Governments." While the full sample of local governments is only administered every five years, all state governments are included in the limited survey taken every year, such that yearly observations from 1970 to 2012 are available for every state. Other state variables of interest are provided at the yearly level on the website of the University of Kentucky's Center for Poverty Research. I estimate state-level recession dates using the Philadelphia Fed's state coincident index. National price level indices are obtained from FRED.

2.1.2 Definition of savings

In order to study the cyclical behavior of state government savings, some definition of 'savings' is naturally required. Four potential definitions are available in the data; I choose to focus on a couple of them for ease of exposition. The first obvious definition of state government savings is the balance of the state's rainy day fund as reported to NASBO. While some amount of heterogeneity exists across funds, and not all states report their RDF balance sep-

arately from their general fund, budget stabilization funds are a useful metric due to their explicit purpose of preparation for adverse shocks. A second, and slightly more expansive, definition includes all end-year balances in a state's general fund; while such a measure will include unplanned revenue and spending shocks, it captures all rainy day fund activity and provides a consistent measure across states.

While the first two potential measures are taken from the NASBO reports, the other two are found in the U.S. Census Bureau's annual Census of Governments dataset. The third potential measure of state government savings is a state's net assets-cash and securities less debt outstanding-not including assets set aside for insurance purposes (pensions, etc.). The fourth measure is all of a state's net assets, including those in insurance-type funds.

My preferred measures of state government savings are measures two and three. These measures, total balances in general funds (including rainy day funds) and net non-insurance assets, provide a nice balance between the ideal features of a savings measurement. They are consistent across states, relatively general, and include a good deal of long-term savings components. Importantly, however, the qualitative results are not altered by the choice of savings measure.

2.2 Three stylized facts

2.2.1 Fact 1: State savings are positive and procyclical

The first stylized fact I identify is the presence of positive and procyclical savings behavior on the part of state governments. Regardless of which measure of savings measure is observed, U.S. states mostly run positive balances. This is not in itself a surprising result; in fact, it is exactly what one might expect given the balanced budget requirement imposed on 49 of the 50 U.S. states.³ Table 1 presents summary statistics for the savings measures of interest, both as a fraction of gross state product and as a fraction of general current

³NCSL, 2010.

Table 1: Measures of state government savings: summary statistics

Savings measure	Mean	Variance	Percentiles		
			50th	10th	90th
BSF over GSP	0.0038	0.0003	0.0012	0	0.0040
Gen. fund balance over GSP	0.0051	0.0003	0.0026	0.0002	0.0083
Net noninsurance assets over GSP	0.0427	0.0139	0.0175	-0.0258	0.1060
Net total assets over GSP	0.1763	0.0208	0.1600	0.0594	0.2823
BSF over expenditures	0.0228	0.0066	0.0102	0	0.0371
Gen. fund balance over expenditures	0.0379	0.0063	0.0242	0.0013	0.0776
Net noninsurance assets over expenditures	0.3367	0.5174	0.1604	-0.2329	0.9355
Net total assets over expenditures	1.5531	0.7419	1.4838	0.6304	2.4441

Note: Moments reported here are over all state-year observations. Data on budget stabilization funds and general fund balance come from the NASBO fiscal survey of the states, data on net assets come from the Census of Governments, and gross state products are obtained from UKCPR. Sample periods are as follows: BSFs from 1985-2016, balances from 1979-2016, both net assets series from 1981-2012.

state government expenditures. Clearly, states run positive levels of savings-0.5 percent or 4.3 percent, depending on the definition-on average, although some observations do record negative savings levels.

In addition to being overwhelmingly positive, state government savings also moves with the business cycle. Figures 1, 2, 3, and 5 show how various percentiles of the distribution of savings across states move over the business cycle for the four measures of state savings, where the shaded regions indicate NBER recession dates. Clearly, savings balances build up in economic expansions and spend down in recessions; this is consistent with the stated purpose of RDFs, which are included in these measures. I interpret this behavior as being indicative of a precautionary savings motive on the part of state governments, induced by the presence of balanced budget rules and the desire of policy makers to smooth expenditures over the cycle.

As a supplemental example, I also plot the series of Kentucky’s balances over GSP alongside its HP-filtered log GSP series in Figure 5. The cyclical behavior of Kentucky’s balances seems acyclical in the 1980s; however, in the early 1990s they come more into line with what would be expected under a precautionary savings motive, building up in state level expansions and spending down during contractions. Notably, the fund doesn’t simply respond to U.S. level decreases in output relative to trend; it experiences a decrease in the mid-1990s and recently in the mid-2010s, both corresponding to downturns in gross state

Figure 1

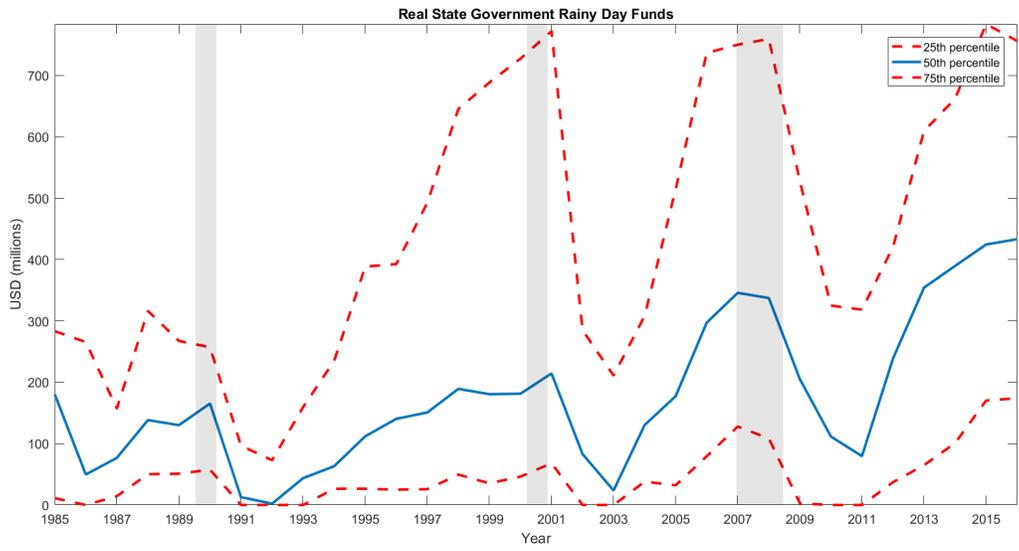


Figure 2

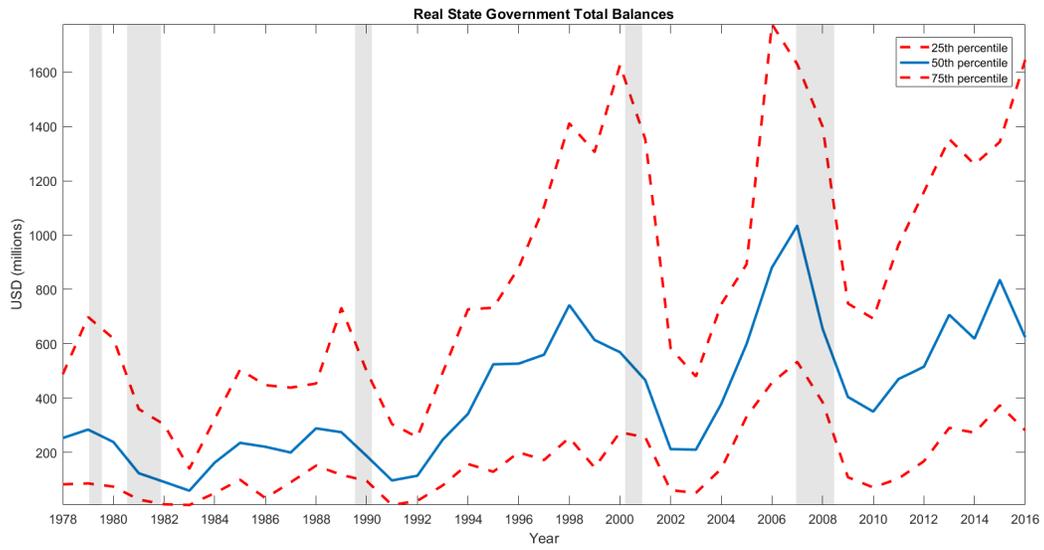


Figure 3

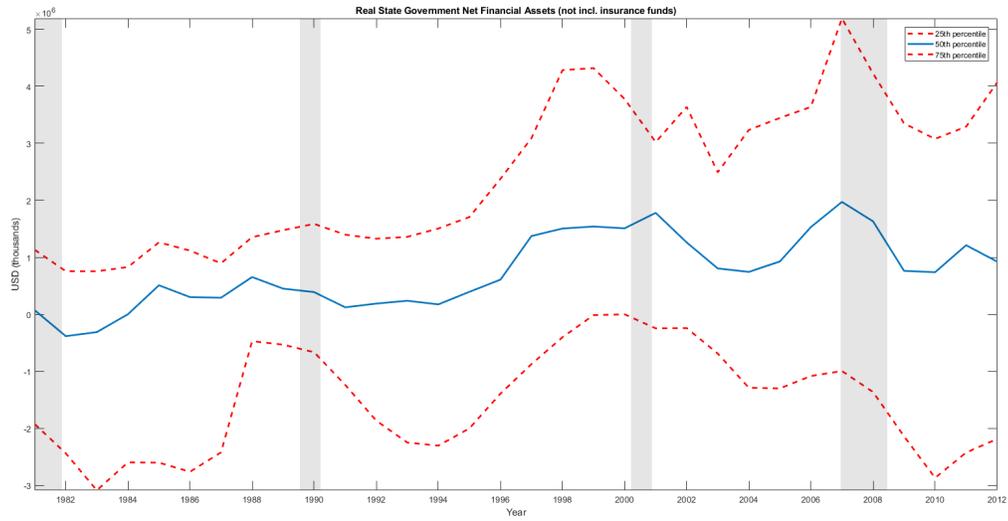


Figure 4

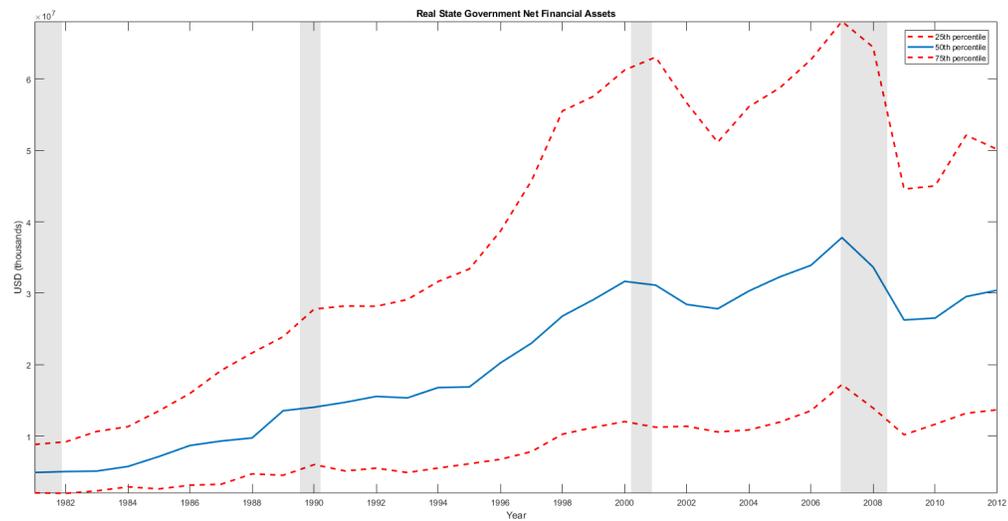
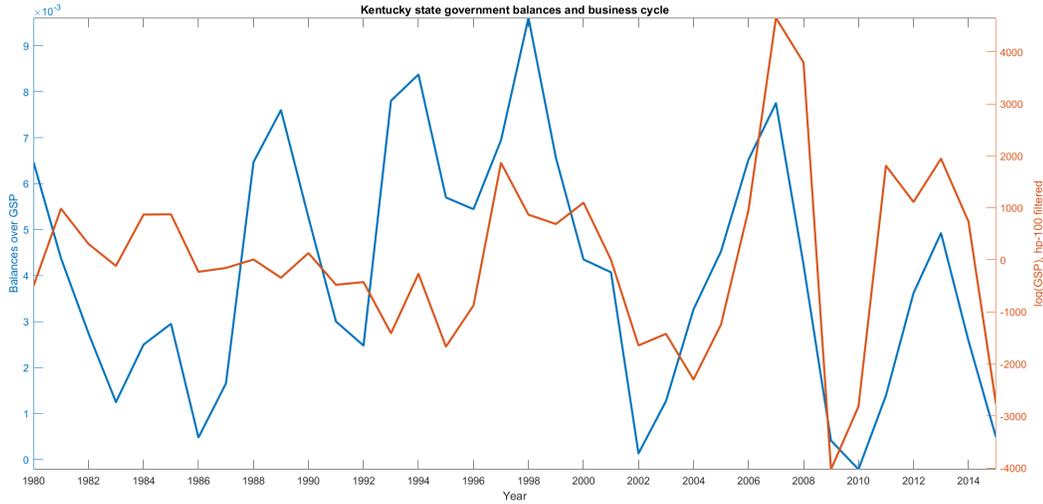


Figure 5



product. Furthermore, note that the mid-2000s recession seems to begin earlier in the state, and the state’s balances begin to respond accordingly before the U.S. as a whole fell into recession. Furthermore, over the entire sample the correlation coefficient of the two series is 0.40, further indication of significant precautionary savings behavior in Kentucky.

2.2.2 Fact 2: Transfer receipts are countercyclical and respond heavily to national cycle

The second stylized fact describes the behavior of state governments’ transfer receipts from the federal government. These transfer payments are countercyclical, as one might expect, but respond quite differently to aggregate and idiosyncratic fluctuations. Specifically, a state government’s transfer receipts from the federal government respond more strongly to the condition of the U.S. economy as a whole than to economic conditions within a state. In other words, Michigan might expect an increase in transfers when the rest of the country goes into recession, even if Michigan is expanding; conversely, Michigan may not expect as much revenue from the federal government when it is contracting, if the rest of the country is doing well.

Table 2: Determinants of state government receipts from federal government

Variable	$\log(pop)$	$\log(GSP_i)$, cyclical	$\log(GSP_{-i})$, cyclical
Coefficient	0.0061	-0.2083**	-0.5068***
(s.e.)	(0.0025)	(0.1042)	(0.1064)

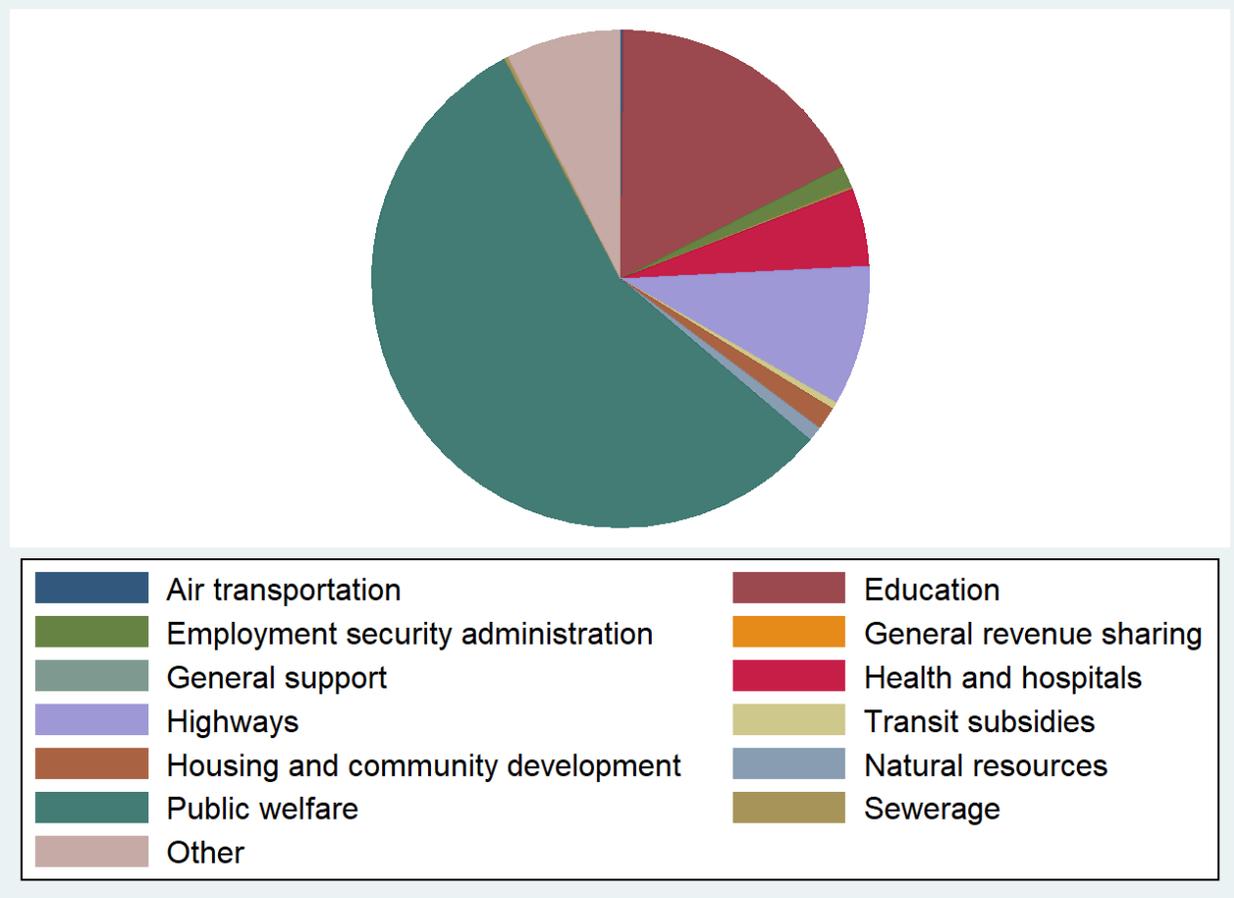
Note: Results from a fixed-effects regression with bootstrapped standard errors. Observations include 46 states for which holes do not exist in the Census of Governments data from 1981 to 2012. * $p \leq 0.10$, ** $p \leq 0.05$, *** $p \leq 0.01$.

Table 2 presents the output from a regression of state receipts from the federal government on population, cyclical GSP, and the cyclical component of the sum of the GSP of the other 49 states. The response of federal transfers to a state respond more strongly to the cyclical component of the *aggregate* economy (less GSP of the state itself) than to the *idiosyncratic* cycle of the individual state. A one percent decrease in a state’s own GSP relative to trend results in a 0.2 percent increase in transfer receipts; at the same time, a similar decrease in other states’ GSP would yield a 0.5 percent increase in the state’s receipts.

For a bit more insight into the composition of these transfer receipts, consider Figure 6. Clearly, the most significant component of federal government transfers to states is the ‘public welfare’ category; this category includes funding for a wide range of public assistance programs that are administered at the state level, Medicaid being the largest among them. the second biggest category is for education funding, and the third for highways. The previous result about the response of federal transfers to the business cycle of the aggregate economy seems to be driven by the two biggest categories, public welfare and education, as both of these transfer categories exhibit the same pattern of greater response to the aggregate business cycle.

It is notable that a category of federal funding to state governments seemingly designed for countercyclical assistance would not to respond to idiosyncratic cycles. Such a system may fail to give financial assistance to state-administered programs when they most need it, and may give undue assistance to states that may not be in such dire straits. The failure of thee transfers to account for state-level fluctuations would not be of any importance if state business cycles lined up perfectly with national business cycles; however, this is not

Figure 6: Composition of federal transfers to states



the case, as state-level recessions tend to be more common than U.S.-level recessions.

To identify state-level recessions, I follow Brown (2017) by using the Philadelphia Fed’s monthly coincident indices, which are available both for states and for the U.S. as a whole. I identify the turning points of the cyclical component of this index using the modified BBQ (MBBQ) algorithm from Engel⁴, and identify recessions for the U.S. and all fifty states therein. While only 4 recessions are identified for the U.S. over the sample period (1979-2017), most states experienced more than four recessions; furthermore, states were in recession for an average of 73 months, compared to just 46 months for the U.S. nationally. These recession differences suggest that U.S. states experience heterogeneous idiosyncratic shocks apart from the U.S. business cycle as a whole.

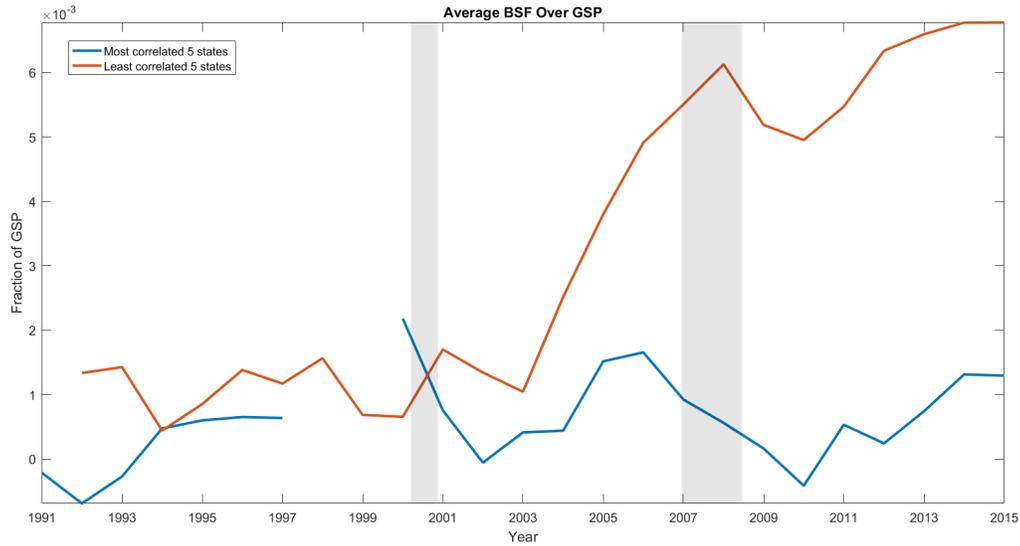
That states’ business cycles differ substantially from the U.S. business cycle suggests the transfer responses identified here will have significant implications for risk in state government budgets. Notably, a state whose business cycle moves more independently from the rest of the country might be exposed to more risk because of the federal transfer system than an otherwise equal state whose cycle moved more in step with the U.S. cycle. Having already identified possible precautionary savings behavior by state governments, one might expect these independent states’ governments to save more relative to other states; indeed, that is what these policy makers do, as I show in the following section.

2.2.3 Fact 3: Less correlated states save more

If the transfer policy of the U.S. federal government to U.S. state governments doesn’t respond as much to idiosyncratic fluctuations, then states whose cycles are less correlated with the rest of the country might be expected to run higher balances of government savings. To evaluate this prediction, I develop a measure of a state’s correlation with the business cycle of the rest of the country. For each state, I apply an HP-100 filter to two annual time series: the state’s annual real GSP series and real U.S. GDP less the state’s GSP. The long-

⁴Engel modifies the BBQ algorithm of Harding and Pagan (2002), and provides code at <http://www.ncer.edu.au/data>.

Figure 7



run correlation of the cyclical component of each of these time series yields the correlation of a state’s business cycle with that of the other 49 states. In this section, I show that this ‘correlation’ measure is negatively associated with precautionary savings behavior on the part of U.S. state governments.

Figures 7, 8, 9, and 10 show the time path of state government savings for the five most correlated and least correlated U.S. states with GDP.⁵ Clearly, states whose business cycles are least correlated with the U.S. business cycle run higher levels of government savings as a percentage of GSP than those which are most correlated. The most stark example is Figure 9, in which the most correlated states on average run slightly negative net assets (not including insurance funds). For a flavor of how correlations vary across the U.S., see Figure ??, in which states whose cyclical GSP is more correlated with U.S. GDP are highlighted.

Of course, there are a multitude of factors determining how correlated a state is with the rest of the country, some of which may also affect a state government’s level of savings. To further demonstrate the relationship between the correlation measure and

⁵For the remainder of the paper, I disregard Alaska. Alaska’s reserve funds are massive in comparison to the other states, and it is the least correlated with the rest of the U.S. The case of Alaska certainly supports my conclusions, but I want to prevent it from driving the results entirely.

Figure 8

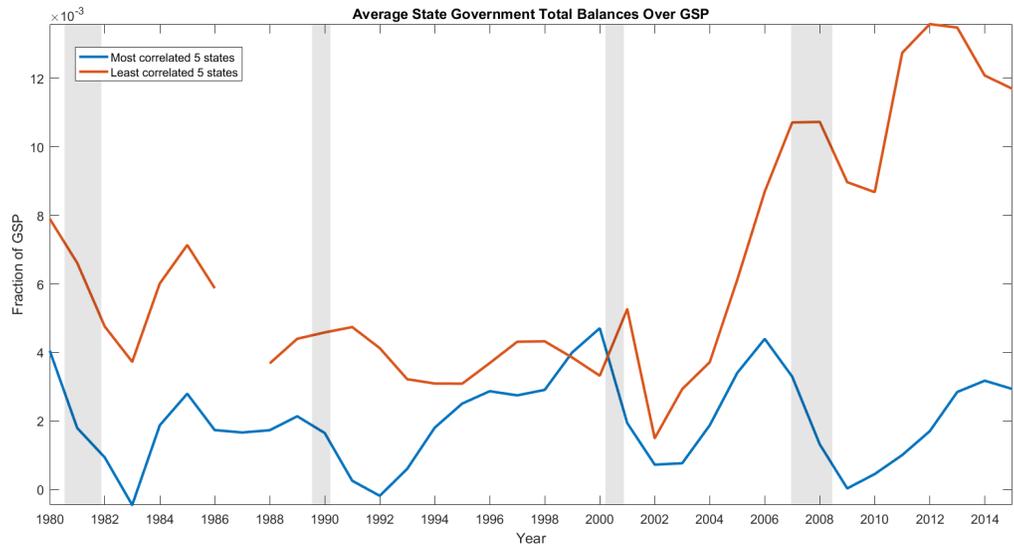


Figure 9

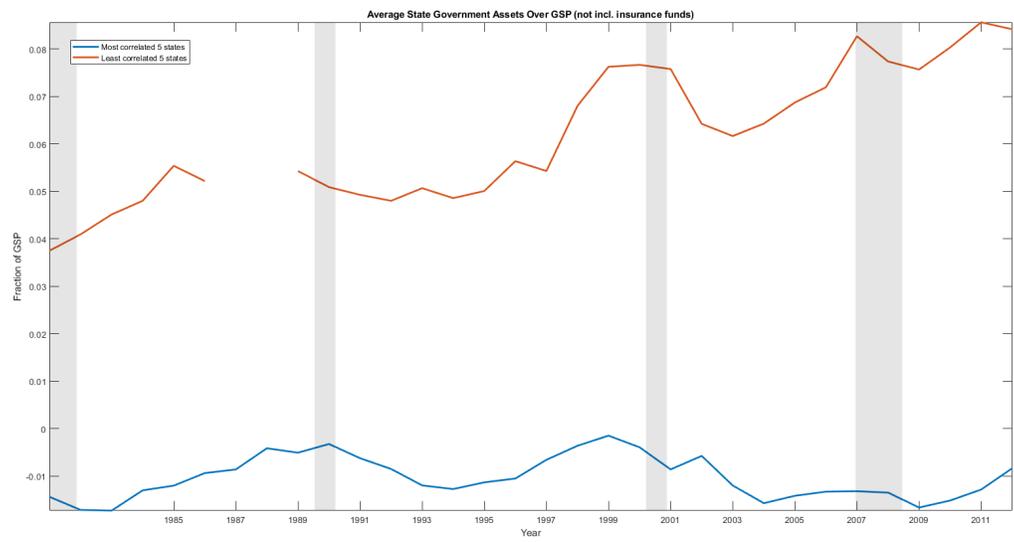


Figure 10

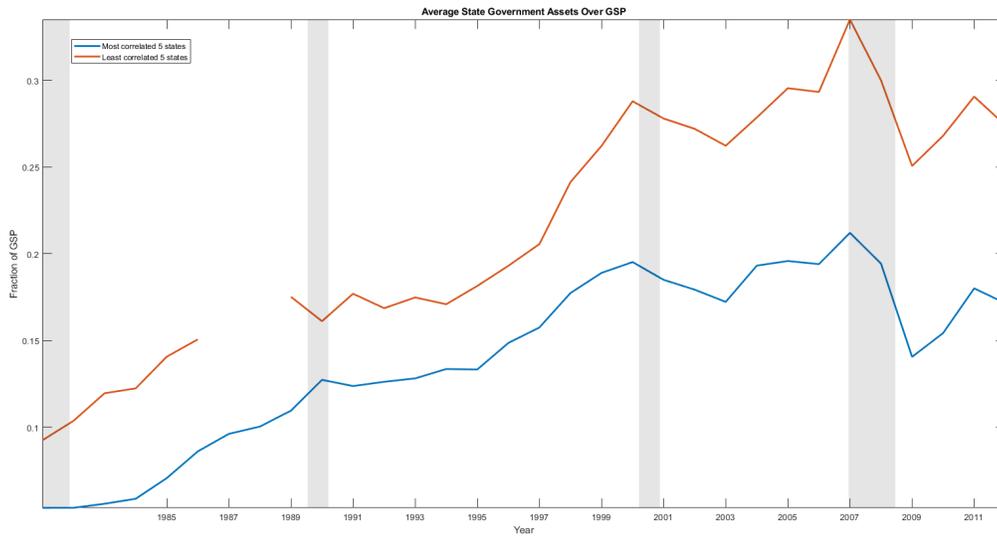
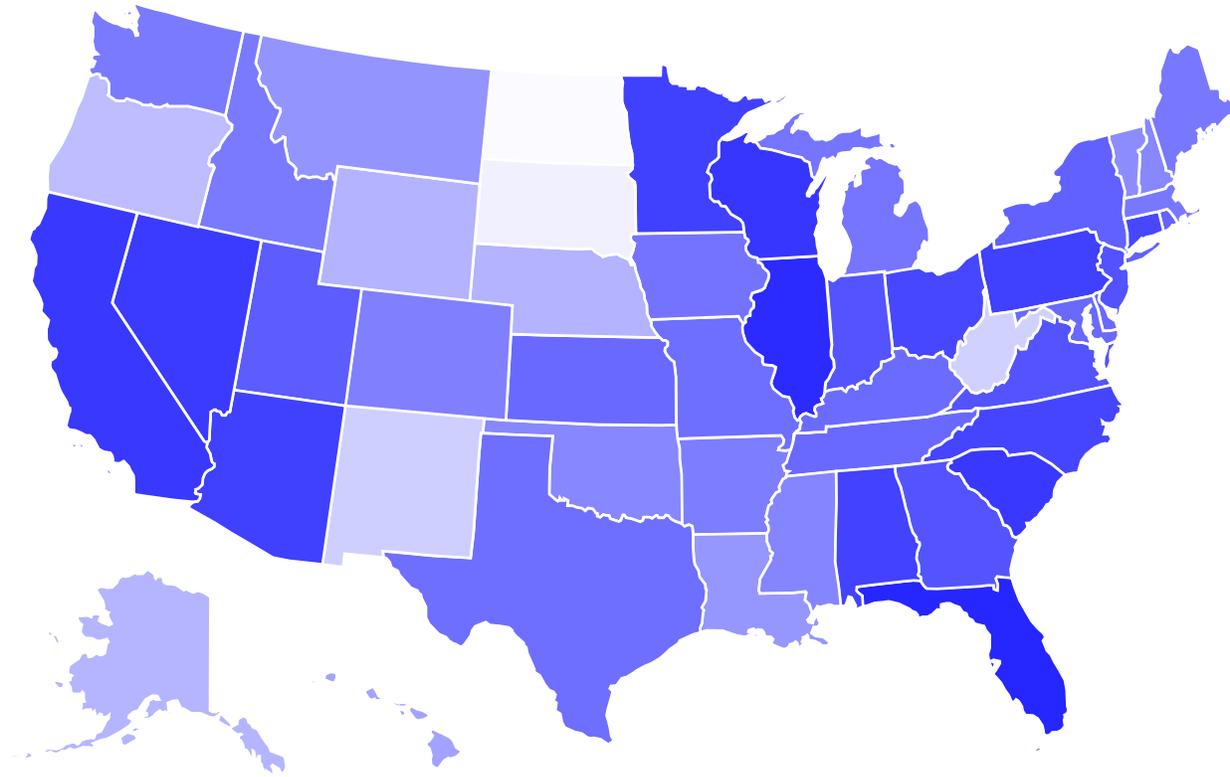


Figure 11: State correlations with the national business cycle



Note: More blue = higher correlation of a state's business cycle with the U.S. business cycle.

Table 3: Determinants of State Government Balances

Dependent variable	log(\sim Real balances)			Real balances / GSP			Real balances / Expenditures		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Business cycle correlation	-0.0920*	-0.0825	-0.0847*	-0.0054***	-0.0038***	-0.0038**	-0.0412***	-0.0262***	-0.0262*
	(0.0535)	(0.0602)	(0.0488)	(0.0015)	(0.0014)	(0.0014)	(0.0143)	(0.0101)	(0.0158)
Log(GSP), cyclical	0.5505***	0.4131*	0.1138	0.0225***	0.0177***	0.0167***	0.3036***	0.2536***	0.2487***
	(0.2084)	(0.2250)	(0.0790)	(0.0045)	(0.0042)	(0.0061)	(0.0354)	(0.0448)	(0.0548)
Log(GSP), cyclical * High correlation	-	-	0.9854*	-	-	0.0031	-	-	0.0155
			(0.5523)			(0.0066)			(0.0684)
Log(GSP)	0.0537***	0.0570***	0.0577***	-	-	-	-	-	-
	(0.0171)	(0.0208)	(0.0167)						
Controls	N	Y	Y	N	Y	Y	N	Y	Y

Note: The sample for these regressions is 49 U.S. states (Alaska not included) for the years 1981-2012. The regressions are according to a random effects model. Standard errors are bootstrapped with 50 replications. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Table 4: Determinants of State Government Net Assets (not incl. insurance funds)

Dependent variable	log(\sim Real net assets)			Real net assets / GSP			Real net assets / Expenditures		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Business cycle correlation	-1.2630***	-1.2365***	-1.2320***	-0.1168***	-0.0605**	-0.0607**	-1.0787***	-0.5528***	-0.5541**
	(0.3736)	(0.4628)	(0.3106)	(0.0311)	(0.0273)	(0.0306)	(0.3177)	(0.2156)	(0.2621)
Log(GSP), cyclical	0.9278**	0.5754	0.0538	0.0513**	0.0581*	0.0202	0.9424***	1.0980***	0.9055***
	(0.3983)	(0.5815)	(0.5995)	(0.0240)	(0.0323)	(0.0339)	(0.0996)	(0.1368)	(0.2301)
Log(GSP), cyclical * High correlation	-	-	1.8011***	-	-	0.1196***	-	-	0.6059**
			(0.6899)			(0.0433)			(0.3051)
Log(GSP)	0.1993***	0.3001***	0.2979***	-	-	-	-	-	-
	(0.0739)	(0.0811)	(0.0838)						
Controls	N	Y	Y	N	Y	Y	N	Y	Y

Note: The sample for these regressions is 49 U.S. states (Alaska not included) for the years 1981-2012. The regressions are according to a random effects model. Standard errors are bootstrapped with 50 replications. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

a state's government savings, I regress the savings measures on a number of state-year characteristics. Because the explanatory variable of interest is a time-invariant object at the state level, I estimate a random-effects model.⁶ Tables 3 and 4 give the output from a selection of these regressions for the two preferred definitions of state government savings.

Clearly, the relationship between 'correlation' and state government savings is negative and significant in all specifications of the estimation model. The interpretation is exactly the stylized fact highlighted in this section: states whose business cycles are less correlated with the rest of the U.S. run higher levels of government savings. Evidence of the procyclicality of these savings balances is also seen in most specifications. Furthermore, among the controls, the variance of a state's cyclical GSP is sometimes positively related with savings levels, lending more evidence to the idea that these savings measures capture precautionary savings behavior. Qualitatively, these results are robust to any of the aforementioned measures of state government savings.

⁶Wooldridge, 2010.

When combined with stylized facts 1 and 2, this third fact hints at an important feature of the effect of the federalist structure of the U.S. on state-level fiscal policy. It seems that, by exposing less correlated state governments to more risk by not insuring them against downturns as much as the more correlated states, the federal government creates stronger incentives for these state governments to engage in precautionary savings behavior. The rest of the paper attempts to expand on this story by putting forth two quantitative models of federalism and government policy over the business cycle that are able to reproduce these facts and others related to public finance over the business cycle. These models will allow counterfactual analysis of policies like balanced budget rules at the state level, and may shed light on the broader debate about the role for states in pursuing countercyclical fiscal policy. For instance, counter to conventional wisdom going back to Oates (1972), if real frictions to centralized fiscal policy are sizeable enough, it may be optimal for lower levels of government to engage in stimulus policy.

3 An information model

To begin thinking about the cyclical behavior of state government savings in a quantitative sense, I put forward a model of multi-tiered governments and information asymmetries. The basic structure of the model is as follows: a state is modeled as a small endowment economy in a fiscal federation, or a ‘region,’ to avoid confusing usage of the word ‘state.’ There are two levels of benevolent governments, regional and central. The regional government must provide a certain level of some public good, but is subject to strict borrowing limits. The central government is not subject to borrowing restrictions and makes tax and transfer policy to help regional governments smooth consumption, but only observes the state of the world with a noisy signal.

The balanced budget restriction at the regional level versus the information friction at the central level is the key trade-off in the allocation of fiscal policy. In a model without frictions, there would be no difference between the provision of financing for the public good

at the local or central level, as either government would be able to perfectly smooth the representative household's consumption over the cycle. Furthermore, in the presence of any sort of balanced budget requirement or a similar disadvantage in smoothing consumption at the local level, it would be optimal for the central government to take over all countercyclical fiscal policy, leaving the local government to simply levy a tax exactly equal to public goods spending. This is the conventional wisdom on federalism and fiscal policy articulated in Oates (1972).

Such specialization in fiscal policy is not observed in the data, however. As noted above, U.S. states seem to engage in some sort of precautionary savings behavior. Therefore, it is likely not the case that centralized fiscal policy is strictly preferred to decentralized policy; there must be some trade-off between policies at regional and central levels. I propose to model this trade-off by way of an information friction on the part of the central government. While the central government has an advantage in its ability to smooth consumption through borrowing and/or compulsory transfers, it does not observe the state of the economy exactly, receiving a noisy signal about the endowment shock. Because of this, the central government will not respond perfectly to region-specific shocks, requiring the regional government to save up funds in order to smooth.

This way of motivating the trade-off is consistent with discussions about the advantages of state and local governments *vis-a-vis* central governments. For example, the CBO references differences in information about citizens' situations and preferences as a reason why local government action might be preferred in some cases.⁷ The 'signal' method of modeling an information friction provided here can be interpreted in a number of ways. The most obvious interpretation is that of a central fiscal authority having imperfect measurement of indicators the regional economy; however, it could also be that a far-away central authority, although receiving accurate measurements, is not as 'tuned in' as local authorities with the effects on local citizens of the observed shocks. Furthermore, an even more reduced form interpretation of the information friction is that it captures other factors which might dampen the ability of a central fiscal authority to respond to local shocks, including political economy

⁷CBO, 2013.

frictions like slow or biased legislatures. The idea that local governments have better ability to know and match the preferences of their constituents goes all the way back to Tiebout (1956), and the political economy friction imposed by a legislature in centralized provision is found most notably in (Besley and Coate, 2003).

3.1 Model environment

3.1.1 Endowment process

Income for the household in region i is allocated exogenously according to an endowment process. The household is passive; it does not engage in any behavior to affect its consumption.⁸ The endowment in period t for region i is given by the following:

$$y_{it} = \bar{y} + \gamma z_{it} + \epsilon_{it}, \quad (1)$$

where \bar{y} is the long-run mean of income, z_{it} is an aggregate component, ϵ_{it} is an idiosyncratic component, and γ multiplies the aggregate component. Both components follow an AR(1) process, for example, the process for ϵ_{it} is given by

$$\epsilon_{it} = \rho_\epsilon \epsilon_{i,t-1} + \xi_{it}^\epsilon, \quad (2)$$

where $\xi^\epsilon \sim N(0, \sigma_\epsilon^2)$.

3.1.2 Regional government

The problem of the government in region i is to choose a stream of taxes and savings, τ_{it} and s_{it} , to finance an exogenous stream of government purchases g_{it} , which generate no utility. The preferences of the government are exactly aligned with those of the representative

⁸The household may be thought to engage in one action, namely, the election of a state government. In the framework here of a representative household, a government whose preferences exactly align with the household's is elected; this is exactly the type of regional government I consider.

household, which has utility over consumption in every period:

$$W(\{c_t\}_{t=0}^{\infty}) = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t U(c_{it}) \right]. \quad (3)$$

The government is subject to the following constraints:

$$c_{it} = y_{it} - \tau_{it}$$

$$g_{it} + s_{it} = \tau_{it} + T_{it} + (1+r)s_{i,t-1}$$

$$s_{it} \geq \phi,$$

where τ_{it} is the lump sump tax chosen in period t , s_{it} is the savings (negative debt) of the regional government, y_{it} is the endowment, g_{it} is required government spending, ϕ is the per-period borrowing constraint, and T_{it} is the fiscal transfer from the federal government, which will be taken as given from the perspective of the regional government.

The transfers will result from the policy function of the central government, $T(s_t, z_t, \theta_t, f_t)$, where θ is a noisy signal of ϵ_t which is unobserved by the regional government, and f_t is its prior belief about ϵ_t ; both of these will be explained shortly. Given the observed state variables, the regional government cannot predict transfers T exactly, and must choose its policy to maximize *expected* utility over the possible realizations of the transfer function, such that its dynamic programming problem can be described by the following equation:

$$\begin{aligned} V^R(s_t, z_t, \epsilon_t) &= \max_{s_{t+1} \geq \phi} \mathbb{E}_t[c_t] + \beta \mathbb{E}_t[V^R(s_{t+1}, z_{t+1}, \epsilon_{t+1})] \\ \text{s.t. } c_t + s_{t+1} &= y_t + T(s_t, z_t, \theta_t, f_t) + (1+r)s_t - g_t. \end{aligned} \quad (4)$$

Note the presence in this problem of an expectation operator on current period consumption; in this model, the local government chooses next period savings before observing the realization of transfers T from the central government, and must use taxes and subsidies to balance the budget at the end of the period.

3.1.3 Central government

In addition to the regional government, the central government features as a second optimizing agent in the model. Its inclusion reflects the fact that, in the context of the U.S., the federal government is not a passive agent with regard to fiscal policy. While much federal spending, such as the Social Security program, is indeed formulaic, the federal government also engages in a large amount of discretionary spending, up to a third of which is a direct transfer to the states. The central government is benevolent, so its optimization problem at first glance is almost equivalent to that of the state government:

$$\begin{aligned}
 V^C(s_t, z_t, \theta_t, f_t) &= \max_{T_t} \mathbb{E}_t[c_t] + \beta \mathbb{E}_t[V^C(s_{t+1}, z_{t+1}, \theta_{t+1}, f_{t+1})] \\
 \text{s.t. } c_t + s(s_t, z_t, \epsilon_t) &= y_t + T_t + (1 + r)s_t - g_t.
 \end{aligned} \tag{5}$$

Note, however, that the differences in the two models are not inconsequential. First, the central government's choice variable is T_t rather than s_t . While the fiscal balance carried over into the next period is chosen by the state governments, the central government chooses how much to give to (or take from) the state governments in the form of transfers in each period. Furthermore, there is no period budget balance necessarily required on the part of the central government. I assume that the central government can perfectly observe the aggregate shock z_t ; as a result, it is able to perfectly insure the states against aggregate shocks. Therefore, for the remainder of the paper I abstract from the aggregate shocks, and consider a model with only idiosyncratic shocks and the recasted policy functions $s(s_t, \epsilon_t, f_t)$ and $T(s_t, \theta_t, f_t)$. Given this recasting, two mechanisms can be employed to discipline the financial behavior of the central government and prevent it from accumulating debt indefinitely.

The first mechanism that can be employed is budget balance over the infinite horizon, i.e., a no-Ponzi-game condition on the central government's assets. In this framework, there is a finite number of regions, and the central government has a stock of assets out of which positive transfers are paid and into which negative transfers are deposited. The

no-Ponzi-game condition, then, is given by

$$\lim_{t \rightarrow \infty} \mathbb{E}_0 \left[\frac{A_t}{(1+r)^t} = 0 \right], \quad (6)$$

where A_t is the stock of assets held by the central government for the purposes of transfers to the regional government.

The second method is to have period budget balance and an infinite number of regional governments. In this setup, the central government in every period takes from some regions and gives to others, such that total transfers (for idiosyncratic shocks) net out to zero:

$$\int T_{it} di = 0. \quad (7)$$

For the simple case in which all regions are identical, solving for the transfer function in this case is analytically equivalent to the solution for the first mechanism. Appendix B.1 shows that both mechanisms result in the simple budget condition $\mathbb{E}_0 [T(s_t, \theta_t, f_t)] = 0$.

In addition to the differences in choice variables, note that the state variables are also different for the central government. The central government observes s_t , but receives a noisy signal θ_t about the state variable ϵ_t :

$$\begin{aligned} \theta_t &= \epsilon_t + \xi_t^\theta \\ \xi_t^\theta &\sim N(0, \sigma_{\xi^\theta}^2). \end{aligned} \quad (8)$$

Here, ξ_t^θ is the noise component of the signal, and σ_{ξ^θ} reflects the relative noisiness of the signal. Finally, the central government brings into the period a prior belief on the distribution of the idiosyncratic component ϵ : $f_t = N(\mu_t, \sigma_{\mu,t}^2)$. After observing the signal, the central government updates this prior to form a posterior with which it forms its expectation for the choice of transfer, then projects this posterior forward into the next period using the law of motion for ϵ . This process is described in further detail in the next section.

3.2 Bayesian updating

The central government begins time period t with a prior belief f_t on the distribution of ϵ_t : $f_t = N(\mu_t, \sigma_{\mu,t}^2)$. Upon observing the noisy signal θ_t , the central government updates its belief to $\hat{f}_t = N(\hat{\mu}_t, \hat{\sigma}_{\mu,t}^2)$ according to the following rules, which mimic the classic signal extraction problem put forth in Lucas (1973):

$$\hat{\mu}_t = \mu_t + \frac{\sigma_{\mu,t}^2}{\sigma_{\mu,t}^2 + \sigma_{\xi^\theta}^2} (\theta_t - \mu_t) \quad (9)$$

$$\hat{\sigma}_{\mu,t}^2 = \frac{\sigma_{\mu,t}^2 \sigma_{\xi^\theta}^2}{\sigma_{\mu,t}^2 + \sigma_{\xi^\theta}^2}. \quad (10)$$

It is this distribution \hat{f}_t that the central government uses to form its expectations when solving for its optimal policy. The extent to which the belief about the mean is updated after observing the signal is determined by the relative variance of the noisy portion of the signal. The noisier the signal, the less weight is attached to it in the process of forming beliefs about the region's endowment.

At the end of the period, the central government must form its belief about ϵ_{t+1} , which is the prior distribution it will bring into the next period as a state variable. These priors for period $t+1$ are formed from applying the known $AR(1)$ process to the posteriors formed in period t :

$$\mu_{t+1} = \rho_\epsilon \hat{\mu}_t \quad (11)$$

$$\sigma_{\mu,t+1}^2 = \rho_\epsilon^2 \hat{\sigma}_{\mu,t}^2 + \sigma_{\xi^\epsilon}^2. \quad (12)$$

Given these laws of motion, the posterior variance $\hat{\sigma}_{\mu,t}^2$ is bounded in the long run, and under certain conditions converges to a single value. In Appendix B.2, I show that the fixed point is

$$\hat{\sigma}_{\mu,\infty}^2 = \frac{(\rho_\epsilon^2 - 1)\sigma_{\xi^\theta}^2 + \sigma_{\xi^\epsilon}^2 + \sqrt{[(\rho_\epsilon^2 - 1)\sigma_{\xi^\theta}^2 + \sigma_{\xi^\epsilon}^2]^2 + 4\rho_\epsilon^2 \sigma_{\xi^\theta}^2 \sigma_{\xi^\epsilon}^2}}{2\rho_\epsilon^2}. \quad (13)$$

In solving the model, I assume that the central government has already reached this value for the posterior variance. This eliminates another state variable and allows the belief about

the distribution of ϵ to be characterized by movements in μ_t .

3.3 Timing and Equilibrium

The timing of events in the model is as follows. In every period t ,

1. All shocks ξ are realized.
2. Regional governments observe the true shock to their endowment ξ_ϵ , but not the private signal θ_t . The central governments observes the noisy signal θ_t .
3. The central government forms its update belief \hat{f}_t from the prior belief f and the signal θ_t .
4. Transfers and next period savings are chosen and committed to simultaneously by the central government and regional governments, respectively.
5. Regional taxes adjust to satisfy the choice of s_{t+1} , given the realization of T_t .
6. The central government uses \hat{f}_t to form f_{t+1} , the prior belief going into the next period.

Definition: *A Markov Perfect Equilibrium is a set of policy functions $\{s(s_t, \epsilon_t, f_t), T(s_t, \theta_t, f_t)\}$ such that, given exogenous processes for ϵ_t , θ_t , and g_t ,*

1. $s(s_t, \epsilon_t, f_t)$ solves the regional government's problem given $T(s_t, \theta_t, f_t)$, and
2. $T(s_t, \theta_t, f_t)$ solves the central government's problem given $s(s_t, \epsilon_t, f_t)$.

Here I define the equilibrium with one region, but in principle there could be many of these regions, each with its own equilibrium with respect to the central government. Since these regional governments are islands in the model, the solutions are separable, and it is helpful to cast the problem in terms of one region only.

3.4 Relation to a political economy model

Thus far, I have specified the main friction to central government policy making as being a matter of imperfect information and learning on the part of the central government. The other compelling source of inefficiency in centralized policy is political in nature. Discretionary transfers to states and state agencies originate from budgetary decisions made in Congress; while noisy signals are likely at play here, political processes and voting are a major determinant of transfers. The federal government may not respond to idiosyncratic shocks simply because other states vote down extra transfers to states in recession.

While the political story is somewhat different, I propose that the information frictions modeled above can be thought of as including political frictions, as well. To see this, consider a simplified version of the information model, in which there is no persistence for the idiosyncratic component ($\rho_\epsilon = 0$) for simplicity. Suppose the noise shock ξ^θ can take on one of two values: $\xi^\theta \in [-\bar{\xi}^\theta, \bar{\xi}^\theta]$, each with probability 1/2, where $\bar{\xi}^\theta > 0$. From the perspective of the regional government, which knows the true ϵ as well as the transfer function $T(s, \theta)$, it could receive one of two values for the transfer. If $\xi^\theta = -\bar{\xi}^\theta$, transfer T will be higher than it would be in a frictionless model, and vice versa if $\xi^\theta = \bar{\xi}^\theta$. So the regional government forms its expectations and policy knowing that, given s and ϵ , its transfer will be either T_{high} or T_{low} , each with probability 1/2.

Now consider a slightly different model, in which the friction to centralized policy making is political, i.e., transfers are voted on by a legislature for the central government. Following the political economy setup of Besley and Coate (2003), suppose there are two regions in the fiscal federation, and utility spillovers of the following type:

$$U_i(c_i, c_{-i}) = (1 - \kappa)u(c_i) + \kappa u(c_{-i}), \kappa \in (0, 1/2).$$

Transfers are chosen by the legislature, which is modeled using the *minimum winning coalition* strategy. In every period, each region has probability 1/2 of being ‘in power,’ i.e., casting the median vote on transfer policy. If a region ends up in power, it receives a

higher-than-efficient transfer, T_{high} , and if it is the minority, it receives a lower-than-efficient transfer T_{low} .⁹ So, just as in the simple model of information, this framework requires the regional government to set policy knowing that it will receive T_{high} or T_{low} with probability 1/2, and we can expect its behavior to be similar to that in the simple information model. Given that a well known and widely used model of political frictions can be mapped into a similar version of the information model presented here, I think it reasonable to think of the ‘information’ friction as potentially including political factors, as well.

4 Quantitative analysis

4.1 Estimation and calibration

In order to examine the properties of the model, it is sufficient to consider the case of a single region. There are no interactions in this model across sectors, and the interactions of interest are between the regional governments and the central government. Given this strategy, I attempt to get some results roughly corresponding to the ‘median’ U.S. state. I estimate an AR(1) model for HP-100 filtered $\log(GSP)$ in all 50 states, and set ρ_ϵ and σ_ϵ to be the respective medians of the AR(1) parameter estimates. I then calibrate $\sigma_{\xi\theta}$ to match the median of $corr(T_t, y_t)$, the correlation of transfer receipts from the federal government with output, at the state level.

Parameters for the baseline case are given in Table 5. I normalize $\bar{y} = 1$ and set $g_t = g = 0.05$ to roughly approximate data on state government spending. Utility is CRRA: $u(c) = \frac{c^{1-\nu}-1}{1-\nu}$, and I let $\nu = 2$. I choose an annual interest rate of 0.04, and set the discount rate such that $\beta < \frac{1}{1+r}$ to keep the region from wanting to increase savings indefinitely.¹⁰ I set $\phi = 0$ to reflect the balanced budget constraints that are present in most U.S. states, and choose a realistic upper bound for state government savings of 0.2. Later, I

⁹Here, as in Besley and Coate’s model, the spillover term κ ensures that the deciding voter does not completely disregard the utility of the other region.

¹⁰Aiyagari, 1994.

Table 5: Baseline parameters, information model

ρ_ϵ	ξ^ϵ	ξ^θ	\bar{y}	g	ν	ϕ	β	r
0.5095	0.0280	0.0671	1	0.05	2	0	0.961	0.04

study the potential welfare effects of lowering ϕ , but 0 is an intuitive choice for the baseline case. I also restrict the transfer policy to respond linearly to the central government’s signal, given its prior beliefs. This does not alter its optimal policy much, but it greatly eases the computation burden involved in solving the problem.

I solve for the equilibrium policy functions by the use of an ‘inner loop, outer loop’ strategy. The ‘inner loop’ refers to the process of solving for each policy function given the policy function of the other government. The regional government policy is solved by value function iteration, and the central government policy is a static optimization problem, since its choices do not affect its future value function. The ‘outer loop,’ then, repeats this process, updating each policy function until both have converged.

4.2 Results

To assess the performance of the model compared to U.S. data, I simulate the calibrated baseline region for 10000 periods and observe its behavior. Table 6 presents some basic moments for some of the key variables of interest. Of the moments which aren’t explicitly targeted, $corr(y_t, s_t)$, the co-movement of state-level government savings and output matches remarkably well. The autocorrelation of transfer receipts is a bit low, but in an acceptable qualitative range. The variability of savings and transfers are low relative to the data, but their relative magnitudes to each other seem to make sense.

The one outlier, of course, is the autocorrelation of savings. In the model, savings is more persistent than it is in the data. One reason for this may be the following: the moments I report in the data are in the extreme long-run case of a static economy. Savings here doesn’t grow over time: once it reaches a desired level, it stays there and simply fluctuates around

Table 6: Business cycle moments

	Model	Data (median of log hp-100 filter at state level)
$corr(y_t, T_t)$	-0.1461	-0.1536
$corr(y_t, s_t)$	0.2264	0.2862
$corr(y_t, y_{t-1})$	0.48	0.5430
$corr(T_t, T_{t-1})$	0.1927	0.3531
$corr(s_t, s_{t-1})$	0.9407	0.1072
$sd(y_t)$	0.0356	0.0281
$sd(T_t)$	0.0175	0.0694
$sd(s_t)$	0.5189	1.16

Note: moments reported here are for the cyclical components of each of these variables.

that level. It may be that, in the real world, U.S. states have not yet ‘settled’ into their desired long-run levels of savings, thus exhibiting more unpredictability.

I also compute the impulse response functions of savings and transfers to state-level idiosyncratic shocks. Figures 12 and 13 give the predictions from the model, while Figures 14 and 15 display results from the data. The IRFs from the data are computed from running a VAR with savings (or transfer), state GSP, and U.S. GDP for all 50 states; I plot the median of the estimated IRFs and the median of the confidence bands. The model predicts a hump-shaped response of savings to an idiosyncratic regional shock, and the data seems to present some weak evidence in favor of this prediction. The complete lack of response of transfers to the regional shock is also consistent with the model, which predicts a response that is stunningly low in magnitude.

The lack of much meaningful transfer response in the model is consistent with the data, and is driven by the massive information cost implied by the baseline calibration. To generate a realistically low correlation between state-level fluctuations and transfer receipts from the federal government, the variability of the noise component of the central government’s signal has to be almost *three times as large* as that of the real idiosyncratic shock. Even though such a sizeable noise shock is necessary to match the data relatively well, information costs that large seem almost incredible. Certainly, in the real world, the frictions at the level of centralized policy making are more diverse; for example, political dynamics

Figure 12

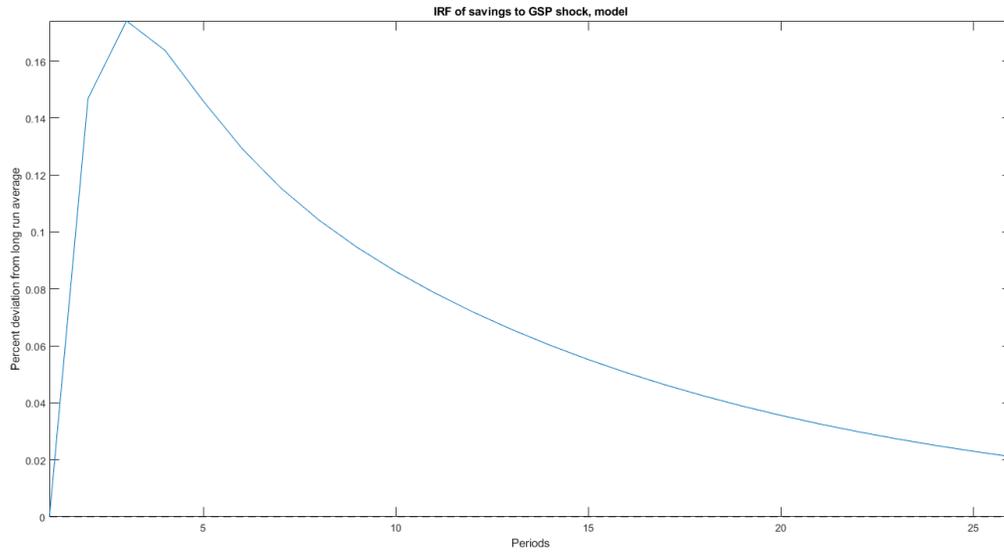


Figure 13

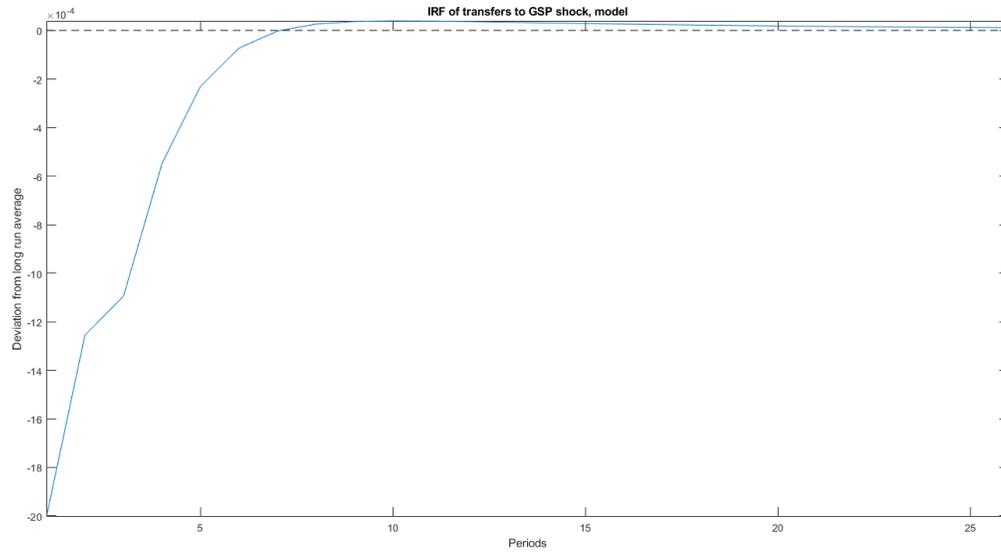


Figure 14

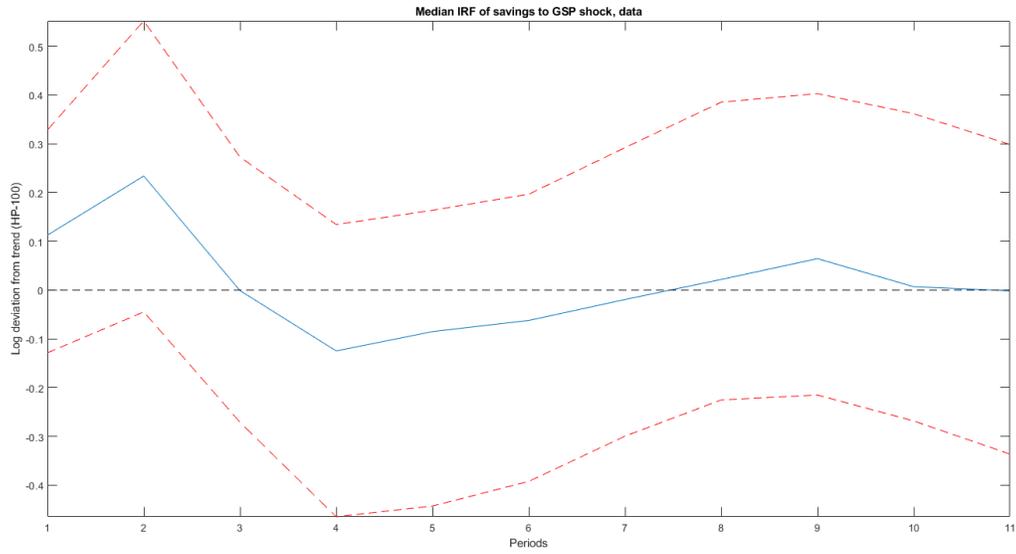
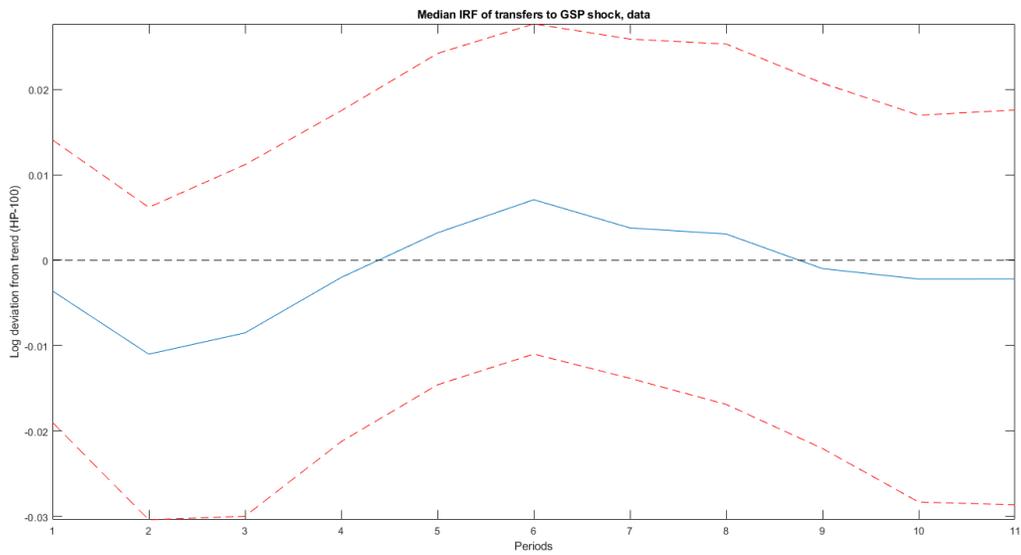


Figure 15



are likely an important part of a central government's inability (or unwillingness) to insure regions against adverse shocks. In the context of this model, all such frictions are being captured by the single noise parameter; nevertheless, its size implies a central government that is quite weak in responding to economic shocks at the state level.

4.3 Welfare analysis

The precautionary savings behavior in the model depends critically on the presence of a strict limit on deficits on the part of the regional governments. This is a realistic feature of the model, given the widespread use of such balanced budget rules in the real world. In this model, such rules are not a result of optimizing behavior, but external parameters imposed on the agents. There are no other frictions on policy at the state level; therefore, were balanced budget rules to be sufficiently relaxed, regional governments would be able to achieve the social planner solution of full consumption smoothing for the household. By studying the effects of removing this constraint, I can say something about the welfare loss imposed by the baseline model relative to the social planner, as well as examine the potential effects of removing a balanced budget constraint.

To compute social welfare in the region, I solve the model and then simulate it starting at $s_0 = 0$, $\epsilon_0 = 0$, with time-0 social welfare being given by the sum of discounted utilities. As ϕ is lowered, the model approaches the social planner outcome quite quickly; when $\phi \leq -0.10$, the model essentially matches the social optimum. The implied welfare loss of the balanced budget rule (combined with the frictions on the central government, of course), then, is about 3.2 percent of welfare in the social planner case. This is, of course, a sizeable number. All of this loss occurs in the early periods, when the regional government increases taxes in order to build up its stock of savings to its desired long-run level. From this point on, the household is *better off* than it would be in the no-balanced-budget case, due to the extra interest income for its government, but these gains are far outweighed by the losses in early periods.

4.4 Implications for fiscal policy

The model presented in this paper implies that significant structural frictions to optimal countercyclical fiscal policy making may exist at centralized levels of government. The magnitude of these frictions suggests that a role exists for states to participate in active fiscal policy over the business cycle. Clearly, as the data show, U.S. state governments do in fact save in order to manage public finances during downturns. The most visible vehicles for such savings are rainy day funds, but states have other avenues, as well.

I have avoided using the word “stimulus” thus far, as Keynesian-type stimulus does not appear in my model. However, it is likely that the presence of these frictions in the making of fiscal policy at the federal level imply that it may be optimal for U.S. states to engage in stimulus policy during recessions; this conclusion runs counter to a conventional wisdom going back at least as far as Oates (1972), though (Gramlich, 1997) does find a stimulus role for states. More work should be done to explicitly model the implications of information and other political frictions for state-level fiscal policy over the business cycle.

Of note as well is the analysis of the effects of balanced budget rules in this model. In the information model presented here, removing the balanced budget rules completely eliminates welfare losses from baseline model, allowing regional governments to smooth completely over the business cycle. Of course, the assumption is that there are no real costs to borrowing, no default risk, etc. It may be that, in the real world, the balanced budget rules for U.S. are optimal responses to real costs of debt finance. In that case, the policy implications of loosening balanced budget rules would be completely the opposite: a welfare loss instead of a welfare gain.

Finally, these results may have something to say about the debate over fiscal policy in Europe. The results in this paper serve as a caution to potential efforts to establish a European fiscal union. While the ability of a such a union to finance spending with a deficit would be an advantage, there may be significant frictions to effective and timely fiscal stimulus along the lines of the frictions identified in this paper. In the presence of

significant information or political frictions to optimal policy, a fiscal union in Europe may have trouble responding to localized shocks, especially as diverse an economic environment as the Eurozone.

5 Conclusion

This paper identified three key facts about the public finances of state governments in the U.S. over the business cycle. First, state governments engage in precautionary savings, in large part due to balanced budget requirements. Second, transfer payments from the federal government tend to respond more strongly to the aggregate business cycle than to state-level economic cycles. Third, states whose business cycles are less correlated with the national cycle tend to save more relative to other states. In light of the first two facts, I interpret the third fact as an indication that federal transfers (or lack thereof) influence state government savings behavior.

To give structure to this interpretation, I turn to a modeling framework in which I consider a U.S. state to be a small open endowment economy in a fiscal federation. Both levels of government, regional and central, may conduct fiscal policy, but each is faced with a different friction. Regional governments face borrowing limits, but central governments are faced with an information friction (which may also be interpreted as a political friction). The information friction prevents the central government from perfectly smoothing over the cycle, and thus regional governments must engage in precautionary savings.

I find that the model fits many qualitative features of the data well, and conclude that it is a useful framework in which to begin thinking about aspects of state government finances. The implied information friction is almost three times as large as the real economic volatility; this is a formidable friction that implies a central government with little ability to smooth over idiosyncratic cycles for states. The baseline calibration also implies a welfare loss of three percent relative to the social optimum, which is quite a sizeable welfare loss. The results of this model imply that frictions to policy making at the central level may be

a significant factor in fiscal policy at the state level.

I conclude, contrary to some conventional wisdom, that space exists for states to actively pursue robust fiscal policies over the business cycle, and that states that save are behaving optimally given their constraints. To the extent that central government policies are constrained by a lack of information or stymied by politics, central governments may not be able to perfectly implement a first-best countercyclical fiscal policy. Of course, more research is needed into the size and nature of these frictions to policy, especially as they relate to recessions and expansions. Federalism, its complexities and mysteries notwithstanding, remains an important vehicle through which policy makers can insure citizens against adverse outcomes.

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A Construction of balances data

I obtain the data on state balances and rainy day funds from “The Fiscal Survey of the States,” a semiannual report published by the National Association of State Budget Officers (NASBO). When available, I use the spring edition of the report for consistency, and I report the “actual” numbers from the previous year, not the “estimated” number for the current year. This report surveys budget officers in U.S. states in order to detail the fiscal health of states across various dimensions. I utilize this specific series because it provides states with the option to report both general fund balances and rainy day fund balances separately.

Unfortunately, the data reported in the Fiscal Survey of the States is not available in electronic format, and must be recorded by hand for each year. The report is released in PDF format every year, and each report gives an end-year balance for the general fund and the rainy day fund for all 50 states. Recording the balances data for the states involves locating the relevant table in each document and pulling the data for each state into a common spreadsheet for analysis. The format tends to be similar across years, although it does change from time to time, with greater frequency of format changes in earlier years.

The basic strategy is to record two figures: the ‘ending balance’ for each state, which is the surplus (or deficit) of the state’s general fund, and the rainy day fund balance. Then ‘total balances’ are reported as the sum of these two numbers. In more recent documents, NASBO constructs and reports total balances but not ending balances, so the data collection involves the collection of total balances and rainy day fund balances. Figures 16, 17, and 18 show the difference between the 2017 report and the 1996 report. In 2017, total balances are reported and therefore do not need to be constructed; in 1996, however, ending balances are reported and total balances need to be constructed.

Note also that certain states report the rainy day fund as a part of their general fund. For these states, I am not able to back out the value of the rainy day fund.¹¹ For

¹¹Sometimes there is a footnote with the value of the rainy day fund. I include these figures with the rainy day fund data when possible.

Figure 16

TABLE 28

Total Balances and Total Balances as a Percentage of Expenditures, Fiscal 2016 to Fiscal 2018

State	Total Balances (\$ in Millions)			Total Balances as a Percent of Expenditures		
	Fiscal 2016	Fiscal 2017	Fiscal 2018	Fiscal 2016	Fiscal 2017	Fiscal 2018
Alabama	\$715	\$814	\$918	9.2%	9.8%	11.0%
Alaska*	7,120	7,033	6,310	130.9	158.4	145.6
Arizona	745	582	485	7.8	6.0	5.0
Arkansas	0	0	0	0.0	0.0	0.0
California*	8,553	7,741	10,404	7.5	6.3	8.5
Colorado***	513	588	676	5.0	5.6	6.1
Connecticut*	236	259	261	1.3	1.4	1.4
Delaware***	568	428	456	14.5	10.5	11.1
Florida	3,246	2,939	2,761	11.1	9.6	8.9
Georgia***	2,131	2,131	2,131	9.7	9.3	9.0
Hawaii	1,129	832	743	16.4	10.8	10.0
Idaho	310	374	361	10.2	11.4	10.4
Illinois*	534	133	133	2.0	0.4	0.4
Indiana	2,244	1,731	1,834	15.0	11.3	11.8
Iowa	773	608	709	10.7	8.3	9.7
Kansas	37	100	217	0.6	1.6	3.5
Kentucky	490	351	179	4.8	3.2	1.6
Louisiana	45	261	286	0.5	2.7	3.0
Maine	193	243	199	5.8	7.1	5.8
Maryland	1,217	926	960	7.5	5.4	5.6
Massachusetts***	1,482	1,345	1,410	3.7	3.2	3.3
Michigan	1,217	1,075	1,012	12.6	10.6	10.0
Minnesota***	3,102	2,720	2,328	15.4	12.6	10.3
Mississippi	356	337	557	6.2	5.8	9.7
Missouri	445	476	414	4.9	5.2	4.3
Montana	255	123	163	11.0	5.2	6.9
Nebraska	1,262	809	707	30.1	18.6	16.1
Nevada	418	484	298	11.6	12.5	7.2
New Hampshire	182	100	97	13.1	6.9	6.5
New Jersey	473	491	493	1.4	1.4	1.4
New Mexico***	146	-67	11	2.3	-1.1	0.2
New York***	8,934	7,232	5,917	13.1	10.4	8.2
North Carolina	2,155	1,794	1,787	10.2	8.1	7.6
North Dakota	983	11	57	32.7	0.4	2.5
Ohio	3,198	2,728	2,200	9.3	7.7	6.5
Oklahoma*	241	N/A	N/A	3.9	N/A	N/A
Oregon	811	1,119	1,021	9.0	12.3	10.6
Pennsylvania	2	-606	4	0.0	-1.9	0.0
Rhode Island	359	272	196	10.1	7.4	5.2
South Carolina***	1,131	856	890	15.8	10.9	11.6
South Dakota	157	160	160	10.8	10.1	9.9
Tennessee	1,958	1,838	801	15.5	13.4	5.5
Texas	14,047	11,783	11,639	26.6	22.3	22.5
Utah	658	501	502	10.4	7.8	7.6
Vermont	78	93	124	5.3	6.1	8.1
Virginia	501	626	297	2.6	3.1	1.5
Washington	1,922	1,951	1,592	10.6	10.0	7.8
West Virginia	1,150	871	853	27.5	19.8	19.2
Wisconsin	612	735	600	3.9	4.3	3.5
Wyoming	1,811	1,481	1,481	109.7	103.0	101.9
Total**	\$80,845	\$69,409	\$67,633	10.3%	8.5%	8.2%
			Median	9.9%	7.7%	7.6%

NOTES: Total balances include both the ending balance and Rainy Day Funds. Fiscal 2016 are actual figures, fiscal 2017 are estimated figures, and fiscal 2018 are recommended figures. N/A indicates data not available. *See notes to Table 28 on page 65. **Fiscal 2017 and Fiscal 2018 total balance amount and total balances as percentage of expenditures exclude Oklahoma, as complete data for these states was not available for this year. ***Ending Balance includes Rainy Day Fund.



Figure 17

TABLE 29

Rainy Day Fund Balances and Rainy Day Fund Balances as a Percentage of Expenditures, Fiscal 2016 to Fiscal 2018

State	Rainy Day Fund Balances (\$ in Millions)**			Rainy Day Fund Balances as a Percent of Expenditures			
	Fiscal 2016	Fiscal 2017	Fiscal 2018	Fiscal 2016	Fiscal 2017	Fiscal 2018	
Alabama	\$530	\$766	\$786	6.8%	9.3%	9.4%	
Alaska*	7,120	7,033	6,310	130.9	158.4	145.6	
Arizona	461	463	468	4.8	4.8	4.8	
Arkansas	N/A	N/A	N/A	N/A	N/A	N/A	
California*	7,573	6,761	9,424	6.6	5.5	7.7	
Colorado	513	589	676	5.0	5.6	6.1	
Connecticut*	236	259	261	1.3	1.4	1.4	
Delaware	215	221	221	5.5	5.4	5.4	
Florida	1,354	1,384	1,417	4.6	4.5	4.6	
Georgia*	2,033	N/A	N/A	9.3	N/A	N/A	
Hawaii	101	311	317	1.5	4.0	4.3	
Idaho	259	259	293	8.5	7.9	8.5	
Illinois	275	0	0	1.0	0.0	0.0	
Indiana	1,468	1,472	1,476	9.8	9.6	9.5	
Iowa	729	607	604	10.1	8.3	8.3	
Kansas	N/A	N/A	N/A	N/A	N/A	N/A	
Kentucky	209	236	179	2.0	2.1	1.6	
Louisiana	359	261	286	4.1	2.7	3.0	
Maine	122	168	173	3.7	4.9	5.1	
Maryland	832	833	860	5.2	4.9	5.0	
Massachusetts	1,292	1,303	1,401	3.2	3.1	3.2	
Michigan	612	709	1,004	6.3	7.0	9.9	
Minnesota	1,947	1,953	1,953	9.7	9.0	8.6	
Mississippi	350	337	440	6.1	5.8	7.7	
Missouri	291	294	309	3.2	3.2	3.2	
Montana	N/A	N/A	N/A	N/A	N/A	N/A	
Nebraska	731	546	503	17.4	12.6	11.4	
Nevada	0	64	62	0.0	1.7	1.5	
New Hampshire	93	100	100	6.7	6.9	6.7	
New Jersey	0	0	0	0.0	0.0	0.0	
New Mexico	146	-67	0	2.3	-1.1	0.0	
New York	1,798	1,798	1,948	2.6	2.6	2.7	
North Carolina	1,575	1,474	1,787	7.4	6.6	7.6	
North Dakota	573	0	0	19.0	0.0	0.0	
Ohio	2,005	2,034	2,034	5.8	5.7	6.0	
Oklahoma*	241	N/A	N/A	3.9	N/A	N/A	
Oregon	550	771	952	6.1	8.5	9.9	
Pennsylvania	0	0	0	0.0	0.0	0.0	
Rhode Island	192	194	196	5.4	5.3	5.2	
South Carolina	505	487	509	7.0	6.2	6.6	
South Dakota	143	157	160	9.8	9.9	9.9	
Tennessee	568	668	800	4.5	4.9	5.5	
Texas	9,715	10,254	10,972	18.4	19.4	21.2	
Utah	493	493	493	7.8	7.7	7.5	
Vermont	78	93	124	5.3	6.1	8.1	
Virginia	236	549	281	1.2	2.7	1.4	
Washington	550	1,340	1,350	3.0	6.9	6.6	
West Virginia	779	635	617	18.7	14.4	13.9	
Wisconsin	281	282	302	1.8	1.7	1.8	
Wyoming	1,811	1,481	1,481	109.7	103.0	101.9	
Total**	\$51,942	\$49,572	\$53,530	6.6%	6.3%	6.7%	
				Median	5.4%	5.5%	6.0%

NOTES: N/A indicates data not available. Fiscal 2016 are actual figures, fiscal 2017 are estimated figures, and fiscal 2018 are recommended figures. *See Notes to Table 29 on page 65. **Total Rainy day fund balance for fiscal 2017 and fiscal 2018 exclude Georgia and Oklahoma, as data were unavailable for these years.

Figure 18

THE FISCAL SURVEY OF STATES: APRIL 1996 25

TABLE A-1

Fiscal 1995 State General Fund, Actual (Millions)

Region/State	Beginning Balance	Revenues	Adjustments	Resources	Expenditures	Adjustments	Ending Balance	Budget Stabilization Fund
NEW ENGLAND								
Connecticut*	\$ 0	\$ 8,480		\$ 8,480	\$ 8,399		\$ 81	\$ 81
Maine*	4	1,672	\$ 37	1,712	1,687	\$ 26	4	10
Massachusetts*	125	15,798		15,923	15,705		179	425
New Hampshire	12	963		975	971	4	0	24
Rhode Island*	4	1,643		1,646	1,641		5	45
Vermont*	0	673	3	675	690		-15	0
MID-ATLANTIC								
Delaware*	313	1,602		1,915	1,541		374	*
Maryland	65	7,068		7,133	7,000		133	286
New Jersey*	1,204	14,898		16,138	14,947	240	952	*
New York*	399	32,296	862	33,557	33,399		158	*
Pennsylvania*	302	15,765	148	16,215	15,732	-54	429	66
GREAT LAKES								
Illinois*	230	17,302	-300	17,232	17,201	-300	331	0
Indiana*	90	7,307	-30	7,367	8,332	356	679	419
Michigan*	0	7,995	44	8,040	8,041	-2	0	1,003
Ohio*	300	15,711		16,011	14,979	962	70	828
Wisconsin*	282	7,946		8,228	7,827		401	*
PLAINS								
Iowa*	0	3,907		3,907	3,616		292	116
Kansas*	454	3,219	4	3,677	3,310		367	5
Minnesota*	903	8,759		9,652	8,605		1,057	*
Missouri	275	5,459		5,734	5,261		473	24
Nebraska*	152	1,706	1	1,858	1,683		176	21
North Dakota*	28	632		660	629		31	0
South Dakota*	0	580	41	622	589	33	0	11
SOUTHEAST								
Alabama	128	4,078		4,206	4,151		54	0
Arkansas*	0	2,400	52	2,453	2,453		0	0
Florida	198	14,179		14,377	14,248		129	282
Georgia*	120	9,625		9,745	9,500		224	288
Kentucky	98	5,188	125	5,411	5,008	144	261	100
Louisiana*	0	4,784	10	4,794	4,729	-132	146	0
Mississippi*	166	2,624		2,790	2,675		115	268
North Carolina*	888	9,972		10,860	10,034		892	*
South Carolina*	407	4,234		4,641	4,051		589	*
Tennessee*	173	5,076	90	5,339	5,174	27	138	*
Virginia*	334	7,174		7,507	7,490		17	80
West Virginia*	69	2,309	3	2,380	2,210	43	127	64
SOUTHWEST								
Arizona	229	4,466		4,695	4,425		270	223
New Mexico*	156	2,892	-60	2,788	2,714	15	0	59
Oklahoma	118	3,513		3,631	3,436		195	45
Texas*	1,929	20,563		22,492	20,640		1,852	9
ROCKY MOUNTAIN								
Colorado*	405	3,996		4,402	3,914	4	484	*
Idaho*	38	1,288	-55	1,271	1,268		3	33
Montana*	50	938	7	995	948		47	NA
Utah	37	2,365		2,402	2,341		61	66
Wyoming*	22	445	35	502	476		26	55
FAR WEST								
Alaska*	0	2,489	83	2,572	2,572		0	2,136
California*	109	42,710	-175	42,644	41,951		683	*
Hawaii	291	2,969		3,259	3,169		90	0
Nevada*	129	1,206	165	1,500	1,103	295	102	100
Oregon*	439	3,390		3,929	3,333		496	*
Washington*	402	8,534	107	9,043	8,484		559	0
TERRITORIES								
Puerto Rico	255	5,211		5,466	5,340		126	62
Total	\$12,077	\$354,584	--	\$367,893	\$352,291	--	\$13,736	\$7,171

NOTE: NA indicates data are not available.

*See Notes to Table A-1.

this reason, and because a few states don't report anything about rainy day funds, I include total balances as a balance of interest for the analysis.

So the overall procedure for collecting the balances data is as follows. First, collect the data for rainy day funds balances for each state/year observation, not including the values reported in footnotes. Second, collect the ending balances, some of which include rainy day fund balances. Third, for those observations whose ending balances include a rainy day fund balance which is reported in a footnote, subtract this value from the ending balance data and add it to the rainy day fund series. Next, construct total balances by adding the ending (less rainy day) and rainy day series together. Finally, replace values in the total balances series with reported total balances from NASBO in years for which these are reported. This is a labor intensive process, but the only way to obtain these data, given the format in which they are published.

B Mathematical appendix

B.1 Equivalence of central government budget mechanisms

In this section I show that the two alternatives for formulating the budget constraint of the central government with respect to smoothing idiosyncratic shocks result in equivalent transfer policy rules. Recall that the first alternative is the no-Ponzi-game condition given in Equation 6:

$$\lim_{t \rightarrow \infty} \mathbb{E}_0 \left[\frac{A_t}{(1+r)^t} = 0 \right].$$

This constraint prevents the central government from accumulating debt indefinitely. This constraint is valid for an economy with any number of regions.

The second alternative assumes that transfers to smooth for idiosyncratic shocks must be paid for in the current period. If regions are indexed by i , this constraint takes the form $\sum_i T_{it} = 0$ for all t , and if there are an infinite number of regions it is expressed as in

Equation 7:

$$\int T_{it} di = 0.$$

Proposition 1. *If all regions are ex ante identical, then the no-Ponzi-game budget constraint is equivalent to the period budget constraint for a continuum of regions, as both result in the condition $\mathbb{E}_0[T(s_t, \theta_t, f_t)] = 0$.*

Proof. Consider first the period budget constraint with an infinite number of *ex ante* identical, but heterogeneous, regions. By the law of large numbers, in every period the distribution over the state variables will yield densities equal to the long-run probabilities of each state. Since the transfer policy function for idiosyncratic shocks will be the same for every region, its distribution will also equal the long run distribution. This results in $\int T(s_{it}, \theta_{it}, f_{it}) di = \mathbb{E}_0[T(s_{jt}, \theta_{jt}, f_{jt})]$ for any region j , which, when plugged into the budget constraint yields $\mathbb{E}_0[T(s_{jt}, \theta_{jt}, f_{jt})] = 0$ for any region j .

Now observe the no-Ponzi-game budget constraint. This allows the central government to hold unlimited assets A_t for the purposes of smoothing idiosyncratic shocks. If regions are *ex ante* identical, then it is without loss of generality to consider a separate fund A_{it} for each region for the purposes of solving for the transfer function $T(s_{it}, \theta_{it}, f_{it})$, which will be the same in each region. The law of motion for A_{it} is given by $A_{it+1} = A_{it}(1+r) + (-T(s_{it}, \theta_{it}, f_{it}))$, which can be expanded and solved to yield $A_{it+1} = -\sum_{j=0}^t T(s_{it-j}, \theta_{it-j}, f_{it-j})(1+r)^j$. Plugging in to the budget constraint, the condition now becomes

$$\begin{aligned} \lim_{t \rightarrow \infty} \mathbb{E}_0 \left[-\sum_{j=0}^{t-1} T(s_{it-(1+j)}, \theta_{it-(1+j)}, f_{it-(1+j)})(1+r)^{(j-t)} \right] &= 0 \\ \Rightarrow \lim_{t \rightarrow \infty} -\sum_{j=0}^{t-1} \mathbb{E}_0 \left[T(s_{it-(1+j)}, \theta_{it-(1+j)}, f_{it-(1+j)}) \right] (1+r)^{(j-t)} &= 0. \end{aligned}$$

This can only be satisfied if $\mathbb{E}_0[T(s_{it}, \theta_{it}, f_{it})] = 0$ for all regions i . □

B.2 Long-run posterior variance

In this section I show that a long-run stable value for the posterior variance exists and solve for its value. First, I derive the law of motion for the posterior variance. Beginning with posterior $\hat{\sigma}_{\mu,t}^2$ in time t , the prior for next period is formed as in Equation 12: $\sigma_{\mu,t+1}^2 = \rho_\epsilon^2 \hat{\sigma}_{\mu,t}^2 + \sigma_{\xi^\epsilon}^2$. Next period's posterior is then formed according to Equation 10: $\hat{\sigma}_{\mu,t+1}^2 = \frac{\sigma_{\mu,t+1}^2 \sigma_{\xi^\theta}^2}{\sigma_{\mu,t+1}^2 + \sigma_{\xi^\theta}^2}$. Combining these two yields the law of motion for the posterior variance:

$$\hat{\sigma}_{\mu,t+1}^2 = \frac{(\rho_\epsilon^2 \hat{\sigma}_{\mu,t}^2 + \sigma_{\xi^\epsilon}^2) \sigma_{\xi^\theta}^2}{\rho_\epsilon^2 \hat{\sigma}_{\mu,t}^2 + \sigma_{\xi^\epsilon}^2 + \sigma_{\xi^\theta}^2}. \quad (14)$$

Defining the function $h(x) = \frac{(\rho_\epsilon^2 x + \sigma_{\xi^\epsilon}^2) \sigma_{\xi^\theta}^2}{\rho_\epsilon^2 x + \sigma_{\xi^\epsilon}^2 + \sigma_{\xi^\theta}^2}$ and taking the limit as $x \rightarrow \infty$, it is clear that $h(x)$ converges to a finite value: $\lim_{x \rightarrow \infty} h(x) = \sigma_{\xi^\theta}^2$. Therefore, we can argue that $\hat{\sigma}_{\mu,t+1}^2 = h(\hat{\sigma}_{\mu,t}^2)$ is bounded on \mathbb{R}^+ . Given this, for simplicity assume that $\hat{\sigma}_{\mu,0}^2$ is initialized in a region such that it converges to the fixed point of $h(x)$; this fixed point then defines the long run value.

Solving for this fixed point is straightforward:

$$\hat{\sigma}_{\mu,\infty}^2 = \frac{(\rho_\epsilon^2 \hat{\sigma}_{\mu,\infty}^2 + \sigma_{\xi^\epsilon}^2) \sigma_{\xi^\theta}^2}{\rho_\epsilon^2 \hat{\sigma}_{\mu,\infty}^2 + \sigma_{\xi^\epsilon}^2 + \sigma_{\xi^\theta}^2}$$

$$\rho_\epsilon^2 (\hat{\sigma}_{\mu,\infty}^2)^2 + (\sigma_{\xi^\epsilon}^2 + \sigma_{\xi^\theta}^2) \hat{\sigma}_{\mu,\infty}^2 = \sigma_{\xi^\theta}^2 \rho_\epsilon^2 \hat{\sigma}_{\mu,\infty}^2 + \sigma_{\xi^\theta}^2 \sigma_{\xi^\epsilon}^2$$

$$\rho_\epsilon^2 (\hat{\sigma}_{\mu,\infty}^2)^2 + (\sigma_{\xi^\epsilon}^2 + \sigma_{\xi^\theta}^2 - \sigma_{\xi^\theta}^2 \rho_\epsilon^2) \hat{\sigma}_{\mu,\infty}^2 - \sigma_{\xi^\theta}^2 \sigma_{\xi^\epsilon}^2 = 0$$

Then, by the quadratic formula (and since the solution must be positive), the result in Equation 13 is obtained:

$$\hat{\sigma}_{\mu,\infty}^2 = \frac{(\rho_\epsilon^2 - 1) \sigma_{\xi^\theta}^2 + \sigma_{\xi^\epsilon}^2 + \sqrt{[(\rho_\epsilon^2 - 1) \sigma_{\xi^\theta}^2 + \sigma_{\xi^\epsilon}^2]^2 + 4 \rho_\epsilon^2 \sigma_{\xi^\theta}^2 \sigma_{\xi^\epsilon}^2}}{2 \rho_\epsilon^2}.$$