Migration, Differential Mobility, and the Choice of Tax Base

William H. Hoyt, University of Kentucky*

November 2017

Abstract

In contrast to federal governments, lower-level governments, states and localities, do not have direct controls on migration. Then absent direct controls on migration they have incentive to account for the impacts of their policies on population flows to or away from their jurisdiction. Importantly, as examined in Hoyt and Lee (2003); Borck (2005); Buettner and Janeba (2016) they have incentive to attract highly-skilled workers who generate a fiscal surplus as the tax revenue collected from them exceeds the cost of providing services to them and discourage migration of low-skilled workers who generate a fiscal deficit.

Unlike these earlier studies that assume either costless mobility or no mobility along the lines of Lehmann, Simula and Trannoy (2014) we incorporate differential mobility costs among the two types of workers. Important for our results, and in contrast to Borck (2005), is that the policy choices of the state depend not only on the relative mobility of the two types of workers but also on how their mobility differs in response to different policies. Thus a tax decrease will lead to a larger increase in high-skilled workers relative to low-skilled workers than will an increase in the public service, the state will underprovide public service; if the reverse is true, the state overprovides services. Analogously, if a decrease in the income tax and an increase in the tax on capital from a starting point of no capital tax leads to an increase in percentage of the population that is high-skilled workers then a tax on capital is optimal; if the reverse is true, capital should be subsidized.

JEL: H20, H71, H73, R12, R28, R41

Keywords: Fiscal Competition, Vertical Externalities, Tax Base Co-occupancy

*Contact information: Department of Economics, Gatton College of Business and Economics, University of Kentucky, Lexington, KY 40506. Email: whoyt@uky.edu.

1 Introduction

Migration, more specifically immigration, has become a major issue in recent national elections and referenda in both Europe and and the United States, perhaps being the galvanizing issue behind the growth of the "populist" movement in both regions. Recently immigration concerns have been addressed in national referenda in Europe, most notably Brexit in the United Kingdom, executive orders in the United States, and legislation in Australia. While some of the concern regarding immigration stems from issues related to cultural identity and security, economic concerns have also played a significant role in the public debate. Extensive coverage of the costs and benefits of immigration is found in media and issued by "think-tanks" and advocacy groups for tightening or relaxing immigration policies. These concerns have generally focused on three specific issues: 1) the impact on wages and employment; 2) the impact on business profits; and 3) the impact on government budgets.

While the impact of aggregate immigration is subject to debate with some studies suggest a positive benefit (OECD, Dustmann and Frattini (2008)) with others more pessimistic (Borjas (2016); **?**), there seems to be a clearer consensus on the costs and benefits of different types of immigrants. This is reflected in federal restrictions to limit and affect the mix of immigrants. Some of the better known policies designed to promote immigration with favorable economic impacts are found in Canada, Australia, and Switzerland. In these countires, with the exception of policies explicitly addressing refugee relocation, immigration is focused on skilled and educated migrants, particularly in fields where there is a "domestic" shortage (at least as seen by potential employers) who would, presumably, have little adverse impact on domestic wages and employment and may actual generate government surpluses. Along these lines, the Trump administration has indicated a preference for a system more like that of Canada with a move immigration focused on worker skills and less on family reunification. The recent study Borjas (2016)provides evidence suggesting that wages of lower skilled workers and government budgets in the United States are more adversely affected by immigration.

While federal governments can explicitly restrict immigration, contrary to recent statements by several state governors in the United States, lower-level governments generally cannot explicitly restrict the flow of migrants – from other countries or states. Migration, not only from abroad but internal to the country as well, will have impacts on wages within these sub-national regions and their government budgets. The budgetary impacts are likely to be more pronounced the more decentralized the budgetary responsibilities as in countries such as Finland, Switzerland, Sweden, and the United States where more revenue is from autonomous taxes and more expenditure responsibilities lay with state and local governments.

In the United States, concerns over both the budgetary impacts of migration and the impacts that government policies have on budgets can be seen at both ends of the income distribution. Recent increases in marginal tax rates for high-income households in New Jersey, Illinois, and Oregon have generated significant attention in the media, think tanks, and scholarly research (Young and Varner (2011); Coomes and Hoyt (2008); **?**) on the impacts of state income taxes on the migration and location of high income households. Evidence from that numerous studies including Blank (1988); McKinnish (2005, 2007) and for immigrants, specifically, Borjas (1999) suggests that more generous welfare benefits lead to an increase in welfare-eligible households in state, a "welfare magnet". Razin and Wahba (2015) finds evidence that immigrants to the European Union are more likely to locate in countries with more generous welfare benefits. Brueckner (1999) summarizes the evidence that states react to changes in neighboring states' welfare policies, consistent with a "race to the bottom" as a result of mobile welfare populations.

While states and municipalities cannot directly influence migration to or from their jurisdiction that is internal to the country or from abroad, they can indirectly influence migration patterns through their setting of public policies, specifically taxes and public service expenditures. It is how these government can influence of migration through their mix and level of taxes and expenditures and how migration responses influence these choices that is the focus of this paper. Key to the results of this paper is that not only might migration responses differ among groups (high-skilled versus low-skilled) but they may differ among policies – that is, the relative impacts on migration of high-skilled workers and low-skilled in response to an increase in the income tax may be different that in response to a sales or income tax. Recently numerous studies including Kleven, Landais and Saez (2013); Akcigit, Baslandze and Stantcheva (2016); Young and Varner (2011); Young et al. (2014); Moretti and Wilson (2014, 2017) suggest that high-income earning specialists are not only regionally-mobile but even internationally mobile. A related study, Agrawal and Hoyt (2017), finds that in metropolitan areas along state borders with pronounced tax differences high-income earners engage in extensive commutes, presumably to avoid either residence or employment in the high-tax state.

This study falls into what has become an extensive literature on fiscal competition. The earliest studies focused on competition between jurisdictions for physical or, perhaps, financial capital with immobile residents (Zodrow and Mieszkowkski (1986) and Wilson (1985)). While Bucovetsky and Wilson (1991), Keen and Marchand (1997), and Huber (1999) consider the models in which labor supply is explicitly modeled and taxed, these models assume labor is immobile with only capital being mobile. Studies including Hoyt (1992, 1993); Wilson (1995); Brueckner (2000) have mobile labor but our framework differs from those found in these papers in several respects. Though labor is mobile in the papers by Hoyt and Wilson, all individuals are identical and the focus is still on the tax on capital, that is, a property tax.

Most closely related to this study are those studies that consider mobile, heterogeneous populations with respect to skills and incomes and how government policies might be strategically chosen to exploit these differences in mobility. Unlike Wildasin (2000) the distinction between skilled and unskilled is not endogenous, that is, it is not a choice of workers but instead it is exogenously determined. Hoyt and Lee (2003) considers the problem facing “rich” suburbs trying to keep lower income households, whose consumption of the public good is being subsidized by higher-income households through an income tax, and show that as a means of deterring entrance of lower-income households to the suburb, a subsidy could be provided to a a luxury good, that is, a commodity. A similar strategy of subsidizing cultural activities, specifically opera in Germany, is modeled in Buettner and Janeba (2016) with empirical support.

Perhaps most closely related to this study is Borck (2005) in which low-skilled households are immobile and high-skilled households are perfectly mobile.[1] In this setting, along the lines of Hoyt and Lee (2003) and Buettner and Janeba (2016), Borck finds that public services are overprovided to high-skilled workers and underprovided for low-skilled workers.

Here, in contrast, I assume that neither low-skilled or high-skilled are perfectly-mobile or completely immobile but instead there is a distribution of mobility as found in Hindriks (1999), Lehmann, Simula and Trannoy (2014), Myers (1990) and Mansoorian and Myers (1993). As well, I relax the assumption that high-skilled workers are stronger complements with capital than low-skilled workers that Borck (2005) makes. These modifications suggest that it is not obvious whether capital should be taxed or subsidized as well as making it less obvious whether it is, in fact, optimal to over-provide public services to the high-skilled workers.

A related literature has focused on how interjurisdictional mobility influences the design of optimal income tax systems. Early contributions include Mirrlees (1982) and Wilson (1982). More recent contributions Hamilton and Pestieau (2005) and Hindriks (1999) consider two distinct, mobile types of individuals. However, the focus of both are on the determination of transfers. Lehmann, Simula and Trannoy (2014); Simula and Trannoy (2010, 2011) considers the strategic determination of an optimal income tax structure with a continuum of individuals characterized by both differences in skills and mobility. While Hamilton and Pestieau (2005) has mobility, it serves as minimum utility constraint on the rich. As mentioned, Hin-

[1] While not the focus, Borck (2005) extends the analysis to consider perfectly-mobile low-skilled households as well.

driks (1999) and Lehmann, Simula and Trannoy (2014), as done here, assume a continuum of differences in mobility. Unlike these studies my focus is on how a region, if restricted to a uniform income tax, might strategically supplement the income tax or alter public service provision in the face of migration.

That my focus here is on multiple tax instruments rather than the only the income tax and that I assume that the income tax is approximately uniform is motivated by current state and local tax policy in the United States. While in the United States, the primary source of tax revenue for local governments is the property tax this is not the case for state governments. As can be seen in *Table 1*, in 2009 local governments collected 73.9 percent of their revenues from the property tax, another 1.2 percent from the corporate net income tax and only 4.4 percent from individual income taxes. In contrast, state governments collected 34.4 percent of their taxes from the individual income tax and only 1.8 percent and 7.3 percent from the property and corporate net income taxes. The major source of revenue for state governments are, in fact, sales taxes with on average 32 percent of state tax revenues derived from general sales taxes another 16 percent from selective sales taxes.

For many U.S. states, a uniform or "flat" tax (with an initial exemption of income) is a good approximation to their tax code. As can be seen in *Table 2* of the 41[2] U.S. states with an income tax on labor earnings, 8 have a single marginal tax rate, 10 have a top bracket that begins at $20,000 or less, and another 8 have a top bracket that begins between $20,000 and $75,000.

The analysis is done for arbitrary degrees of mobility for the two groups, with the possibility that skilled workers are more or less mobile than unskilled workers. However, evidence on the relationship between educational achievement or skill and geographical mobility suggests that in the U.S. households with higher levels of education tend to be more mobile. As can seen in *Table* 3, which gives the one year mobility rates in the U.S. from 2015 to 2016, this is particularly true for interstate moves with 2.7 percent of those with a college degree making a move to a different state or abroad per year in contrast to only 1.2 percent for high school graduates. Of course, when analyzing the determination of government policies in response to mobility it is not mobility per se that is relevant but mobility in response to changes in government tax and service policies. In the case of mobile skilled workers, the focus of policy might be the impacts of taxation on the residential location of the highly-skilled; in the case of mobile unskilled workers, redistributive policies that may influence the location of lower skilled workers are the policy focus.

Critical to the determination of government policies is their objective function. Here I

[2]Alaska, Florida, South Dakota, Texas, Washington, and Wyoming have no income tax. New Hampshire and Tennessee only tax dividend and interest income.

begin with the state government choosing its policies to maximize a very general form of a social welfare function that includes as arguments the utility of the two types of labor (skilled and unskilled) and land rents within the state. While results regarding state policies are obtained for this general form of the social welfare function, much more understandable and intuitively appealing results are obtained when I consider two specific formulations of the social welfare function. One case, which I refer to "Rent Maximization," has the government choosing its policies to maximize a utilitarian welfare function that is simply the sum of the utility of the two types of labor (skilled and unskilled) and land rents within the state. The second case, "Utility Maximization," is simply the sum of the utility of the two types of labor.

Given that the policies of these regional governments will not, in general, result in the efficient provision of public services, I then consider whether, if given the option, regions will choose to tax or, possibly, subsidize capital. Whether regions will tax or subsidize capital depends on the relationships between capital and the two types of labor, specifically whether and how strong a substitute or complement with capital each may be. The other determinant of the level and sign of a tax on capital is the relative impact of a tax increase on the number of workers in each group in the region. While it may be argued that the findings of models of fiscal competition for physical capital apply to the case of mobile human capital as well, there are some important and obvious distinctions between the two cases. As extensions I also consider and show that if the types of labor spend different shares of income on a commodity, specifically, if the low-skilled households spend more of their income on it than high-skilled households. Finally, along the lines of Borck (2005) I extend the model to consider the provision of public services provided specifically to low-skilled and high-skilled workers.

The paper proceeds as follows: in *Section 2* I present the basic model and derive comparative static results of the effects of the tax and public service policies on wages and population. *Section 3* generates the policy choices of a centralized government, that is, a government that faces no mobility of its labor force but the possibility that individual labor supply is elastic. The policies of the central government serve as a contrast to those of the regional governments that do face a mobile labor force and one, specifically, that differs in mobility. In *Section 4* I derive the optimal regional policies and contrast them to those of a centralized government, emphasizing the implications of differential heterogeneity in the labor force. *Section B* discusses extensions to a voting equilibria and *Section* 5 presents some numerical examples. *Section 6* concludes.

2 A Model of Regional Policy Determination with Heterogeneity

I consider regional government policies when there are two types of resident/workers, earning different wages and differing in both their taste for a public service and inter-regional mobility, that referred to as skilled workers and unskilled workers and denoted by superscripts *s* and *u,* respectively.

Utility for individuals of type *i, i=u,s* is given by the $U^i = x_1^i + z^i\left(x_2^i\right) + h^i(l^i) + m^i(g) + \chi^i\left(N^i\right),\ i = s, u$ where $x_1^i$and x_2^i are traded commodities, g is the public service, and l^i is labor supply with the standard conditions applying – $\frac{\partial z^i}{\partial x_2^i} > 0$, $\frac{\partial^2 z^i}{\partial x_2^{i2}} < 0$, $\frac{\partial h^i}{\partial l^i} < 0$, $\frac{\partial^2 h^i}{\partial l^{i2}} < 0$, $\frac{\partial m^i}{\partial g} > 0$, and $\frac{\partial^2 m^i}{\partial g^2} < 0$. The term $\chi^i\left(N^i\right)$ is the mobility cost where I assume that $\frac{\partial \chi^i}{\partial N} < 0$ – as the number workers (N^i) in each group increases, the "attachment to home" for the marginal worker decreases. Following Lehmann, Simula and Trannoy (2014) $\chi^i\left(N^i\right)$ is the difference in utility obtained in the region and the utility that the individual could receive elsewhere $\overline{U}^i$ and we define $\chi^i\left(\overline{N}^i\right) = 0$ where $\overline{N}^i$ is the equilibrium population of type i in the region with all residents of the region having $\chi^i > 0$ and any type i not in the region having $\chi^i < 0$. [3]

The indirect utility function, then, is given by $U^i = V^i\left(w^i\left(1 - \tau_I^i\right), \tau_s\right) + m^i\left(g\right) + \chi^i\left(N^i\right),\ i = s, u$ where w^i is the wage rate for group i, τ_I is the income tax rate, and τ_s is a tax on commodity 2.[4] As discussed, following the institutional practices of many regional (state) governments, I restrict the income tax to a single rate (τ_I).

Production is characterized by $f(L^s, L^u, K, Q)$ where labor supply for each group in the region is the product of the number of workers and labor per worker, $L^i = N^i l^i,\ i = s, u$. The amount of capital is given by K with ρ being the net-of-tax rate of return on capital. Finally, land (Q), inelastically supplied in the region is used in production as well. I assume that in equilibrium that the wages (w^j, $j = s, u$) are such that $w^s > w^u$. Then with competitive firms profits will equal zero and land rent (R), then, is the difference between output and payments to labor and capital or

[3]The primary role of attachment of home in this framework is to allow for incomplete capitalization with the extent of capitalization varying with the extent of the attachment. An alternative means of generating differential capitalization would be to allow for a sufficiently small number of regions such that the extent of capitalization varies with the number of regions. It is important to note that in Myers (1990) and Lehmann, Simula and Trannoy (2014), for example, there are only two regions so the term is the taste for one region relative to the other region. Here, I do not wish to consider how individual region's policies affect wage rates in other regions when setting policies. This being the case, a large number of regions is necessary. Then cannot be interpreted strictly as preference for one region relative to another single region but rather relative to one of the *J-1* other regions. I assume that with a large N we can treat this as a continuous function. The assumption of imperfect mobility allows for incomplete capitalization of policies into wages.

[4]I normalize the net-of-tax price of both commodities to unity.

$$R = f\left(L^s, L^u, K, Q\right) - L^s w^s - L^u w^u - \left(\rho + \tau_k\right) K \tag{2.1}$$

where τ_k is the tax rate on a unit of capital.

The nature of public service cost functions, specifically the extent to which costs are proportional to population, varies with the service and the level of government. For many public services at the federal level, most notably, national defense, costs are relatively independent of population; for state and local public services, police and fire protection, educational services, for example, estimates of cost functions suggests that there are congestion costs but also economies of scale (Craig (1987); Hayes and Slottje (1987); Blesse and Baskaran (2013); Gronberg, Jansen and Taylor (2011)). As others, including Boadway and Flatters (1982) and Wilson (1995), have considered the implications of public service costs not proportional to population, my focus is on the case in which costs are proportional to population. The general form of the cost function, then, is $c(N^s, N^u, g) = (N^S + N^u)g$.

The government budget constraint can now be expressed as

$$S\left(\tau_I, \tau_k, \tau_s, g\right) \equiv \tau_I\left(N^s w^s l^s + N^u w^u l^u\right) + \tau_s\left(N^s x_2^s + N^u x_2^u\right) + \tau_k K - \left(N^s + N^u\right) g = 0 \tag{2.2}$$

where, to simplify expressions and reduce notation, I assume that both aggregate income and population sum to one, that is, $N^s w^s l^s + N^u w^u l^u = N^s + N^u = 1$ in equilibrium. While there are assumed to be more unskilled workers than skilled workers, $N^u > N^s$, skilled workers have a greater share of labor income, $\phi^s > \phi^u$, where $\phi^i \equiv \frac{N^i w^i l^i}{N^s w^s l^s + N^u w^u l^u},\ i = u, s$.

2.1 Government Objective

In this simple characterization of regional economies there are three constituencies (groups) in the economy: skilled workers, unskilled workers, and owners of land. To provide some characterization of expected government tax policies requires a characterization of the government objective, that is, a social welfare process or, alternatively, a political process that determines policy. Here I posit a general form of a social welfare function,

$$W\left(\tau_I \tau_k, g\right) = \omega^R R + \omega^S \overline{N}^s U^s + \omega^u \overline{N}^u U^u \tag{2.3}$$

where $\omega^i = 1,\ i = u,\ s,\ R$ are the weights on the three groups in the economy.

While a general social welfare function allows for flexibility on the welfare weights associated with each group, it does not allow much to be said about specific policy outcomes. This being the case, I focus first on the impacts of the policies on the two groups of workers and the landowners (rent) whose welfare are the arguments in the welfare function. Then, while I state the policies that maximize a general social welfare function, my focus is on two special cases of social welfare. The first is one in has equal weights on all three arguments of the social welfare function. As the $\overline{N}^i$ is the given (equilibrium) population of type i workers, this might be interpreted as the socially-optimal policies maximize utility for current residents and landowners. This objective, in a sense, is an analog to land value maximization found, for example, in Hoyt (1991), Wilson (1995) and Henderson (1985) with the objective being maximization inelastic factors in the region but in this case both land rent and the utility of infra-marginal residents. Alternatively, having $\omega^R = 0$ and $\omega^s = \omega^u = 1$ would be consistent with only the utility of current residents being considered when setting policy and the welfare of possibly absentee landowners ignored.

2.2 Equilibrium Conditions

Equilibrium requires that labor and capital markets clear in each of the regions,

$$w^i = f_i\left(L^s, L^u, K\right), \; i = s, u \tag{2.4a}$$

$$\rho + \tau_k = f_K\left(L^s, L^u, K\right), \tag{2.4b}$$

where $f_i = \frac{\partial f}{\partial N^i}$, $i = s, u$ and $f_K = \frac{\partial f}{\partial K}$. As well the marginal worker in each group must receive the same utility that she can obtain elsewhere,

$$V^i\left(w^i\left(1 - \tau_I^i\right), \tau_s\right) + m^i\left(g\right) + \chi^i\left(N^i\right) = \overline{U}^i, \; i = s, u. \tag{2.4c}$$

Finally the zero profit condition (2.1) must be satisfied. The equilibrium conditions will be used to determine the impacts of changes in government policies on wages, populations, the government budget, and worker utility.

2.3 Comparative Statics

2.3.1 Wage and Population Gradients

The extent that the impacts of taxes and public services are capitalized into wage rates and how this capitalization may differ between the two types of labor is of critical importance in determining both the optimal regional and central government policies. Capitalization not only affects the distribution of welfare impacts among the workers and landowners but also the impact of policy changes on the budget constraint. Differentiating (2.4a) - (2.4c) with respect to the policies τ_I, τ_k, τ_s and g yields the (percentage) changes in the wage rate

$\hat{w}^i_x,\ i = u, s;\ x = \tau_I,\ \tau_k,\ \tau_s,\ g.$

$$\hat{w}^i_{\tau_I} = \frac{(\gamma^j + \theta - \eta_{jj})(\theta + \gamma^i) + \eta_{ij}(\theta + \gamma^j)}{|A|(1-\tau_I)} = \frac{(\gamma^j + \theta - \eta_{jj})\gamma^i + \eta_{ij}\gamma^j}{|A|(1-\tau_I)} + \theta\frac{(\gamma^j + \theta - \eta_{jj}) + \eta_{ij}}{|A|(1-\tau_I)} \tag{2.5a}$$

$$\hat{w}^i_{\tau_k} = \frac{(\gamma^j + \theta - \eta_{jj})\eta_{ik} + \eta_{ij}\eta_{jk}}{|A|(\rho + \tau_k)} \tag{2.5b}$$

$$\hat{w}^i_{\tau_s} = \frac{\left[\ (\gamma^j + \theta - \eta_{jj})(\gamma^i + \theta) B^i_2 + \eta_{ij}\ \ (\gamma^j + \theta) B^j_2\right]}{|A|(1-\tau_I)},\ and \tag{2.5c}$$

$$\hat{w}^i_g = -\frac{\left[\ (\gamma^j + \theta - \eta_{jj})\gamma^i \frac{MRS^i}{w^i l^i} + \eta_{ij}\gamma^j \frac{MRS^j}{w^j l^j}\ \right]}{|A|(1-\tau_I)}. \tag{2.5d}$$

where $\gamma^i = \frac{dN^i}{d[w^i(1-\tau^i)]}\frac{w^i\left(1-\tau^i\right)}{N^i} = -\frac{1}{\chi^{j'}}\frac{w^j(1-\tau)}{N^i} > 0$ is, as in Lehmann, Simula and Trannoy (2014), the elasticity of migration for group j with respect to their consumption; $\eta_{ji} = \frac{\partial L^j}{\partial w^i}\frac{w^i}{L^j},\ i, j = s, u$ is the elasticity of demand for workers of type j with respect to the wages of workers of type i and $\eta_{jk} = \frac{\partial N^j}{\partial (r+\tau_k)}\frac{r+\tau_k}{N^j},\ j = s, u$ is the elasticity of demand for workers of type j with respect to the (gross) price of capital. The term $\theta = \frac{\partial l^i}{\partial [w^i(1-\tau^i)]}\frac{w^i\left(1-\tau^i\right)}{l^i}$ is the (uncompensated) elasticity of labor supply with respect to the net wage. In (2.5c) $B^i_2 = \frac{x^i_2}{w^i l^i}$ is the budget share of commodity 2. Finally, $|A| = (\gamma^s + \theta - \eta_{ss})(\gamma^u + \theta - \eta_{uu}) - \eta_{su}\eta_{us} > 0$ by the second order condition. Then the associated population changes are given by

$$\hat{N^i_{\tau_I}} = \ \gamma^i \frac{\left[\eta_{ii}\left(\gamma^j + \theta - \eta_{jj}\right) + \eta_{ij}\left(\theta + \gamma^j + \eta_{ji}\right)\right]}{|A|(1-\tau_I)} \tag{2.6a}$$

$$\hat{N^i_{\tau_k}} = \gamma^i \frac{(\gamma^j + \theta - \eta_{jj})\eta_{ik} + \eta_{ij}\eta_{jk}}{|A|(\rho + \tau_k)} \tag{2.6b}$$

$$\hat{N}^i_{\tau_s} = -\frac{\gamma^i}{|A|(1-\tau_I)}\left[\ (-\eta_{jj}(\gamma^j + \theta - \eta_{jj}) - \eta_{su}\eta_{us}) B^i_2 - \eta_{ij}\ \ \left(\gamma^j + \theta\right) B^j_2\right],\ i, j = s, u; i \neq j \tag{2.6c}$$

$$\hat{N^i_g} = \ \gamma^i \left[\frac{\left[(\theta - \eta_{ii})\left(\gamma^j + \theta - \eta_{jj}\right) - \eta_{su}\eta_{us}\right]\frac{MRS^i}{w^i l^i} - \eta_{ij}\gamma^j \frac{MRS^j}{w^j l^j}}{|A|(1-\tau_I)}\right] \tag{2.6d}$$

Finally, the impact of the changes in land rent are

$$dR_x = -\left[N^s l^s \hat{w}^s_x + N^u l^u \hat{w}^u_x\right],\ x = \tau_I, \tau_s, g \tag{2.7a}$$

$$dR_{\tau_k} = -\left[K + N^s l^s \hat{w}^s_{\tau_k} + N^u l^u \hat{w}^u_{\tau_k}\right] \tag{2.7b}$$

More details on the derivation of (2.5a) - (2.5d) and (2.6a) - (2.6d) are found in the *Appendix* A.1.

2.3.2 Welfare Impacts

$$\frac{dU^i}{d\tau_I} = \frac{w^i l^i}{|A|}\left[(\eta_{ii} + \eta_{ij})\left(\gamma^j + \theta\right) - (\eta_{ii}\eta_{jj} - \eta_{su}\eta_{us})\right] < 0 \tag{2.8}$$

$$\frac{dU^i}{d\tau_k} = \frac{w^i l^i}{|A|\,(\rho + \tau_k)}\left[\left(\gamma^j + \theta - \eta_{jj}\right)\eta_{ik} + \eta_{ij}\eta_{jk}\right] \tag{2.9}$$

$$\frac{dU^i}{d\tau_s} = \frac{w^i l^i}{|A|}\left[\ (\eta_{ii}\left(\gamma^j + \theta - \eta_{jj}\right) + \eta_{su}\eta_{us})\, B^i_2 + \eta_{ij}\ \ \left(\gamma^j + \theta\right) B^j_2\right] < 0 \ and \tag{2.10}$$

$$\frac{dU^i}{dg} = \frac{w^i l^i}{|A|}\left[\ ((\theta - \eta_{ii})\left(\gamma^j + \theta - \eta_{jj}\right) - \eta_{su}\eta_{us})\,\frac{MRS^i}{w^i l^i} - \eta_{ij}\ \ \left(\gamma^j + \theta\right)\frac{MRS^j}{w^j l^j}\right] > 0 \tag{2.11}$$

3 Optimal Centralized Policies

To provide a benchmark for the policies choice of independent regions faced with mobile labor, I first briefly outline the policy choices of a central (federal) government choosing uniform policies for all regions. As it is assumed that labor is only mobile between the regions within the country, labor mobility will play no role in the federally-determined policies . I continue to assume that capital is internationally mobile. Then with no labor mobility ($\gamma^i = 0,\ i = s, u$) using (2.5a) - (2.5d) I obtain the wage gradients,

$$\hat{w}^i_{\tau_I}\Big|_{\gamma^i=0} = \frac{\theta[\theta - \eta_{jj} + \eta_{ij}]}{|A^C|(1-\tau_I)},\ \ \hat{w}^i_{\tau_k}\Big|_{\gamma^i=0} = \frac{(\theta - \eta_{jj})\eta_{ik} + \eta_{ij}\eta_{jk}}{|A^C|(\rho + \tau_k)},\ \ and\ \ \hat{w}^i_g\Big|_{\gamma^i=0} = 0. \tag{3.1}$$

where $\left|A^C\right| = (\theta - \eta_{ss})(\theta - \eta_{uu}) - \eta_{su}\eta_{us} > 0$. As can be seen in (3.1), income taxes only affect wages to the extent that labor supply is elastic with the $sign\left\{\hat{w}^i_{\tau_I}\big|_{\gamma^i=0}\right\} = sign\left\{\theta\right\}$ – if a decrease (increase) in gross wage increases labor supply then the net wage will decrease (increase). The tax on capital will affect wages even with an inelastic labor supply with the impact determined by the cross-price elasticity of labor with respect to capital with$sign\left\{\hat{w}^i_{\tau_k}\big|_{\gamma^i=0}\right\} = sign\left\{\eta_{iK}\right\}$. As changes in the level of the public service will not have any impact on labor supply when labor is immobile there is no impact of the public service on wages.

3.1 Optimal Income Tax

I first consider centralized tax policy when the government is restricted to using a uniform income tax rate. Then with the central government maximizing welfare (**??**) subject to the budget constraint (2.2) with $\tau_k = 0$, the first order conditions that can be expressed as

$$\tfrac{\partial W}{\partial g} = \omega^s N^s \tfrac{MRS^s}{w^s l^s} + \omega^u \phi^u \tfrac{MRS^u}{w^u l^u} = \lambda. \tag{3.2a}$$

where $\overline{w}_x = \phi^s \hat{w}_x^s + \phi^u \hat{w}_x^u$, the income-weighted average wage capitalization with respect to policy x and $\widetilde{\omega w}_x = \left((1-\tau_I)\,\omega^u - \omega^R\right)\phi^u \hat{w}^u_{\tau_I} + \left((1-\tau_I)\,\omega^s - \omega^R\right)\phi^s \hat{w}^s_{\tau_I}$ is the direct impact of wage capitalization on social welfare. When the government is rent-maximizing $(\omega^s = \omega^u = \omega^R = 1)$ $\widetilde{\omega w}_x$ simplifies to $-\tau_I \overline{w}_x$ and when the government is utility maximizing $(\omega^s = \omega^u = 1,\ \omega^R = 0)$, $\widetilde{\omega w}_x$ simplifies to $(1-\tau_I)\,\overline{w}_x$.

$$\frac{\partial W}{\partial \tau_I} = \ -(\phi^s + \phi^u) + \widetilde{\omega w}_{\tau_I} + \lambda\left[1 + \tau_I\left((1+\theta)\,\overline{w}_{\tau_I} - \tfrac{\theta}{(1-\tau_I)}\right)\right] \ = 0 \tag{3.3}$$

Then using (3.4) to substitute for λ in (3.2a) gives

$$\omega^s \phi^s \frac{MRS^s}{w^s l^s} + \omega^u \phi^u \frac{MRS^u}{w^u l^u} = \lambda = \begin{array}{c} (\omega^s \phi^s + \omega^u \phi^u)\left[1 - \frac{\tau_I\left(\overline{w}_{\tau_I} + \theta\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)\right)}{\left[1+\tau_I\left((1+\theta)\overline{w}_{\tau_I} - \frac{\theta}{(1-\tau_I)}\right)\right]}\right] \\ + \frac{\widetilde{\omega w}_{\tau_I}}{\left[1+\tau_I\left((1+\theta)\overline{w}_{\tau_I} - \frac{\theta}{(1-\tau_I)}\right)\right]} \end{array} \tag{3.4}$$

Note that if labor supply is inelastic $(\theta = 0)$ then (3.4) simplifies to $\omega^s \phi^s \frac{MRS^s}{w^s l^s} + \omega^u \phi^u \frac{MRS^u}{w^u l^u} = \omega^s \phi^s + \omega^u \phi^u$, the first-best outcome. In general, determining when the public service level is above or below the first-best outcome is complicated by the indeterminant sign of the term $\widetilde{\omega w}_{\tau_I}$ However, if the two cases of rent and utility maximization can be considered "bounds" on the weights of the welfare function, then as discussed below, if labor has a positive elasticity $(\theta > 0)$ the public service is less than the first-best level; if labor supply has a negative elasticity $(\theta < 0)$ the public service level is above the first-best level.

In the case of rent maximization (3.4) simplifies to

$$N^s MRS^s + N^u MRS^u = \lambda = 1 - \frac{\theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)}{\left(1 + \tau_I \overline{w}_{\tau_I} + \theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)\right)}. \tag{3.5a}$$

As $N^s MRS^s + N^u MRS^u = 1$ characterizes the efficient allocation in this case, the value of θ determines whether the public service is underprovided $(\theta > 0)$ or overprovided $(\theta < 0)$. For utility maximization (3.4) simplifies to

$$N^s MRS^s + N^u MRS^u = \lambda = 1 - \frac{\theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) - \overline{w}_{\tau_I}}{\left[1+\tau_I\left((1+\theta)\,\overline{w}_{\tau_I} - \frac{\theta}{(1-\tau_I)}\right)\right]}. \tag{3.5b}$$

While (3.5b) differs from (3.5a), like (3.5a), the public service is underprovided when labor supply elasticity is positive ($\theta > 0$) and therefore $\lambda > 1$ and overprovided when the elasticity is negative ($\theta < 0$) and $\lambda < 1$. However, difference between λ, the marginal cost of funds, and the cost of providing the public service *(1)* is greater (in absolute value) with utility maximization because of the additional term ($\overline{w}_{\tau_I}$)than with rent maximization (3.5a). This suggests a greater difference between the utility-maximizing level of public service and the efficient level than the difference between the rent-maximizing and the efficient levels, not surprising given that utility-maximizing governments ignore the impacts on land rents.

3.2 Optimal Taxation with Capital and Income Taxation

The focus of much of the literature on fiscal competition has been on capital taxation. The most prototypical models have assumed mobile capital and immobile labor. In this section, I follow Bucovetsky and Wilson (1991) and consider immobile but elastic labor. While I replicate some of their results I do so to provide a comparison for capital taxation as set by regional governments faced with a differently-mobile population. Then if the central government can tax both capital and wage income, the first order conditions are

$$\frac{\partial W}{\partial \tau_I} = -\left(\omega^s\phi^s + \omega^u\phi^u\right) + \widetilde{\omega w}_{\tau_I} + \lambda\left[1+\tau_I\left((1+\theta)\,\overline{w}_{\tau_I} - \frac{\theta}{(1-\tau_I)}\right) + \tau_k K\hat{K}_{\tau_I}\right] = 0 \tag{3.6a}$$

$$\frac{\partial W}{\partial \tau_K} = \left(-K + \widetilde{\omega w}_{\tau_k}\right) + \lambda\left[K + \tau_I\,(1+\theta)\,\overline{w}_{\tau_K} + \tau_K\hat{K}_{\tau_K}\right] = 0 \tag{3.6b}$$

where $\hat{K}_{\tau_I} = \eta_{ks}\hat{w}^s_{\tau_I} + \eta_{ku}\hat{w}^u_{\tau_I}$ and $\hat{K}_{\tau_k} = \frac{\eta_{kk}}{\rho+\tau_k}$. If both types of labor are substitutes (complements) with capital ($\eta_{uk} > (<)\ 0,\ \eta_{sk} > (<)\ 0$), then $\overline{w}_{\tau_K} > (<)\ 0$. Again the first order condition with respect to the public service is given by (3.2a). To determine whether it is optimal to tax capital, evaluate (3.6b) with $\tau_k = 0$ to give

$$\left.\frac{\partial W}{\partial \tau_K}\right|_{\tau_k=0} = (\lambda-1)\,K + \lambda\tau_I\,(1+\theta)\,\overline{w}_{\tau_K} + \widetilde{\omega w}_{\tau_k} \tag{3.7}$$

In general, the sign of (3.7) is not obvious. More can be said about the rent and utility maximizing cases. In these cases, if $\theta > (<)\ 0$ then $\lambda > (<)\,1$. For rent maximization, as $\widetilde{\omega w}_{\tau_k} = -\tau\overline{w}_{\tau_k}$ then the last two terms of (3.7) can be expressed as $\tau_I\,(\lambda\,(1+\theta)-1)\,\overline{w}_{\tau_K}$ that is, if capital and labor are substitutes, positive if $\theta > 0$ and is negative if $\theta < 0$. Then as $\lambda > (<)1$ when $\theta > (<)\ 0$ it follows that $\left.\frac{\partial W}{\partial \tau_K}\right|_{\tau_k=0} > (<)\,0$ when $\theta > (<)\ 0$. When capital and labor are substitutes under rent maximization capital will be taxed when labor supply has a positive elasticity ($\theta > 0$), subsidized when labor has a negative supply elasticity ($\theta < 0$), and neither taxed nor subsidized when labor supply is inelastic ($\theta = 0$). When labor and capital are complements, the sign of (3.7) is indeterminant as it the terms $\lambda - 1$ and $\tau_I \left(\lambda \left(1+\theta\right) - 1\right) \overline{w}_{\tau_K}$ will be of opposite signs.

With utility maximization, the last two terms of (3.7) equal $\left[1 + \tau_I \left(\lambda \left(1+\theta\right) - 1\right)\right] \overline{w}_{\tau_k}$. For $\theta > 0$, the bracketed term is clearly positive and given plausible values for λ, θ, and τ_I it seems reasonable that it is positive when $\theta < 0$ as well. If labor and capital are substitute, making $\overline{w}_{\tau_k} > 0$ both terms of (3.7) are positive when $\theta > 0$ as $\lambda > 1$ and it is optimal to tax capital $\left(\frac{\partial W}{\partial \tau_K}\Big|_{\tau_k=0} > 0\right)$; if the labor and capital are complements, making $\overline{w}_{\tau_k} < 0$, then if $\theta < 1$ both terms are negative and it is optimal to subsidize capital $\left(\frac{\partial W}{\partial \tau_K}\Big|_{\tau_k=0} < 0\right)$. Unlike the case of rent-maximization, when labor supply is inelastic ($\theta = 0$), it will still be optimal to tax capital when labor and capital are substitutes and subsidize them when they are complements because of the positive benefits of wage appreciation to labor and the fact that the negative effects on land rent are not considered.

The following proposition provides a summary of the key results of this section.

Proposition 1. *Let the elasticity of labor be represented by θ, the level of public service that satisfies the condition $\omega^s \phi^s MRS^s + \omega^u \phi^u \frac{MRS^u}{w^u l^u} = \omega^s \phi^s + \omega^u \phi^u$ by $g^*$and the public service level chosen by the central government when it only taxes labor income be given by g_L^c. Then:*
a) In the absence of capital taxation, the public service level of the central government, $g_L^C < (=) > g^*$as $\theta > (=) < 0$.
b) With rent-maximizing governments:

i) Capital will be taxed in addition to income when $\theta > 0$ and capital is a substitute for labor ($\eta_{sk} > 0,\ \eta_{uk} > 0$).

ii) Capital will be subsidized $\theta < 0$ and capital is a substitute for labor ($\eta_{sk} > 0,\ \eta_{uk} > 0$).

c) With utility maximizing governments:

i) Capital will be taxed in addition to income when $\theta \geq 0$ and capital is a substitute for labor ($\eta_{sk} > 0,\ \eta_{uk} > 0$).

ii) Capital will be taxed in addition to income when $\theta \leq 0$ and capital is a complement for labor ($\eta_{sk} < 0,\ \eta_{uk} < 0$).

4 Optimal Regional Policies

4.1 Optimal Regional Income Tax Policy

I begin by considering the problem facing a region when it taxes income but cannot tax capital. I then consider the possibility of taxing capital as well as labor income. In the absence of any fixed costs ($F = 0$) when the regions only taxes income, the first order conditions can be expressed as

$$\frac{\partial W}{\partial \tau_I} = -\left(\omega^s \phi^s + \omega^u \phi^u\right) + \widetilde{\omega} w_{\tau_I} + \lambda \left[1 + \tau_I \overline{w}_{\tau_I} + \theta \tau_I \left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + \ N^s\left(\tau_I w^s l^s - g\right) \ \left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right)\right] = 0 \quad (4.1a)$$

$$\frac{\partial W}{\partial g} = \omega^s \phi^s \frac{MRS^s}{w^s l^s} + \omega^u \phi^u \frac{MRS^u}{w^u l^u} + \widetilde{\omega} w_g + \lambda \left[-1 + \tau_I (1+\theta) \overline{w}_g + N^s \left(\tau_I w^s l^s - g\right) \left(\hat{N}^s_g - \hat{N}^u_g\right)\right] = 0. \quad (4.1b)$$

Equations (4.1a) and (4.1b) are simplified by using the budget constraint, (2.2), to substitute $-N^s\left(\tau w^s l^s - g\right)$ for $N^u\left(\tau w^u l^u - g\right)$. The key distinction between (4.1a) and the first order with respect to the tax rate for the central government, (**??**), is the impact of migration on net revenue, $N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right)$. The distinction between (4.1b) and the first order condition for the public service for the central government, (3.2a), are the changes in net revenue because of migration induced by changes in the public service, $N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}^s_g - \hat{N}^u_g\right)$ and the impact of changes in wages $\left(\lambda \tau_I (1+\theta) \overline{w}_g + \widetilde{\omega} w_g\right)$. Alternatively we can express the first order conditions by dividing (4.1b) by (4.1a) to obtain

$$\left.\frac{d\tau_I}{dg}\right|^R_{W=\overline{W}} = \frac{\omega^s \phi^s \frac{MRS^s}{w^s l^s} + \omega^u \phi^u \frac{MRS^u}{w^u l^u} + \widetilde{\omega} w_g}{\left(\omega^s \phi^s + \omega^u \phi^u\right) - \widetilde{\omega} w_{\tau_I}}$$

$$= \frac{1 - \tau_I (1+\theta) \overline{w}_g - N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}^s_g - \hat{N}^u_g\right)}{1 + \tau_I \overline{w}_{\tau_I} + \theta \tau_I \left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + \ N^s\left(\tau_I w^s l^s - g\right) \ \left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right)} = \left.\frac{d\tau_I}{dg}\right|^R_{R=\overline{R}}. \quad (4.2)$$

where (4.2) is simply the condition that the marginal rate of welfare substitution, the slope of the social-indifference curve equals the marginal tax cost of the public service, the slope of the budget constraint. In obvious contrast to the first order condition for the central government, (**??**), the mobility of the labor force affects the slope of the budget constraint, $\left(\left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}\right)$, with it becoming steeper if a tax increase leads to a greater (percentage) reduction in the population of skilled workers than unskilled workers $\left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right)$ because of the fiscal surplus they generate $\left(\tau_I w^s l^s - g\right)$ but decreases if the public service increase leads to a greater increase in skilled workers $\left(\hat{N}^s_g - \hat{N}^u_g\right)$.

4.1.1 A Comparison of Tax Policies

To better understand the policies of regional governments and the impacts that differentially mobility has upon them, I consider the regional public service level $\left(g^R\right)$ relative to the public service level chosen by the central government (g^c). To determine whether g^R is above or below g^C, I determine the conditions at which the marginal rate of welfare substitution for the regional government, $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R}$, is greater or less than the marginal tax cost of the public service, $\left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{R}$ when evaluated at the tax/public service policy of the central government, $\{\tau_I^c, g^c\}$. If $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R} > \left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{R}$ at $\{\tau_I^c,\ g^c\}$ at then $g^R > g^c$; if $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R} < \left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{R}$ then $g^R < g^c$. For the general case, a direct comparison of $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R}$ to $\left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{R}$ is not very enlightening. Instead, I compare $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R}$ to $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{c}$ and $\left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{R}$ to $\left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{c}$ to be able to provide some conditions on the relative values of the public services for the two governmental settings. For the rent and utility-maximizing policies, comparisons are more straightforward.

Proposition 2. *Let $\{g^c, \tau_I^c\}$ denote the level of public service and the associated income tax rate chosen by a central government and let g^R denote the level of public service chosen by a regional government.*

a) If $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R} > (=) < \left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{c}$ *and* $\left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{R} < (=) > \left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{c}$ *or, equivalently,*

$$\frac{\left[\theta\tau_I^*\overline{w}_g+N^s\left(\tau_I^*w^sl^s-g\right)\left(\hat{N}_g^s-\hat{N}_g^u\right)\right]}{1-\tau_I^*\overline{w}_g} > (=) < \frac{-\left[\theta\tau_I^*\left(\overline{w}_{\tau_I}-\frac{1}{(1-\tau_I)}\right)+N^s\left(\tau_I^*w^sl^s-g\right)\left(\hat{N}_{\tau_I}^s-\hat{N}_{\tau_I}^u\right)\right]}{1+\tau_I^*\overline{w}_{\tau_I}} \tag{4.3a}$$

then $g^R > (=) < g^*$.
ii) if $\theta = 0$ and if at g^* $\frac{MRS^s}{w^sl^s} > (=) < \frac{MRS^u}{w^ul^u}$ then $g^R > (=) < g^*$.
c) If costs are given by $C(N^s, N^u, g) = (N^s + N^u)\,g$ and

$$\frac{\left[\theta\tau_I^*\overline{w}_g+N^s\left(\tau_I^*w^sl^s-g\right)\left(\hat{N}_g^s-\hat{N}_g^u\right)\right]}{1-\tau_I^*\overline{w}_g} > (=) < \frac{-\left[\theta\tau_I^*\left(\overline{w}_{\tau_I}-\frac{1}{(1-\tau_I)}\right)+N^s\left(\tau_I^*w^sl^s-g\right)\left(\hat{N}_{\tau_I}^s-\hat{N}_{\tau_I}^u\right)+Z\right]}{1+\tau_I^*\overline{w}_{\tau_I}} \tag{4.3b}$$

then $g^R > (=) < g^C$.where $Z = -\theta\frac{\tau_I\left(\overline{w}_{\tau_I}^C-\frac{1}{(1-\tau_I)}\right)}{1-\tau_I^*\overline{w}_g}\left[\frac{\theta\tau_I^*\left(\overline{w}_{\tau_I}-\frac{1}{(1-\tau_I)}\right)+N^s\left(\tau_I^*w^sl^s-g\right)\left(\hat{N}_{\tau_I}^s-\hat{N}_{\tau_I}^u\right)+Z}{\left(1+\tau_I\overline{w}_{\tau_I}^C+\theta\tau_I\left(\overline{w}_{\tau_I}^C-\frac{1}{(1-\tau_I)}\right)\right)}\right]$.

Proof of *Proposition 2* is found in *Appendix A.3.2*. It is worth noting that part a) of the proposition is simply a special case of part b) when the absence of fixed cost *(F)* allows for the substitution of $-N^s\left(\tau_I^*w^sl^s-g\right)$ for $N^u\left(\tau_I^*w^ul^u-g\right)$. Then focusing on part a), (4.3a) is derived using the expressions for the slopes of the budget constraint, $\left(\left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{R}\right)$, and the indifference curves $\left(\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R}\right)$ from (4.2) evaluated at $\{\tau_I^*, g^*\}$ and is simply the condition that if $\left.\frac{d\tau_I}{dg}\right|^{R}_{W=\overline{W}} > (=) < \left.\frac{d\tau_I}{dg}\right|^{R}_{R=\overline{R}}$ then $g^R > (=) < g^*$. These terms can be interpreted as the ratio of the "distortionary" budgetary changes arising from an elastic labor supply and the inter-regional mobility of labor and the non-distortionary change in budget, that is, the change in budget in the absence of elastic and differentially-mobile labor. Focusing on the impacts of labor mobility, key to whether the public service is over-provided or under-provided is the difference in mobility between skilled and unskilled workers with respect to changes in the public service *relative* to the difference with respect to the income tax. If increases in public service stimulates a more positive (less negative) increase in the population of the skilled labor relative to unskilled labor than is achieved through a decrease in the income tax the region will set a public service level above the efficient level; if a decrease in the income tax leads to a greater percentage change in the population of skilled labor than an increase in the level of the public service, the region will set a public service below the efficient level. The two cases are illustrated in *Figure 1.* In the figure is an indifference curve tangent to the budget constraint in the absence of any distortion at $\{\tau_I^*, g^*\}$. Then also illustrated are two budget constraints, one reflecting the case in which $\hat{N}^s_g - \hat{N}^u_g > \hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}$ with the other reflect the case of $\hat{N}^s_g - \hat{N}^u_g < \hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}$. As can be seen in the figure when $\hat{N}^s_g - \hat{N}^u_g > \hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}$ $\left.\frac{d\tau_I}{dg}\right|^{R}_{W=\overline{W}} > \left.\frac{d\tau_I}{dg}\right|^{R}_{R=\overline{R}}$ and utility will increase with an increase in the public service and tax rate. The reverse is true when $\hat{N}^s_g - \hat{N}^u_g < \hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}$.

As shown in *Appendix A.3.2* when $\theta = 0$, the condition of whether the public service will be greater or less than the efficient level depends entirely on whether $\frac{MRS^s}{w^s l^s} > (=) < \frac{MRS^u}{w^u l^u}$, the "demand" for the public service relative to its tax price for each type of labor, at $\{\tau_I^*, g^*\}$. This is the case because when $\theta = 0$, the changes in population due to public service increases are proportional to the changes in population due to income tax decreases.[5] If there is a fixed cost, *part b)* of the proposition demonstrates only the marginal revenue and marginal costs with respect to population determine whether the public service is underprovided or overprovided.

Finally, comparison with respect to the public service provided by the central government when $\theta \neq 0$ require accounting for the public service level $\left(g^C\right)$ not being the efficient level but above or below it. The term Z in (4.3b) is the difference in the marginal cost of funds when labor supply is inelastic ($\lambda = 1$) and when it is elastic $(1 - \lambda^c_L)$ as defined by (**??**) and the product of the ratio of marginal tax revenue by regional governments and the marginal tax

[5]I am being somewhat imprecise here as shown in the proof in *Appendix A.3.2.* With non-zero cross-price elasticities between the two types of labor, $\hat{N}^i_g$ depends on both $\frac{MRS^s}{w^s l^s}$ and $\frac{MRS^u}{w^u l^u}$ but the result in part a.ii) of the proposition does not depend on cross-price elasticities equal to zero.

revenue for a centralized government. The key is whether $\theta > (<)\ 0$: if $\theta > 0$ then $g^C < g*$ and if the regional government overprovides the public service relative to the efficient level than it will overprovide relative to the central government outcome. However, if $\theta < 0$ then $g^C > g*$ and if the regional government provides a level of public service exceeding the efficient level it will not necessarily be providing a level exceeding the level chosen by a centralized government.

4.2 Regional Policies with Capital Taxation and Income Taxation

Wilson (1995) demonstrates that with homogeneous but mobile labor and no fixed costs for the public service, the optimal tax on capital was zero. I now examine whether this is the case with mobile, heterogeneous workers. The first order conditions are given by

$$\frac{\partial W}{\partial \tau_I} = \begin{array}{c} (\lambda-1)\left(1+\tau_I \overline{w}_{\tau_I}\right)+\theta\lambda\tau_I\left(\overline{w}_{\tau_I}-\frac{1}{(1-\tau_I)}\right) \\ +\lambda\left[N^s\left(\tau_I w^s l^s-g\right)\hat{N}^s_{\tau_I}+N^u\left(\tau_I w^u l^u-g\right)\hat{N}^u_{\tau_I}+\tau_k K\hat{K}_{\tau_I}\right] \end{array} = 0 \tag{4.4a}$$

$$\frac{\partial W}{\partial \tau_k} = \begin{array}{c} (\lambda-1)\left(\tau_I \overline{w}_{\tau_K}+K\right)+\theta\lambda\tau_I\overline{w}_{\tau_K} \\ +\lambda\left[N^s\left(\tau_I w^s l^s-g\right)\hat{N}^s_{\tau_k}+N^u\left(\tau_I w^u l_u-g\right)\hat{N}^u_{\tau_k}+\tau_k K\hat{K}_{\tau_k}\right] \end{array} = 0 \tag{4.4b}$$

$$\frac{\partial W}{\partial g} = \begin{array}{c} \phi^s\frac{MRS^s}{w^s l^s}+\phi^u\frac{MRS^u}{w^u l^u}-\lambda+\lambda\left[\tau_I(\lambda(1+\theta)-1)\overline{w_g}\right]+ \\ \lambda\left[N^s\left(\tau_I w^s l^s-g\right)\hat{N}^s_g+N^s\left(\tau_I w^u l^u-g\right)\hat{N}^u_g+\tau_k K\hat{K}_g\right] \end{array} = 0. \tag{4.4c}$$

where $\hat{K}_{\tau_I} = \eta_{ku}\hat{w}^u_{\tau_I} + \eta_{ks}\hat{w}^s_{\tau_I}$. Then based on these first order conditions, the following propositions summarizes conditions when regional governments will tax or subsidize capital.

Proposition 3. *Let g^L and τ_I^L denote the public service and tax rate satisfies (4.1a) and (4.1b), the first order conditions in the absence of capital taxation.*
a) Then if the cost function for the public service is $C(N^s, N^u, g) = (N^s + N^u)\, g$ and

$$\frac{\left[\theta\tau_I^L\left(\overline{w}_{\tau_I}-\frac{1}{\left(1-\tau_I^L\right)}\right)+N^s\left(\tau_I^L w^s l^s-g^L\right)\left(\hat{N}^s_{\tau_I}-\hat{N}^u_{\tau_I}\right)\right]}{1+\tau_I^L\overline{w}_{\tau_I}} >(=)< \frac{\theta\tau_I^L\overline{w}_{\tau_K}+N^s\left(\tau_I^L w^s l^s-g^L\right)\left(\hat{N}^s_{\tau_k}-\hat{N}^u_{\tau_k}\right)}{\tau_I^L\overline{w}_{\tau_K}+K} \ . \tag{4.5a}$$

then $\tau_k > (=) < 0$,
b) Assume there exists a fixed cost of the public service (F) and let τ_k^F be the tax rate such that $\tau_k^F K = F$. Then if

$$\frac{\left[\theta\tau_I^L\left(\overline{w}_{\tau_I}-\frac{1}{\left(1-\tau_I^L\right)}\right)+N^s\left(\tau_I^L w^s l^s-g^L\right)\left(\hat{N}^s_{\tau_I}-\hat{N}^u_{\tau_I}\right)+\tau_k^F K\hat{K}_{\tau_I}\right]}{1+\tau_I^L\overline{w}_{\tau_I}} > (=) < \frac{\theta\tau_I^L\overline{w}_{\tau_K}+N^s\left(\tau_I^L w^s l^s-g^L\right)\left(\hat{N}^s_{\tau_k}-\hat{N}^u_{\tau_k}\right)+\tau_k^F K\hat{K}_{\tau_K}}{\tau_I^L\overline{w}_{\tau_K}+K}. \tag{4.5b}$$

then $\tau_k > (=) < \tau_k^F$,

c) If $\frac{R_{g\tau_k}}{R_g} - \frac{R_{\tau_I\tau_k}}{R_{\tau_I}} > 0$ *then with an increase in the tax on capital* (τ_k) *the tax on income is reduced* $\left(\frac{d\tau_I}{d\tau_k} < 0\right)$ *and the level of public service increase* $\left(\frac{dg}{d\tau_k} > 0\right)$ *where and* $R_g = \frac{\partial R}{\partial g} < 0$, $R_{\tau_I} = \frac{\partial R}{\partial \tau_I} > 0$, $R_{g\tau_k} = \frac{\partial^2 R}{\partial g \partial \tau_k}$, $R_{\tau_I\tau_k} = \frac{\partial^2 R}{\partial \tau_I \partial \tau_k}$ *where* $R(\tau_I, \tau_k, g)$ is the budget constraint as defined by (2.2).

Proof of the proposition is in *Appendix A.3.3.* In part a, the expression *(4.5a)* is the condition that if at $\tau_k = 0$ $\left.\frac{d\tau_I}{d\tau_k}\right|_{W=\overline{W}} < (=) > \left.\frac{d\tau_I}{d\tau_k}\right|_{R=\overline{R}}$ then $\tau_k > (=) < 0$. As was the case when considering the trade off between the income tax rate and the public service level, the terms can be interpreted as the ratio of the distortionary impact of the tax increase on revenue relative to the nondistortionary impact on revenue. Then if the distortionary component for the capital tax relative to its base is more positive or less negative than the distortionary component of the income tax then it is optimal to have a positive tax on capital. If the reverse is true, then capital should be subsidized.

To provide more intuition on what determines when capital is subsidized consider with the elasticity of labor and the elasticity of substitution between the two types of labor both equal to zero ($\theta = \eta_{su} = 0$). Then in this case $\hat{N}^i_{\tau_k} = (1-\tau_I)\frac{\eta_{ik}}{\eta_{ii}}\hat{N}^i_{\tau_I}$ making $\hat{N}^s_{\tau_k} - \hat{N}^u_{\tau_k} = (1-\tau_I)\left(\frac{\eta_{sk}}{\eta_{ss}}\hat{N}^s_{\tau_I} - \frac{\eta_{uk}}{\eta_{uu}}\hat{N}^u_{\tau_I}\right)$. Then if $\frac{\eta_{sk}}{\eta_{ss}} = \frac{\eta_{uk}}{\eta_{uu}}$ the optimal tax on capital is zero – the change in population from an increase in the capital is proportionate to the change from an increase in the income tax rate. However if $\frac{\eta_{sk}}{\eta_{ss}} < \frac{\eta_{uk}}{\eta_{uu}}$ perhaps with $\eta_{sk} > 0$ and $\eta_{su} < 0$ then an increase in the capital tax will increase the population of skilled workers relative to unskilled workers, making it optimal to tax capital; if $\frac{\eta_{sk}}{\eta_{ss}} > \frac{\eta_{uk}}{\eta_{uu}}$ then an increase in the capital tax would reduce the population of skilled workers relative to unskilled workers making it optimal to subsidize capital.

Part b) of the proposition addresses the taxation of capital when a fixed cost exists. If if capital and labor supply were both inelastic and the population changes for both types of labor are the same for both taxes $\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I} = \hat{N}^s{}_{\tau_k} - \hat{N}^u_{\tau_k} = 0$ then the tax on capital $\left(\tau_k^F\right)$ would be such that set equal to the fixed cost as in Wilson (1995). However, with differential mobility and elastic labor and capital supply, the capital tax will be above or below τ_k^F based on the same consideration as in part a).

Part c) of the proposition is obtained by considering the impact of exogenous increases in τ_k on the values of τ_k, g, and λ as determined by the budget constraint (2.2) and the first order conditions with respect to τ_I and g, (4.4a) and (4.4c). The term $R_{\tau_I\tau_k}$ is the impact of an increase in the tax on capital on the marginal revenue from an increase in the income tax. There are two distinct effects: there is there is the effect of an increase in τ_k on the impact of an increase in income tax on fiscal surpluses and deficits, $N^s(\tau_I w^s l^s - g)\hat{N}^s_{\tau_I}\hat{N}^s_{\tau_k} + N^u(\tau_I w^u l^u - g)\hat{N}^u_{\tau_I}\hat{N}^u_{\tau_k} + \tau_I \left[\frac{d[N^s w^s l^s]}{d\tau_k}\hat{N}^s_{\tau_I} + \frac{d[N^u w^u l^u]}{d\tau_k}\hat{N}^u_{\tau_I}\right]$ and the effect on the impact of an increase in the income tax on capital tax revenue, $K\left(1 + \tau_k \hat{K}_{\tau_K}\right)\hat{K}_{\tau_I}$. Analogously, $R_{g\tau_k} = N^s(\tau_I w^s l^s - g)\hat{N}^s_g\hat{N}^s_{\tau_k} + N^u(\tau_I w^u l^u - g)\hat{N}^u_g\hat{N}^u_{\tau_k} + \tau_I \left[\frac{d[N^s w^s l^s]}{d\tau_k}\hat{N}^s_g + \frac{d[N^u w^u l^u]}{d\tau_k}\hat{N}^u_g\right]$. Critical to determining the sign of these impacts is the relationship between capital and labor. If they are substitutes, the increase in the capital tax will increase wages and, therefore population. The effect of this on the budget is ambiguous as increasing the population of the increasing the high-skilled workers makes income tax increases more costly $N^s(\tau_I w^s l^s - g)\hat{N}^s_{\tau_I}\hat{N}^s_{\tau_k} < 0$ if $\hat{N}^s_{\tau_k} > 0$ with the reverse true for the effects on low-skilled workers $N^u(\tau_I w^u l^u - g)\hat{N}^u_{\tau_I}\hat{N}^u_{\tau_k} > 0$ if $\hat{N}^u_{\tau_k} > 0$. Given the complicated nature of these relationships, I employ numerical simulations to gather further insights.

Figure 2 provides an illustration of part a) of the proposition showing the case when it is optimal to tax capital and when it is optimal to subsidize capital.

4.3 Regional Policy with Sales and Income Taxation

While capital and income taxation have received the more attention in the theoretical tax competition literature, in fact, regional governments (states), use taxes on consumption as a major source of revenue as well. Ass an example, as seen in *Table* 1, the general sales tax accounts for 32 percent of state tax revenue. While the *VAT* system primarily used in Europe is a broad-based tax across most consumption, the sales and selective excise taxes employed in the United States have a much narrower base and has been criticized as being regressive as lower-income households generally consume a larger share of their income on goods and services subject to sales taxation.[6]

Here, following the approach used in *Section 4.2,* I consider how different mobility responses to a sales tax increase between the two types of labor generates the incentive for regional governments to employ a sales tax (τ_s).
where the tax rate on commodity x_1 is denoted by τ_s and $B^i_1 = \frac{x^i_1}{w^i l^i}$ is the budget share of on x_1 for group i.

Before deriving the optimal conditions for regional tax policy when a sales tax is available, it is worth noting that if the impact of a sales tax on labor supply is proportionate to the

[6]The *Institute for Taxation and Economic Policy* regularly produces a report *Who Pays?* calculating state and local income taxation by income. There most recent (report) states "The average state's consumption tax structure is equivalent to an income tax with a 7 percent rate for the poor, a 4.7 percent rate for the middle class, and a 0.8 percent rate for the wealthiest taxpayers."

budget share on the tax good and the labor supply elasticities are the same for both types of labor, as assumed here, there is no gain to employing a sales tax in addition to the income tax in the absence of interregional labor mobility. The central government cannot exploit any difference in the relationship between direct revenue and the distortionary impact on labor supply that differs between the income tax and the sales tax.

Then considering the policies of a regional government, the first order condition for the income tax with the inclusion of capital taxation is now slightly modified to allow for the sales tax we have

$$\frac{\partial W}{\partial \tau_I} = \begin{array}{c} (\lambda - 1)\left(1 + \tau_I \overline{w}_{\tau_I}\right) + \theta\lambda\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + \\ \lambda\left[\begin{array}{c} N^s\left(\tau_I w^s l^s + \tau_s x_1^s - g\right)\hat{N}_{\tau_I}^s + N^u\left(\tau_I w^u l^u + \tau_s x_1^u - g\right)\hat{N}_{\tau_I}^u \\ +\tau_k K\hat{K}_{\tau_I} + \tau_s\varepsilon_{1w}\left(\phi^s B_1^s\left(\hat{w}_{\tau_I}^s - \frac{1}{(1-\tau_I)}\right) + \phi^u B_1^u\left(\hat{w}_{\tau_I}^u - \frac{1}{(1-\tau_I)}\right)\right) \end{array}\right] \end{array} = 0 \quad (4.6a)$$

where $\varepsilon_{1w} = \frac{\partial x_1^i}{\partial w^i(1-\tau_I)}\frac{w^i(1-\tau_I)}{x_1^i}$, the elasticity of demand for x_1 with respect to income, which for simplicity we assume to the be the same for both groups. The first order condition with respect the sales tax gives

$$\frac{\partial W}{\partial \tau_s} = \begin{array}{c} (\lambda - 1)\left(\phi^s B_1^s + \phi^u B_1^u\right) + \tau_I\left((1+\theta)\lambda - 1\right)\overline{w}_{\tau_s} \\ +\lambda\left[\begin{array}{c} N^s\left(\tau_I w^s l^s + \tau_s x_1^s - g\right)\hat{N}_{\tau_s}^s + N^u\left(\tau_I w^u l^u + \tau_s x_1^u - g\right)\hat{N}_{\tau_s}^u \\ +\tau_k K\hat{K}_{\tau_s} + \frac{\tau_s}{1+\tau_s}\varepsilon_{11}\left(\phi^s B_1^s + \phi^u B_1^u\right) \end{array}\right] \end{array} = 0 \quad (4.6b)$$

where $\varepsilon_{11} = \frac{\partial x_1^i}{\partial(1+\tau_s)}\frac{(1+\tau_s)}{x_1^i}$, the own-price elasticity of demand for x_1. Then as with the income and capital tax, the optimality of the sales tax depends on its direct effects on revenues and the costs to consumers, $(\lambda - 1)\left(\phi^s B_1^s + \phi^u B_1^u\right)$, its effects through wage appreciation, $\tau_I\left((1+\theta)\lambda - 1\right)\overline{w}_{\tau_s}$, and indirect revenue impacts including changes in the capital tax base and population. As with the capital tax, we can determine if it is optimal to employ a sales tax by determining if, at $\tau_s = 0$, $\left.\frac{d\tau_I}{d\tau_s}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{R=\overline{R}} > (=) < 0$, that is, whether the marginal rate of welfare substitution between the two taxes is more or less than the marginal rate of revenue substitution (budget balance) using (4.6a) and (4.6b). *Proposition 4* summarizes the conditions under which it is optimal to employ a sales tax on x_1.

Proposition 4. *Consider the implementation of a sales tax on a select commodity,* x_1. *It will be optimal to have a positive tax on* x_1 $(\tau_s > 0)$ *if:*

$$a) \quad \frac{\begin{bmatrix} \theta\tau_I\overline{w}_{\tau_s} + N^s\left(\tau_I w^s l^s - g\right)\hat{N}^s_{\tau_s} \\ +N^u\left(\tau_I w^u l^u - g\right)\hat{N}^u_{\tau_s} + \tau_k K\hat{K}_{\tau_s} \end{bmatrix}}{\phi^s B^s_1 + \phi^u B^u_1 + \tau_I\overline{w}_{\tau_s}} > \frac{\begin{bmatrix} \theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + N^s\left(\tau_I w^s l^s - g\right)\hat{N}^s_{\tau_I} \\ +N^u\left(\tau_I w^u l^u - g\right)\hat{N}^u_{\tau_I} + \tau_k K\hat{K}_{\tau_I} \end{bmatrix}}{1+\tau_I\overline{w}_{\tau_I}}. \tag{4.7}$$

b) if, the share of income spent on the tax good (x_1) *is greater for unskilled workers than skilled workers,* $B^u_1 > B^s_1$ *when labor supply elasticity,* θ*, equals zero and there is no tax on capital expenditures.*

Analogous to the interpretation of (4.5a) for the capital tax, an interpretation of (4.7) is that if the increase in revenue from the changes in tax base due to changes in capital, labor supply, and labor mobility relative to the non-distorted base are greater for the sales tax than for the income tax when the $\tau_s = 0$ then it is optimal to employ a sales tax on commodity

To focus the impact that differential mobility has on the implementation of a sales tax assume that $\theta = 0$ and a capital tax is not employed ($\tau_k = 0$). Further assume that $\eta_{us} = \eta_{su} = 0$. In this case $\hat{w}^i_{\tau_s} = B^i_1\hat{w}^i_{\tau_I}$ and $\hat{N}^i_{\tau_s} = B^i_1\hat{N}^i_{\tau_I}$ and $\overline{w}_{\tau_s} = \phi^s B^s_1\hat{w}^s_{\tau_I} + \phi^u B^u_1\hat{w}^u_{\tau_I}$. Then making these substitutions and simplifying gives $\left.\frac{d\tau_I}{d\tau_s}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{R=\overline{R}} > (=) < 0$ if

$$\begin{array}{c} \underbrace{\left(\phi^s B^s_1 + \phi^u B^u_1 - B^s_1 + \tau_I\left(\phi^u(B^u_1 - B^s_1)\hat{w}^u_{\tau_I}\right)\right)}_{(a)} \quad \hat{N}^s_{\tau_I} > (=) < \\ \underbrace{\left(\phi^s B^s_1 + \phi^u B^u_1 - B^u_1 + \tau_I\left(\phi^s\left(B^s_1 - B^u_1\right)\hat{w}^s_{\tau_I}\right)\right)}_{(b)} \quad \hat{N}^u_{\tau_I} \end{array}. \tag{4.8}$$

Then if $B^u_1 > B^s_1$, the unskilled workers spend greater share of their income on x_1 then term (a) of (4.8) is negative and term (b) is positive. Then as both $\hat{N}^s_{\tau_I}$ and $\hat{N}^u_{\tau_I}$ are negative, the product of (a) and $\hat{N}^s_{\tau_I}$ is positive and the product of (b) and $\hat{N}^u_{\tau_I}$ is negative making $\left.\frac{d\tau_I}{d\tau_s}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{R=\overline{R}}$ positive and for it to be optimal to employ a sales tax.

4.4 Multiple Public Goods and Transfers

Following Borck (2005), consider the existence of multiple public goods, some consumed by the unskilled workers and some consumed by the skilled workers, that provide additional instruments to address the fiscal deficit generated by the uniform income tax. Let the public service consumed strictly by unskilled be denoted by g_u and the one consumed by the skilled by g_s. Assume that both are produced at constant cost with respect to population $C(N^i, g_i) = N^i g_i$. In this case the first order conditions are

$$\frac{\partial W}{\partial \tau_I} = \left(\lambda\left(1-\tau_I\frac{\theta}{(1-\tau_I)}\right)-1\right)+\tau_I\left(\lambda\left(1+\theta\right)-1\right)\overline{w}_{\tau_I}+\lambda\ N^s\left(\tau_I w^s l^s-g_s\right)\ \left(\hat{N}^s_{\tau_I}-\hat{N}^u_{\tau_I}\right)=0 \quad (4.9a)$$

$$\frac{\partial W}{\partial g_i} = \phi^i\frac{MRS^i}{w^i l^i}-\lambda N^i+\tau_I\left(\lambda\left(1+\theta\right)-1\right)\phi^i\hat{w}^i_{g_s}+\lambda\left[\ N^i\left(\tau_I w^i l^i-g_i\right)\ \hat{N}^i_{g_i}\right]\ =0,\ i=s,\ u. \quad (4.9b)$$

where (4.9a) is simplified by using the budget constraint to substitute $-N^s\left(\tau_I w^s l^s-g_s\right)$ for $N^u\left(\tau_I w^u l^u-g_u\right)$. Unlike the case with uniform public service provision, the sign of $\tau_I w^i l^i-g_i$ is not obvious. as can be seen by rewriting this expression as $w^i l^i\left(\tau_I-\frac{g^i}{w^i l^i}\right)$, i=u,s and that whether $\tau_I w^s l^s-g_s>(=)<0$ depends on whether the share of income for the public service for the skilled $\left(\frac{g^s}{w^s l^s}\right)$ is less than or greater than that for the unskilled. Here I assume that share is lower for skilled workers, making $\tau_I-\frac{g^i}{w^i l^i}>0$. Then, to facilitate comparisons and simplify results, consider the optimal provision of the two public goods when $\theta=0$ and $\eta_{su}=0$. In this case, $\hat{w}^i_{g_i}=-\frac{MRS^i_i}{w^i l^i}\hat{w}^i_{\tau_i}$ and $\hat{N}^i_{g_i}=-\frac{MRS^i_i}{w^i l^i}\hat{N}^i_{\tau_I}$ Then making these substitutions into (4.9b) gives

$$MRS^i=\frac{\lambda}{\left[1-(\lambda-1)\tau_I\hat{w}^i_{\tau_I}-\lambda\left(\tau_I-\frac{g_i}{w^i l^i}\right)\hat{N}^i_{\tau_I}\right]} \quad (4.10)$$

$$MRS^s-MRS^u=\frac{\lambda}{D^s D^u}\left[(\lambda-1)\,\tau_I\left(\hat{w}^s_{\tau_I}-\hat{w}^u_{\tau_I}\right)+\lambda\left(\left(\tau_I-\frac{g_s}{w^s l^s}\right)\hat{N}^s_{\tau_I}-\left(\tau_I-\frac{g_u}{w^u l^u}\right)\hat{N}^u_{\tau_I}\right)\right] \quad (4.11)$$

where $D^i=1-(\lambda-1)\,\tau_I\hat{w}^i_{\tau_I}-\lambda\left(\tau_I-\frac{g_i}{w^i l^i}\right)\hat{N}^i_{\tau_I}$. With skilled workers generating a surplus $\left(\tau_I-\frac{g_s}{w^s l^s}>0\right)$ if $\hat{N}^s_{\tau_I}>\hat{N}^u_{\tau_I}$, that is, the skilled workers are less responsive to a change in the income tax rate, from ("4.9a) $\lambda<1$.[7]. As the terms in the denominator of (4.10) are positive then $MRS^s<1$, the public service for the skilled workers is overprovided relative to the first-best. As it is assumed that the unskilled workers generate a fiscal deficit, the term $-\lambda\left(\tau_I-\frac{g_i}{w^i l^i}\right)\hat{N}^i_{\tau_I}$ in the denominator of (4.10) is negative making the magnitude of the denominator ambiguous. If, however, the skilled are more responsive to a tax increase, $\hat{N}^s_{\tau_I}>\hat{N}^u_{\tau_I}$ we have $\hat{w}^s_{\tau_I}>\hat{w}^u_{\tau_I}$ and $\lambda>1$. Whether MRS^s is greater or less than one is unclear from (4.10) but as the unskilled workers generate a deficit from (4.10) it is clear that the public service for them is underprovided.

As can be seen in (4.11) the *relative* differences in the marginal rates of substitution are ambiguous. While the second term, reflecting the impacts of the fiscal surplus from additional skilled workers and deficit for unskilled workers favors increasing the public service of the skilled relative to that of the unskilled, the impact on capitalization is always positive. However, as will become clear from the numerical examples, this impact, the product of

[7]From (4.9a) $\lambda=\frac{1+\tau_I\overline{w}_{\tau_I}}{1+\tau_I\overline{w}_{\tau_I}+\ N^s\left(\tau_I w^s l^s-g_s\right)\ \left(\hat{N}^s_{\tau_I}-\hat{N}^u_{\tau_I}\right)}$

$\lambda - 1$, the tax rate (τ_I) and difference in impacts of a tax increase on wages is small relative to the sum of the impacts on the fiscal surplus and deficit.

4.4.1 Transfers to the Unskilled

As a special case, consider the public service being a transfer to the unskilled workers financed by a tax on both skilled and unskilled workers. We can apply use (4.9b) to describe the optimal transfer with $MRS^u = 1$ in this case. In addition to the transfer assume that the government continues to provide the two public services, g_s and g_u, consumed by skilled and unskilled workers, respectively. Let the budget constraint for the government be given by $C(N^s, N^u, g_s, g_u) = F + N^s g_s + N^u (g_u + T_u)$.

Then making this substitution into (4.9b) gives

$$\frac{\partial W}{\partial \tau_I} = \left(\lambda\left(1 - \tau_I \frac{\theta}{(1-\tau_I)}\right) - 1\right) + \tau_I (\lambda(1+\theta) - 1)\,\overline{w}_{\tau_I} + \lambda\left[\ N^s(\tau_I w^s l^s - g_s)\ \left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right) - F\hat{N}^u_{\tau_I}\right] = 0 \quad (4.12a)$$

$$\frac{\partial W}{\partial T_u} = (1-\lambda)(1 - \tau_I\ \hat{w}^u_{T_u}) + \lambda\left[\ \theta\tau_I\hat{w}^u_{T_u} + \left(\tau_I - \frac{g_u + T_u}{w^u l^u}\right)\ \hat{N}^u_{T_u}\right]\ = 0. \quad (4.12b)$$

where $-N^s(\tau_I w^s l^s - g_s) - F$ is substituted for $N^u(\tau_I w^u l^u - g_u - T_u)$ in (4.12a). Using (4.12a) we can solve for λ,

$$\lambda = \frac{1 + \tau_I \overline{w}_{\tau_I}}{1 + \tau_I \overline{w}_{\tau_I} +\ N^s(\tau_I w^s l^s - g_s)\ (\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}) - F\hat{N}^u_{\tau_I}} \quad (4.12c)$$

Then if the revenues from additional unskilled worker exceeds the cost of supply the public service and transfer to that worker, net revenue . If $\theta \leq 0$, tax revenue will increase as well. Finally, if the marginal cost of funds λ is less than or equal to one, the case with $\hat{N}^s_{\tau_I} > \hat{N}^u_{\tau_I}$, $\theta \leq 0$, and there is a fiscal surplus for the skilled workers, a transfer to the unskilled workers is optimal.

The findings of *Section 4.4* are summarized below.

Proposition 5.
a) Let $g_i,\ i = u, s$ be a public service consumed only by type i workers with a marginal cost of N^i and assume that in equilibrium that $\tau_I - \frac{g_s}{w^s l^s} > 0$. Then:
i) If $\hat{N}^s_{\tau_I} > \hat{N}^u_{\tau_I}$ then the public service for the skilled workers is overprovided relative to the efficient level, that is, $MRS^s < 1$.
ii) If $\hat{N}^s_{\tau_I} < \hat{N}^u_{\tau_I}$ then the public service for the unskilled workers is underprovided relative to the efficient level, that is, $MRS^u > 1$.

b) Let T_i, $i = u, s$ be a transfer to type i workers. Then if λ^T, as defined by (4.12c) is less than one and $N^i (\tau_I w^i l^i - g_i) > 0$ it is optimal to have a transfer to type i workers.

5 Numerical Examples

In *Table* 4, the results of a simple numerical parameterization of the general equilibrium outcome when regions can only tax income.This model is only intended to be illustrative as I made no attempt to find empirically-based values for the parameters. In this simple model, utility for a worker is in group i is $U^i = w^i (1 - \tau_I) + a^i lng(g) + \gamma^i (N^i)$ $i = s, u$ where I vary the values for a^i and γ^i the elasticity of mobility. The parameters were chosen so that for all specifications, the efficient outcome has an income tax rate of $\tau_I = .1333$ and public service level of g = .2. In equilibrium, the wage of the high skilled workers is 2 and that of the low skilled workers is 1 with the equilibrium rate of return on capital of one as well. As well, in equilibrium there are equal numbers of high-skilled and low-skilled workers ($N^s = N^u = .5$) and the equilibrium amount of capital is 1 in each region. In column (a) the taste for the public service is equal for both groups. Then when mobility is equal for the two groups ($\gamma^s = \gamma^u = 3$, column a.1), the relative impact of a tax increase on the number of workers is equal but the impact of a reduction of the public service is greater for the unskilled workers . This being the case, the public service is underprovided (g = .1972). When the skilled workers are relatively more mobile than the unskilled workers ($\gamma^s = 5.5, \gamma^u = 0.5$, column a.2) while they are both more responsive to increases in tax and public services than the unskilled workers and , the difference is greater for taxes making it optimal for regions to underprovide the public service. When unskilled workers are relatively more mobile (column a.3), the impact of the taxes is still relatively greater for the skilled workers, making it again optimal to underprovide the public service. Note that regardless of the specification of tastes and mobility, as expected, the decentralized outcome results in higher utility for skilled workers and lower utility for unskilled workers than the centralized outcome though the differences in total social welfare are extremely small. It is interesting to note that in these examples whether skilled workers are relatively more or less responsive to taxes or public services determines whether the public service is underprovided or overprovided. If then the relative responsiveness of skilled workers to taxes exceeds their relative responsiveness to public services and a balanced-budget decrease in taxes reduces the ratio of unskilled to skilled workers making it optimal for the region to underprovide the public service. Analogously, if then a balanced-budget increase in taxes reduces the ratio of unskilled to skilled workers making it optimal for the region to overprovide the public service.

To provide some indication of how the opportunity to tax capital affects public service provision, utility of the two groups, and overall welfare, the general equilibrium simulations reported in Table 2.1 are modified to allow regions to tax capital as well as income. In *Tables 5-7* , the results of these simulations with capital taxation are presented. The parameterization is identical to the earlier simulations reported in Table 2.1 except now the cross price elasticities between labor and capital are non-zero. Four alternative sets of values for these cross-price elasticities (η_{sk}=.5, η_{uk}=0; η_{sk}=-.5, η_{uk}=0; η_{sk}=0,η_{uk}=.5; η_{sk}=0, η_{uk}=-.5) are considered. The three tables differ in the taste parameters for the public service with *Table 5* having equal valuations of the public service for the two groups (a^s=a^u =.2); *Table 6* reports the results for when only the skilled workers value the public service (a^s=.4; a^u =0); and *Table 7* reports when only unskilled workers value the public service (a^s=0; a^u =.4). In addition to varying the taste parameters and the elasticity of substitution between the two types of labor and capital, simulations with different degrees of mobility for the two groups are performed as well. Perhaps the most interesting results from these simulations is the sign and magnitude of the tax on capital. Capital is positively taxed when skilled labor and capital are substitutes (column (a) of Tables 5-7) except when unskilled labor is much more mobile than skilled labor ($\gamma^s = .5$; $\gamma^u = 0$) in which case it subsidized. When unskilled labor is a complement with capital (column (d)) it is always taxed regardless of the relative mobility of the two types of labor. As expected, capital is subsidized whenever skilled labor is a complement with capital (column (b)) and when unskilled labor is a complement with it, though not when skilled labor is relatively more mobile (column (c)). Public service provision with capital taxation exceeds that when only the income tax is used when the tax on capital is positive though it is not necessarily the case that it exceeds the efficient level of the public service (g = .20). When the tax on capital is negative, the level of public service is below the level provided in the absence of capital taxation. Relative total welfare is quite close to that found with only the income tax in all cases. However, utility for both skilled and unskilled workers is higher with capital taxation when the tax on capital is positive and lower when the tax is negative. That welfare can be quite similar in both cases and utility quite different in some cases is explained by the impact of capital taxation on the return to capital owners. If the tax on capital is positive, the return is lower; a negative tax increases the return to capital owners. The results of these simulations confirm that the sign and magnitude of any tax on capital imposed by regions depends on the elasticity of substitution between the two types of labor and capital and, to a less extent, the relative mobility of the two types of labor. The tax on capital appears to have little effect on overall welfare, though the impact on the level of public service, the welfare of the workers, and the returns to capital owners are directly related to whether capital is taxed or subsidized which, as mentioned, depends on the relationships of the two types of labor with capital.

6 Conclusion

In contrast to federal governments, lower-level governments, states and localities, do not have direct controls on immigration or emigration. Then absent direct controls on migration they have incentive to account for the impacts of their policies on population flows to or away from their jurisdiction. Importantly, as examined in Hoyt and Lee (2003); Borck (2005); Buettner and Janeba (2016) they have incentive to attract highly-skilled workers who generate a fiscal surplus as the tax revenue collected from them exceeds the cost of providing services to them and discourage migration of low-skilled workers who generate a fiscal deficit.

Unlike these studies that assume either costless mobility or no mobility, along the lines of Lehmann, Simula and Trannoy (2014) Ie incorporate differential mobility costs among the two types of workers. Important for our results is that the policy choices of the state depend not only on the relative mobility of the two types of workers but also on how their mobility differs in response to different policies. Thus a tax decrease will lead to a larger increase in high-skilled workers relative to low-skilled workers than will an increase in the public service, the state will underprovide public service; if the reverse is true, the state overprovides services. Analogously, if a decrease in the income tax and an increase in the tax on capital from a starting point of no capital tax leads to an increase in percentage of the population that is high-skilled workers then a tax on capital is optimal; if the reverse is true, capital should be subsidized.

While there are obvious challenges, empirical work devoted not only to better determining how different income groups respond, with respect to migration, to sub-federal policy changes but on whether, as suggested here, on whether these responses influence the choices of policy would seem to be an important goal of future work in the field.

References

Agrawal, David R., and William H. Hoyt. 2017. “Commuting and Taxes: Theory, Empirics, and Welfare Implications.” *Economic Journal*.

Akcigit, Ufuk, Salome Baslandze, and Stefanie Stantcheva. 2016. “Taxation and the International Mobility of Inventors.” *American Economic Review*, 106(10): 2930–2981.

Blank, Rebecca. 1988. “The Effects of Welfare and Wage Levels on the Location Decisions of Femaile-Headed Households.” *Journal of Urban Economics*, 24: 186 – 211.

Blesse, Sebastian, and Thushyanthan Baskaran. 2013. “Do Municipal Mergers Reduce Costs? Evidence from a German Federal State.” SSRN Working Paper.

Boadway, Robin, and Frank Flatters. 1982. "Efficiency and Equalization Payments in a Federal System of Government: A Synthesis and Extension of Recent Results." *The Canadian Journal of Economics*, 15(4): 613–622.

Borck, Rainald. 2005. "Fiscal Competition, Capital-Skill Complementarity, and the Composition of Public Spending." *FinanzArchiv*, 61(4): 488 – 499.

Borjas, George J. 1999. "Immigration and Welfare Magnets." *Journal of Labor Economics*, 17(4 (Part 1)): 607 – 637.

Borjas, George J. 2016. *We Wanted Workers: Unraveling the Immigration Narrative.* W.W. Norton & Company.

Brueckner, Jan. 1999. "Welfare Reform and the Race to the Bottom: Theory and Evidence." *Southern Economic Journal*, 66(2): 505–525.

Brueckner, Jan K. 2000. "Welfare Reform and the Race to the Bottom: Theory and Evidence." *Southern Economic Journal*, 66(3): 505–525.

Bucovetsky, Sam, and John Douglas Wilson. 1991. "Tax Competition with Two Tax Instruments." *Regional Science and Urban Economics*, 21(3): 333–350.

Buettner, Thiess, and Eckhard Janeba. 2016. "City Competition for the Creative Class." *Journal of Cultural Economics*, 40: 413 – 451.

Coomes, Paul A., and William H. Hoyt. 2008. "Income Taxes and the Destination of Movers to Multistate MSAs." *Journal of Urban Economics*, 63(3): 920–937.

Craig, Steven G. 1987. "The Impact of Congestion on Local Public Good Production." *Jounral of Public Economics*, 32(3): 331–353.

Dustmann, Christian, and Tommaso Frattini. 2008. "The Fiscal Effects of Immigration to the UK." *The Economic Journal*, 124: 593 – 643.

Gronberg, Timothy J., Dennis W. Jansen, and Lori L. Taylor. 2011. "The Adequacy of Education Cost Functions: Lessons from Texas." *Peabody Journal of Education*, 86(1): 3–27.

Hamilton, Jonathan, and Pierre Pestieau. 2005. "Optimal Income Taxation and the Ability Distribution: Implications of Migration Equilibrium." *International Tax and Public Finance*, 12: 29–45.

Hayes, Kathy J., and Daniel J. Slottje. 1987. "Measures of Publicness Based on Demographic Scaling." *The Review of Economics and Statistics*, 714– – 718.

Henderson, J. Vernon. 1985. "The Tiebout Model: Bringing Back the Entrepreneurs." *Journal of Political Economy*, 93(2): 248–264.

Hindriks, Jean. 1999. "The Consequences of Labour Mobility for Redistribution: Tax vs. Transfer Competition." *Joural of Public Economics*, 74: 215–234.

Hoyt, William H. 1991. "Property Taxation, Nash Equilibrium and Market Power." *Journal of Urban Economics*, 30(1): 123–131.

Hoyt, William H. 1992. "Market Power of Large Cities and Policy Differences in Metropolitan Areas." *Regional Science and Urban Economics*, 22(4): 539–558.

Hoyt, William H. 1993. "Tax Competition, Nash Equilibria, and Residential Mobility." *Journal of Urban Economics*, 34(3): 358–379.

Hoyt, William H., and Kangoh Lee. 2003. "Subsidies as Sorting Devices." *Journal of Urban Economics*, 53(3): 436 – 457.

Huber, Bernd. 1999. "Tax Competition and Tax Coordination in an Optimun Income Tax Model." *Joural of Public Economics*, 71(3): 441–458.

Keen, Michael, and Maurice Marchand. 1997. "Fiscal Competition and the Pattern of Public Spending." *Joural of Public Economics*, 66: 33–53.

Kleven, Henrik Jacobsen, Camille Landais, and Emmanuel Saez. 2013. "Taxation and International Migration of Superstars: Evidence from the European Football Market." *American Economic Review*, 103(5): 1892–1924.

Lehmann, Etienne, Laurent Simula, and Alain Trannoy. 2014. "Tax Me If You Can! Optimal Nonlinear Income Tax Between Competing Governments." *Quarterly Journal of Economics*, 129(4): 1995–2030.

Mansoorian, Arman, and Gordon M. Myers. 1993. "Attachment to Home and Efficient Purchases of Population in a Fiscal Externality Economy." *Journal of Public Economics*, 52(1): 117 – 132.

McKinnish, Terra G. 2005. "Importing the Poor." *Journal of Human Resources*, 40(1): 57 – 76.

McKinnish, Terra G. 2007. "Cross-Border Welfare Migration: New Evidence from Micro-Data." *Journal of Public Economics*, 91(3-4): 437–450.

Mirrlees, James A. 1982. "Migration and Optimal Income Taxes." *Joural of Public Economics*, 18: 319–341.

Moretti, Enrico, and Daniel J. Wilson. 2014. "State Incentives for Innovation, Star Scientists and Jobs: Evidence From Biotech." *Journal of Urban Economics*, 79: 20–38.

Moretti, Enrico, and Daniel Wilson. 2017. "The Effect of State Taxes on the Geographical Location of Top Earners: Evidence from Star Scientists." *American Economic Review*, 107(7): 1858–1903.

Myers, Gordon M. 1990. "Optimality, Free Mobility, and Regional Authority in a Federation." *Joural of Public Economics*, 43(107-121).

Razin, Assaf, and Jacklin Wahba. 2015. "Welfare Magnet Hypothesis, Fiscal Burden, and Immigration Skill Selectivity." *The Scandinavian Journal of Economics*, 117(2): 369 – 402.

Simula, Laurent, and Alain Trannoy. 2010. "Optimal Income Tax under the Threat of Migration of Top-Income Earners." *Journal of Public Economics*, 94: 163 – 173.

Simula, Laurent, and Alain Trannoy. 2011. "Shall We Keep the HiHigh-Skilled at Home? The Optimal Income Tax Perspective." *Social Choice and Welfare*, 1–32.

Wildasin, David E. 2000. "Labor Market Integration, Investment in Risky Capital, and Fiscal Competition." *The American Economic Review*, 90(1): 73–95.

Wilson, John D. 1982. "Optimal Linear Income Taxation in the Presence of Emigration." *Journal of Public Economics*, 18(3): 363 – 379.

Wilson, John D. 1985. "Optimal Property Taxation in the Presence of Interregional Capital Mobility." *Journal of Urban Economics*, 18: 296–315.

Wilson, John Douglas. 1995. "Mobile Labor, Multiple Tax Instruments, and Tax Competition." *Journal of Urban Economics*, 38(3): 333–356.

Young, Cristobal, and Charles Varner. 2011. "Millionaire Migration and State Taxation of Top Incomes: Evidence from a Natural Experiment." *National Tax Journal*, 64(2): 255–284.

Young, Cristobal, Charles Varner, Ithai Lurie, and Rich Prisinzano. 2014. "Millionaire Migration and the Demography of the Elite: Implications for American Tax Policy." NTA Conference Paper.

Zodrow, George R., and Peter Mieszkowkski. 1986. "Pigou, Tiebout, Property Taxation, and the Underprovision of Local Public Goods." *Journal of Urban Economics*, 19: 356–370.

A Appendix

A.1 derivation of Wage and Population Gradients

Differentiating (2.4a) and (2.4b)yields

$$\begin{bmatrix} f_{ss} & f_{su} & f_{sk} \\ f_{us} & f_{uu} & f_{uk} \\ f_{ks} & f_{ku} & f_{kk} \end{bmatrix} \begin{bmatrix} dL^s \\ dL^u \\ dK \end{bmatrix} = \begin{bmatrix} dw^s \\ dw^u \\ d\tau_k \end{bmatrix}. \tag{A.1}$$

Solving (A.1) gives

$$dL^i = \frac{\widetilde{f_{ii}}}{|f|}dw^i + \frac{\widetilde{f_{ji}}}{|f|}dw^j + \frac{\widetilde{f_{ki}}}{|f|}d\tau_k, \;\; i,j = u,s; \;\; i \neq j \;\; and \tag{A.2a}$$

$$dK = \frac{\widetilde{f_{ik}}}{|f|}dw^i + \frac{\widetilde{f_{jk}}}{|f|}dw^j + \frac{\widetilde{f_{kk}}}{|f|}d\tau_k, \;\; i,j = u,s; \;\; i \neq j \tag{A.2b}$$

where $\widetilde{f_{ii}} = f_{jj}f_{kk} - f_{jk}^2$, $\widetilde{f_{ji}} = -\left(f_{su}f_{kk} - f_{kj}f_{ik}\right)$, $\widetilde{f_{ki}} = f_{su}f_{jk} - f_{us}f_{ik}$, $\widetilde{f_{ik}} = f_{us}f_{kj} - f_{ki}f_{jj}$, $\widetilde{f_{kk}} = f_{ss}f_{uu} - f_{su}^w$, and $|f| = f_{ss}\widetilde{f_{ss}} + f_{us}\widetilde{f_{us}} + f_{ks}\widetilde{f_{ks}}, \;\; i,j = s,u; \; i \neq j.$
Then equivalently (A.2a) and (A.2b) can be expressed as

$$\hat{L}_x^s = \eta_{ss}\hat{w}_x^s + \eta_{su}\hat{w}_x^u + \eta_{sk}\hat{\tau}_k, \tag{A.3a}$$

$$\hat{L_x^u} = \eta_{us}\hat{w}_x^s + \eta_{uu}\hat{w}_x^u + \eta_{uk}\hat{\tau}_k \tag{A.3b}$$

$$\hat{K_x} = \eta_{ks}\hat{w}_x^s + \eta_{ku}\hat{w}_x^u + \eta_{kk}\hat{\tau}_k \tag{A.3c}$$

where $\hat{w}_x^i = \frac{dw^i}{dx}\frac{1}{w^i}$, $x = \tau_I, \, \tau_k, \, g$ and $\hat{\tau}_k = \frac{d\tau_k}{\rho+\tau_k}$, the percentage change in variables w_k and the change in τ_k as a fraction of the gross price of capital, respectively. The term $\eta_{ij} = \frac{\widetilde{f}_{ij}}{|f|}\frac{w^i}{L^j}$ is the price elasticity for input i with respect to the price of input j, i, j=s,u with $\eta_{kk} = \frac{\widetilde{f}_{kk}(\rho+\tau_k)}{|f|K}$, $\eta_{kj} = \frac{\widetilde{f}_{kj}}{|f|}\frac{w^j}{K}$, and $\eta_{jk} = \frac{\widetilde{f}_{jk}}{|f|}\frac{(\rho+\tau_k)}{L^j}$.

Then as $L^i = N^i l^i$ it follows that

$$dL_x^i = dN_x^i l^i + N^i dl_x^i$$

where $dl_x^i = \frac{\partial l^i}{\partial[w^i(1-\tau_I)]}\left[dw_x^i\left(1-\tau_I\right) - w^i\right] = \frac{\theta}{l^i}\left(\widehat{w}_x^i - \frac{d\tau_i}{(1-\tau_i)}\right)$ and $\theta = \frac{\partial l^i}{\partial[w^i(1-\tau_I)]}\frac{w^i(1-\tau_I)}{l^i}$. Then we have

$$\hat{L}_x^i = \hat{N}_x^i + \theta\left(\hat{w_x^i} - \hat{\tau}_I^i\right), \;\; i = s,u \tag{A.4}$$

A.1.1 Wage Gradients

Then substituting for $\hat{L}^i,\ i = s, u$ using (A.3a) or (A.3b) gives

$$\hat{N_x}^i = (\eta_{ii} - \theta)\,\hat{w}_x + \eta_{ij}\hat{w}_x^j + \theta\frac{d\tau_I}{(1-\tau_I)} + \eta_{ik}\hat{\tau}_k,\ i, j = s, u; i \neq j \tag{A.5}$$

Differentiating (2.4c) with respect to policy x, $x = \tau_I$, τ_s, τ_k, g gives

$$dw^i l^i (1-\tau_I) + \frac{\phi^{i'}}{V_y^i}dN^i + U_x^i = 0,\ i = 1, 2 \tag{A.6}$$

then we can express (A.6) by

$$\widehat{w}_x^i + \frac{\chi^{i'}N^i}{w^i(1-\tau_I)}\widehat{N}_x^i + \frac{U_x^i}{w^i(1-\tau_I)} \tag{A.7}$$

which becomes

$$\hat{N_x}^i = \gamma^i\left(\hat{w_x}^i + \frac{U_x^i}{w^i l^i (1-\tau_I^i)}\right) \tag{A.8}$$

where $\gamma^i = \frac{dN^i}{d[w^j(1-\tau)]}\frac{w^i l^i(1-\tau_I^i)}{N^i} = -\frac{V_y^i w^i l^i (1-\tau_I^i)}{\phi^{i'}N^i}$, $U_{\tau^i}^i = -w^i l^i$, $U_g^i = \frac{\partial U^i}{\partial g} \equiv MRS^i$, and $U_{\tau_s}^i = -x_1^i$. Then using (A.5) and (A.8) gives

$$(\eta_{ii} - \theta)\,\hat{w}_x^i + \eta_{ij}\hat{w}_x^j + \theta^i\hat{\tau}_I^i + \eta_{ik}\hat{\tau}_k = \gamma^i\left(\hat{w_x}^i + \frac{U_x^i}{w^i l^i(1-\tau_I^i)}\right),\ i, j = u, s;\ i \neq j \tag{A.9}$$

which can be expressed in matrix form as

$$\underbrace{\begin{bmatrix} \gamma^s + \theta - \eta_{ss} & -\eta_{su} \\ -\eta_{us} & \gamma^u + \theta - \eta_{uu} \end{bmatrix}}_{A} \begin{bmatrix} \hat{w}_x^s \\ \hat{w}_x^u \end{bmatrix} = \begin{bmatrix} \theta\frac{d\tau_I}{(1-\tau_I)} + \eta_{sk}\hat{\tau}_k - \gamma^s\frac{U_x^s}{w^s l^s(1-\tau^s)} \\ \theta\frac{d\tau_I}{(1-\tau_I)} + \eta_{uk}\hat{\tau}_k - \gamma^u\frac{U_x^u}{w^u l^u(1-\tau^u)} \end{bmatrix} \tag{A.10}$$

where $|A| = (\gamma^s + \theta - \eta_{ii})(\gamma^u + \theta - \eta_{ii}) - \eta_{us}^2 > 0$. Then from (A.10) we obtain

$$\hat{w}_x^i = \frac{1}{|A|}\left[\ (\gamma^j + \theta - \eta_{jj})\left(\theta\frac{d\tau_I}{(1-\tau_I)} + \eta_{ik}\hat{\tau}_k - \gamma^i\frac{U_x^i}{w^i l^i(1-\tau_I^i)}\right) + \eta_{ij}\left(\theta\frac{d\tau_I}{(1-\tau_I)} + \eta_{jk}\hat{\tau}_k - \gamma^j\frac{U_x^j}{w^j l^j(1-\tau_I^j)}\right)\ \right],\ i, j = s, u; i \neq j \tag{A.11}$$

Then replacing in (A.11) gives

$$\hat{w}_{\tau_I}^i = \frac{(\gamma^j + \theta)(\theta + \gamma^i) - \eta_{jj}(\theta + \gamma^i) + \eta_{ij}(\theta + \gamma^j)}{|A|(1-\tau_I)} > 0, \tag{A.12a}$$

$$\hat{w}_{\tau_k}^i = \frac{(\gamma^j + \theta - \eta_{jj})\,\eta_{ik} + \eta_{ij}\eta_{jk}}{|A|(\rho + \tau_k)}, \tag{A.12b}$$

$$\hat{w}^i_{\tau_s} = \frac{\left[\ (\gamma^j + \theta - \eta_{jj})\,\gamma^i B^i_2 + \eta_{ij}\gamma^j B^j_2 \ \right]}{|A|\,(1-\tau_I)} > 0, \ \textit{and} \tag{A.12c}$$

$$\hat{w}^i_g = -\frac{\left[\ (\gamma^j + \theta - \eta_{jj})\,\gamma^i \frac{MRS^i}{w^i l^i} + \eta_{ij}\gamma^j \frac{MRS^j}{w^j l^j} \ \right]}{|A|\,(1-\tau_I)} < 0. \tag{A.12d}$$

A.1.2 Population Gradients

Using (A.12a) - (A.12d) in (A.8) gives the population gradients. For τ_I we have

$$\hat{N}^i_{\tau_I} = \gamma^i \left[\frac{(\gamma^j + \theta - \eta_{jj})\,(\theta + \gamma^i) + \eta_{ij}\,(\theta + \gamma^j)}{|A|\,(1-\tau_I)} - \frac{1}{(1-\tau_I)} \right] \tag{A.13}$$

which gives

$$\hat{N}^i_{\tau_I} = \frac{\gamma^i}{|A|\,(1-\tau_I)} \left[\left(\gamma^j + \theta - \eta_{jj}\right)\left(\theta + \gamma^i\right) + \eta_{ij}\left(\theta + \gamma^j\right) - \left[(\gamma^s + \theta - \eta_{ss})\,(\gamma^u + \theta - \eta_{uu}) - \eta^2_{us} \right] \right] \tag{A.14}$$

which simplifies to

$$\hat{N}^i_{\tau_I} = \frac{\gamma^i}{|A|\,(1-\tau_I)} \left[\eta_{ii}\left(\gamma^j + \theta - \eta_{jj}\right) + \eta_{ij}\left(\theta + \gamma^j + \eta_{ji}\right) \right] < 0. \tag{A.15a}$$

Analogously, we have

$$\hat{N}^i_{\tau_k} = \frac{\gamma^i}{|A|\,(\rho + \tau_k)} \left[\eta_{ik}\left(\gamma^j + \theta - \eta_{jj}\right) + \eta_{ij}\eta_{jk} \right], \tag{A.15b}$$

$$\hat{N}^i_{\tau_s} = \frac{\gamma^i}{|A|\,(1-\tau_I)} \left[\left((\eta_{ii} - \theta)\left(\gamma^j + \theta - \eta_{jj}\right) + \eta^2_{us} \right) B^i_2 + \eta_{ij}\gamma^j B^j_2 \right] < 0, \ \textit{and} \tag{A.15c}$$

$$\hat{N}^i_g = \frac{-\gamma^i}{|A|\,(1-\tau_I)} \left[\left((\eta_{ii} - \theta)\left(\gamma^j + \theta - \eta_{jj}\right) + \eta^2_{us} \right) \frac{MRS^i}{w^i\,(1-\tau_I)} + \eta_{ij}\gamma^j \frac{MRS^j}{w^j\,(1-\tau_I)} \right] > 0. \tag{A.15d}$$

A.1.3 Utility Gradients

From (A.8) it can easily be seen that the impact of a change in policy on utility is simply

$$dU^i_x = \frac{1}{\gamma^i}\hat{N}^i_x = \left(\hat{w_x}^i + \frac{U^i_x}{w^i l^i\,(1-\tau^i_I)} \right). \tag{A.16}$$

Then using (A.15a) - (A.15d) in (A.16) gives

$$dU^i_{\tau_I} = \frac{1}{|A|(1-\tau_I)}\left[\eta_{ii}\left(\gamma^j+\theta-\eta_{jj}\right)+\eta_{ij}\left(\theta+\gamma^j+\eta_{ji}\right)\right] < 0. \tag{A.17}$$

Analogously, we have

$$dU^i_{\tau_k} = \frac{1}{|A|(\rho+\tau_k)}\left[\eta_{ik}\left(\gamma^j+\theta-\eta_{jj}\right)+\eta_{ij}\eta_{jk}\right], \tag{A.18}$$

$$\hat{U}^i_{\tau_s} = \frac{1}{|A|(1-\tau_I)}\left[\left((\eta_{ii}-\theta)\left(\gamma^j+\theta-\eta_{jj}\right)+\eta^2_{us}\right)B^i_2+\eta_{ij}\gamma^j B^j_2\right] < 0,\ and \tag{A.19}$$

$$\hat{U}^i_g = \frac{-1}{|A|(1-\tau_I)}\left[\left((\eta_{ii}-\theta)\left(\gamma^j+\theta-\eta_{jj}\right)+\eta^2_{us}\right)\frac{MRS^i}{w^i(1-\tau_I)}+\eta_{ij}\gamma^j\frac{MRS^j}{w^j(1-\tau_I)}\right] > 0. \tag{A.20}$$

Then

$$\frac{dU^i_g}{dU^i_{\tau_I}} = -\frac{\left[\left((\eta_{ii}-\theta)\left(\gamma^j+\theta-\eta_{jj}\right)+\eta^2_{us}\right)\frac{MRS^i}{w^i(1-\tau_I)}+\eta_{ij}\gamma^j\frac{MRS^j}{w^j(1-\tau_I)}\right]}{\left[\eta_{ii}\left(\gamma^j+\theta-\eta_{jj}\right)+\eta_{ij}\left(\theta+\gamma^j+\eta_{ji}\right)\right]} > 0, \tag{A.21}$$

$$\frac{dU^i_g}{dU^i_{\tau_k}} = -\frac{\left[\left((\eta_{ii}-\theta)\left(\gamma^j+\theta-\eta_{jj}\right)+\eta^2_{us}\right)\frac{MRS^i}{w^i(1-\tau_I)}+\eta_{ij}\gamma^j\frac{MRS^j}{w^j(1-\tau_I)}\right]}{\left[\eta_{ik}\left(\gamma^j+\theta-\eta_{jj}\right)+\eta_{ij}\eta_{jk}\right]}\frac{(\rho+\tau_k)}{(1-\tau_I)} > 0, \tag{A.22}$$

A.1.4 Impact on Land Rents

From (2.7a) and (2.7b) and using (A.12a) - (A.12d) we can obtain the impacts of policy changes on land rents,

$$\hat{R}_{\tau_I} = -\frac{(\gamma^s+\theta)(\theta+\gamma^u)-\phi^s\eta_{uu}(\theta+\gamma^s)-\phi^u\eta_{ss}(\theta+\gamma^u)+\eta_{su}(\theta+\phi^s\gamma^u+\phi^u\gamma^s)}{|A|(1-\tau_I)} < 0, \tag{A.23}$$

$$\hat{R}_{\tau_k} = -\frac{\phi^s\left(\gamma^u+\theta-\eta_{uu}+\phi^u\eta_{su}\right)\eta_{sk}+\phi^u\left(\gamma^s+\theta-\eta_{ss}+\phi^s\eta_{su}\right)\eta_{uk}}{|A|(\rho+\tau_k)}, \tag{A.24}$$

$$\hat{R}_{\tau_s} = -\frac{\left[\ \left(\phi^s\left(\gamma^u+\theta-\eta_{uu}\right)+\phi^u\eta_{us}\right)\gamma^s B^s_2+\left(\phi^u\left(\gamma^s+\theta-\eta_{ss}\right)+\phi^s\eta_{su}\right)\gamma^u B^u_2\ \right]}{|A|(1-\tau_I)} < 0,\ and \tag{A.25}$$

$$\hat{R}_g = \frac{\left[\left(\phi^s \left(\gamma^u + \theta - \eta_{uu}\right) + \phi^u \eta_{us}\right) \gamma^s \frac{MRS^s}{w^s l^s} \quad + \left(\phi^u \left(\gamma^s + \theta - \eta_{ss}\right) + \phi^s \eta_{su}\right) \gamma^u \frac{MRS^u}{w^u l^u} \right]}{|A| \left(1 - \tau_I\right)} > 0. \tag{A.26}$$

A.1.5 Budgetary Impacts

Totally differentiating the budget constraint gives

A.1.6 Impacts on the Budget Constraint

Differentiating the government budget constraint (2.2) with respect to each of the government policies yields

$$\begin{aligned} \frac{dS}{d\tau_I} = \; & N^s l^s w^s + N^u l^u w^u + \tau_I \left(1 + \theta\right) \left(N^s l^s w^s \hat{w}^s_{\tau_I} + N^u l^u w^u \hat{w}^u_{\tau_I}\right) - \theta \tfrac{\tau_I}{(1-\tau_I)} \left(N^s l^s w^s + N^u l^u w^u\right) \\ & + \tau_k K \left(\eta_{ks} \hat{w}^s_{\tau_I} + \eta_{ku} \hat{w}^u_{\tau_I}\right) + + N^s \left(\tau_I l^s w^s + \tau_s x^s_2 - g\right) \hat{N}^s_{\tau_I} + N^u \left(\tau_I l^u w^u + \tau_s x^u_2 - g\right) \hat{N}^u_{\tau_I} \end{aligned} \tag{A.27a}$$

$$\frac{dS}{d\tau_k} = \theta \tau_I \left(N^s l^s w^s \hat{w}^s_{\tau_k} + N^u l^u w^u \hat{w}^u_{\tau_k}\right) + K \left(1 + \tau_k \eta_{kk}\right) + N^s \left(\tau_I l^s w^s - g\right) \hat{N}^s_{\tau_k} + N^u \left(\tau_I l^u w^u - g\right) \hat{N}^u_{\tau_k} \tag{A.27b}$$

$$\frac{dS}{d\tau_s} = \begin{array}{c} \theta \tau_I \left(N^s l^s w^s \hat{w}^s_{\tau_s} + N^u l^u w^u \hat{w}^u_{\tau_s}\right) + \left(N^s x^s_2 \left(1 + \epsilon^s_2\right) + N^u \left(1 + \tau_s \epsilon^u_2\right)\right) \\ + N^s \left(\tau_I l^s w^s + \tau_s x^s_2 - g\right) \hat{N}^s_{\tau_s} + N^u \left(\tau_I l^u w^u + \tau_s x^u_2 - g\right) \hat{N}^u_{\tau_s} \end{array} \quad and \tag{A.27c}$$

$$\frac{dS}{dg} = \begin{array}{c} -\left(N^s + N^u\right) + \theta \tau_I \left(N^s l^s w^s \hat{w}^s_g + N^u l^u w^u \hat{w}^u_g\right) \\ + N^s \left(\tau_I l^s w^s + \tau_s x^s_2 - g\right) \hat{N}^s_g + N^u \left(\tau_I l^u w^u + \tau_s x^u_2 - g\right) \hat{N}^u_g \end{array} \tag{A.27d}$$

A.2 Derivations for *Section 3*

A.2.1 Derivation of First Order Conditions

Maximizing social welfare (??) subject to the government budget constraint (2.2) yields the first order conditions

$$\frac{\partial W}{\partial \tau_I} = -\left[\omega^s N^s w^s l^s + \omega^u N^u w^u l^u\right] + \left(\left(1-\tau_I\right)\omega^u - \omega^R\right) N^u w^u l^u \hat{w}^u_{\tau_I} + \left(\left(1-\tau_I\right)\omega^s - \omega^R\right) N^u w^u l^u \hat{w}^s_{\tau_I}$$
$$\lambda \left[N^s w^s l^s + N^u w^u l^u + \tau_I \left(1+\theta\right)\left[N^s w^s l^s \hat{w}^s_{\tau_I} + N^u w^u l^u \hat{w}^u_{\tau_I}\right] - \frac{\theta \tau_I}{(1-\tau_I)} \left[N^s w^s l^s + N^u w^u l^u\right] \right] = 0 \tag{A.28a}$$

where we use the fact that $\frac{d\pi}{d\tau_I} = \left(f_{L^s} - w^s\right)\frac{dL^s}{d\tau_I} + \left(f_{L^u} - w^u\right)\frac{dL^u}{d\tau_I} - N^s w^s l^s \hat{w}^s_{\tau_I} - N^u l^u w^u \hat{w}^u_{\tau_I}$ where the first two terms equal zero by the equilibrium condition (2.4a) and the fact that $\frac{dV^i}{d\tau_I} = -\frac{\partial V}{\partial y}\left[\left(1-\tau_I\right) w^i l^i \hat{w}^i_{\tau_I} - w^i l^i\right]$. Then dividing by total labor income $\left(N^s w^s l^s + N^u w^u l^u\right) = 1$, using the assumption $N^s + N^u = 1$ and rearranging gives

$$\frac{\partial W}{\partial \tau_I} = -\overline{\Delta w} + \widetilde{\omega w}_{\tau_I} + \lambda \left[1 + \tau_I \left(1+\theta\right)\overline{w}_{\tau_I} - \frac{\theta \tau_I}{(1-\tau_I)} \right] = 0 \tag{A.28b}$$

where $\overline{\Delta w} = \omega^s \phi^s + \omega^u \phi^u,\ \phi^i = N^i w^i,\ i = s, u$ the change in weighted aggregate earnings, $\overline{w}_x = \phi^s \hat{w}^s_x + \phi^u \hat{w}^u_x$, the income-weighted average wage capitalization with respect to policy x and $\widetilde{\omega w}_x = \left(\left(1-\tau_I\right)\omega^u - \omega^R\right)\phi^u \hat{w}^u_{\tau_I} + \left(\left(1-\tau_I\right)\omega^s - \omega^R\right)\phi^s \hat{w}^s_{\tau_I}$ is the direct impact of wage capitalization on social welfare. For the public service we obtain

$$\frac{\partial W}{\partial g} = \omega^s N^s MRS^s + \omega^u N^u MRS^u - \left[\omega^s N^s w^s l^s \hat{w}^s_g + \omega^u N^u l^u w^u \hat{w}^u_g\right] + \left(1-\tau_I\right)\left[N^s w^s l^s \hat{w}^s_g + N^u w^u l^u \hat{w}^u_g\right]$$
$$\lambda \left[- 1 + \tau_I \left(1+\theta\right)\left[N^s w^s l^s \hat{w}^s_g + N^u w^u l^u \hat{w}^u_g\right] \right] = 0 \tag{A.28c}$$

that simplifies to

$$\frac{\partial W}{\partial g} = \overline{MRS} - \lambda = 0 \tag{A.28d}$$

where $\overline{MRS} = \omega^s \phi^s \frac{MRS^s}{w^s l^s} + \omega^u \phi^u \frac{MRS^u}{w^u l^u}$. We obtain (A.28d) using the fact that with a central government $\overline{w}_g = 0$. Then using ((A.28b)) to solve for λ we obtain

$$\lambda = \frac{\overline{\Delta w} - \widetilde{\omega w}_{\tau_I}}{1 + \tau_I \left(1+\theta\right)\overline{w}_{\tau_I} - \frac{\theta \tau_I}{(1-\tau_I)}} \tag{A.28e}$$

substitute for λ in (A.28d) we get

$$\overline{MRS}^{C} = \frac{\overline{\Delta w} - \widetilde{\omega w}^{C}_{\tau_I}}{1+\tau_I \overline{w}^{C}_{\tau_I} + \theta\tau_I\left(\overline{w}^{C}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)} \tag{A.28f}$$

for the general case. When the government is rent-maximizing $(\omega^s = \omega^u = \omega^R = 1)$ $\widetilde{\omega w}_x$ simplifies to $-\tau_I \overline{w}_x$ and when the government is utility maximizing $(\omega^s = \omega^u = 1,\ \omega^R = 0)$, $\widetilde{\omega w}_x$ simplifies to $(1-\tau_I)\,\overline{w}_x$. Then for these two cases,

$$\overline{MRS}^{C_{Rent}} = \frac{1+\ \tau_I \overline{w}^{C}_{\tau_I}}{1+\tau_I \overline{w}^{C}_{\tau_I} + \theta\tau_I\left(\overline{w}^{C}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)} = 1 - \frac{\theta\tau_I\left(\overline{w}^{C}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)}{1+\tau_I \overline{w}^{C}_{\tau_I} + \theta\tau_I\left(\overline{w}^{C}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)} > (=) < 1\, if\ \theta < (=) > \tag{A.28g}$$

$$\overline{MRS}^{C_{Utility}} = \frac{1-\ (1-\tau_I)\,\overline{w}^{C}_{\tau_I}}{1+\tau_I \overline{w}^{C}_{\tau_I} + \theta\tau_I\left(\overline{w}^{C}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)} = 1 - \frac{\hat{w}^{C}_{\tau_I} + \theta\tau_I\left(\overline{w}^{C}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)}{1+\tau_I \overline{w}^{C}_{\tau_I} + \theta\tau_I\left(\overline{w}^{C}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)} \tag{A.28h}$$

Alternatively, we can express equilibrium as the condition $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}} = -\frac{\frac{dW}{dg}}{\frac{dW}{d\tau_I}} = -\frac{\frac{dS}{dg}}{\frac{dS}{d\tau_I}} = \left.\frac{d\tau_I}{dg}\right|_{S=\overline{S}}$, the slope of the indifference curve in $\{\tau_I, g\}$ equals the slope of the budget constraint. For the three cases, the slope of the indifference curves are

$$\left.\frac{d\tau_I}{dg}\right|^{C}_{W=\overline{W}} = \frac{\overline{MRS}}{[\omega^s\phi^s + \omega^u\phi^u] - \widetilde{\omega w}^{C}_{\tau_I}} \tag{A.28ia}$$

$$\left.\frac{d\tau_I}{dg}\right|^{C_{Rent}}_{W=\overline{W}} = \frac{\overline{MRS}}{1+\ \tau_I \overline{w}^{C}_{\tau_I}} \tag{A.28ib}$$

$$\left.\frac{d\tau_I}{dg}\right|^{C_{Utility}}_{W=\overline{W}} = \frac{\overline{MRS}}{1-\ (1-\tau_I)\,\overline{w}^{C}_{\tau_I}} \tag{A.28ic}$$

with

$$\left.\frac{d\tau_I}{dg}\right|^{C_{Utility}}_{W=\overline{W}} > (=) < \left.\frac{d\tau_I}{dg}\right|^{C_{Rent}}_{W=\overline{W}} \iff \begin{array}{c} \frac{\overline{MRS}}{1-\ (1-\tau_I)\overline{w}^{C}_{\tau_I}} > (=) < \frac{\overline{MRS}}{1+\ \tau_I \overline{w}^{C}_{\tau_I}} \\ 0 > -\overline{w}^{C}_{\tau_I} \end{array} \tag{A.28id}$$

and therefore $\left.\frac{d\tau_I}{dg}\right|^{C_{Utility}}_{W=\overline{W}} > \left.\frac{d\tau_I}{dg}\right|^{C_{Rent}}_{W=\overline{W}}$.
From (A.28b) and (A.28d) we obtain the slope of the budget constraint,

$$\left.\frac{d\tau_I}{dg}\right|_{S=\overline{S}}^{C}=\frac{1}{1+\tau_I\left(1+\theta\right)\overline{w}_{\tau_I}^{C}-\ \frac{\theta\tau_I}{(1-\tau_I)}} \tag{A.28j}$$

then using (A.28j), (A.28ia) - (A.28ic) provides the equibrium conditions,

$$\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{C}=\frac{\overline{MRS}}{\overline{\Delta w}-\widetilde{\omega w}_{\tau_I}^{C}}=\frac{1}{1+\tau_I\left(1+\theta\right)\overline{w}_{\tau_I}^{C}-\ \frac{\theta\tau_I}{(1-\tau_I)}}=\left.\frac{d\tau_I}{dg}\right|_{S=\overline{S}}^{C} \tag{A.28k}$$

$$\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{C_{Rent}}=\frac{\overline{MRS}}{1+\ \tau_I\overline{w}_{\tau_I}^{C}}=\frac{1}{1+\tau_I\left(1+\theta\right)\overline{w}_{\tau_I}^{C}-\ \frac{\theta\tau_I}{(1-\tau_I)}}=\left.\frac{d\tau_I}{dg}\right|_{S=\overline{S}}^{C}\ and \tag{A.28l}$$

$$\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{C_{Utility}}=\frac{\overline{MRS}}{1-\ \left(1-\tau_I\right)\overline{w}_{\tau_I}^{C}}=\frac{1}{1+\tau_I\left(1+\theta\right)\overline{w}_{\tau_I}^{C}-\ \frac{\theta\tau_I}{(1-\tau_I)}}=\left.\frac{d\tau_I}{dg}\right|_{S=\overline{S}}^{C}. \tag{A.28m}$$

A.3 Derivations for *Section 4*

A.3.1 Derivation of the First Order Condition (Income Tax)

The first order conditions for regional governments with a uniform tax are analogous for the central government with an additional term reflecting the impact of differential migration

$$\frac{\partial L}{\partial\tau_I}=-\overline{\Delta w}+\widetilde{\omega\hat{w}_{\tau_I}}+\lambda\left[1+\tau_I\left(1+\theta\right)\overline{w}_{\tau_I}-\theta\frac{\tau_I}{(1-\tau_I)}+\Delta S_{\tau_I}\right]=0 \tag{A.28na}$$

$$\frac{\partial L}{\partial g}=\overline{MRS}+\widetilde{\omega\hat{w}_g}+\lambda\left[-1+\tau_I\left(1+\theta\right)\overline{w}_g+\Delta S_g\right]=0 \tag{A.28nb}$$

Solving (A.28na) for λ gives

$$\lambda=\frac{\overline{\Delta w}-\widetilde{\omega\hat{w}_{\tau_I}}}{1+\tau_I\left(1+\theta\right)\overline{w}_{\tau_I}-\theta\frac{\tau_I}{(1-\tau_I)}+\Delta S_{\tau_I}} \tag{A.28nc}$$

Then using (A.28nb) gives

$$\overline{MRS}=\frac{\left[1-\tau_I\left(1+\theta\right)\overline{w}_g-\Delta S_g\right]}{1+\tau_I\left(1+\theta\right)\overline{w}_{\tau_I}-\theta\frac{\tau_I}{(1-\tau_I)}+\Delta S_{\tau_I}}\left[\overline{\Delta w}-\widetilde{\omega\hat{w}_{\tau_I}}\right]-\widetilde{\omega\hat{w}_g} \tag{A.28nd}$$

Then for rent maximization we have

$$\overline{MRS} = 1 + \frac{[1-\tau_I(1+\theta)\overline{w}_g - \Delta S_g]}{1+\tau_I(1+\theta)\overline{w}_{\tau_I} - \theta\frac{\tau_I}{(1-\tau_I)} + \Delta S_{\tau_I}}\left[1+\tau_I\overline{w}_{\tau_I}\right] - \left[1-\tau_I\overline{w}_g\right]$$
$$= 1 + \frac{[1-\tau_I\overline{w}_g]\Delta S_{\tau_I} - \left[1+\tau_I\overline{w}_{\tau_I}\right]\Delta S_g - \tau_I\theta\left(\overline{w}_g + \overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)}{1+\tau_I\overline{w}_{\tau_I} - \theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + \Delta S_{\tau_I}} \quad \text{(A.28ne)}$$

Then for the three cases we have the slopes of the indifference curves given by

$$\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R} = \frac{\overline{MRS} + \widetilde{\omega\hat{w}_g}}{\Delta w - \widetilde{\omega\hat{w}_{\tau_I}}}, \; \left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R_{Rent}} = \frac{\overline{MRS} - \tau_I\overline{w}_g}{1+\tau_I\overline{w}_{\tau_I}}, \; and \; \left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R_{Utility}} = \frac{\overline{MRS} + (1-\tau_I)\,\overline{w}_g}{1-(1-\tau_I)\,\overline{w}_{\tau_I}}. \quad \text{(A.28o)}$$

Then using (A.28o) gives

$$\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{Utility} > (=) < \left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{Rent} \Longleftrightarrow \frac{\overline{MRS} + (1-\tau_I)\overline{w}_g}{1-(1-\tau_I)\overline{w}_{\tau_I}} > (=) < \frac{\overline{MRS} - \tau_I\overline{w}_g}{1+\tau_I\overline{w}_{\tau_I}}. \quad \text{(A.28p)}$$

that can be simplified to

$$(1-\tau_I)\,\overline{w}_g + \tau_I\overline{w}_{\tau_I}\left(\phi^s\frac{MRS^s}{w^s l^s} + \phi^u\frac{MRS^u}{w^u l^u}\right) > (=) < -(1-\tau_I)\,\overline{w}_{\tau_I}\overline{MRS} - \tau_I\overline{w}_g \quad \text{(A.28q)}$$

and further to

$$\overline{w}_g + \overline{w}_{\tau_I}\overline{MRS} > (=) < 0 \quad \text{(A.28r)}$$

Then using (A.11) to substitute for $\overline{w}_g$ and $\overline{w}_{\tau_I}$ gives

$$-\frac{\phi^s\gamma^s\left(\Gamma^s + \frac{\phi^u}{\phi^s}\eta_{us}\right)\frac{MRS^s}{w^s l^s} + \phi^u\gamma^u\left(\Gamma^u + \frac{\phi^s}{\phi^u}\eta_{su}\right)\frac{MRS^u}{w^u l^u}}{\phi^s(\gamma^s+\theta)\left(\Gamma^s + \frac{\phi^u}{\phi^s}\eta_{us}\right) + \phi^u(\gamma^u+\theta)\left(\Gamma^u + \frac{\phi^s}{\phi^u}\eta_{su}\right)} + \left(\phi^s\frac{MRS^s}{w^s l^s} + \phi^u\frac{MRS^u}{w^u l^u}\right) > (=) < 0 \quad \text{(A.28s)}$$

where $\Gamma^i = (\gamma^j + \theta - \eta_{jj})\,,\; i \neq j.$ Further simplifying gives

$$\frac{-\phi^s\gamma^s\left(\Gamma^s + \frac{\phi^u}{\phi^s}\eta_{us}\right)\frac{MRS^s}{w^s l^s} - \phi^u\gamma^u\left(\Gamma^u + \frac{\phi^s}{\phi^u}\eta_{su}\right)\frac{MRS^u}{w^u l^u} + \left[\phi^s(\gamma^s+\theta)\left(\Gamma^s + \frac{\phi^u}{\phi^s}\eta_{us}\right) + \phi^u(\gamma^u+\theta)\left(\Gamma^u + \frac{\phi^s}{\phi^u}\eta_{su}\right)\right]\left(\phi^s\frac{MRS^s}{w^s l^s}\right.}{\phi^s(\gamma^s+\theta)\left(\Gamma^s + \frac{\phi^u}{\phi^s}\eta_{us}\right) + \phi^u(\gamma^u+\theta)\left(\Gamma^u + \frac{\phi^s}{\phi^u}\eta_{su}\right)} \quad \text{(A.28t)}$$

and further to

$$\frac{\phi^s\phi^u\left[(\gamma^s+\theta)\left(\Gamma^s+\frac{\phi^u}{\phi^s}\eta_{us}\right)\frac{MRS^u}{w^u l^u}+(\gamma^u+\theta)\left(\Gamma^u+\frac{\phi^s}{\phi^u}\eta_{su}\right)\frac{MRS^s}{w^s l^s}\right]+\phi^{s^2}\theta\left(\right)\left(\Gamma^s+\frac{\phi^u}{\phi^s}\eta_{us}\right)\frac{MRS^s}{w^s l^s}+\phi^{u^2}\theta\left(\Gamma^u+\frac{\phi^s}{\phi^u}\eta_{su}\right)\frac{MR}{w^u}}{\phi^s\left(\gamma^s+\theta\right)\left(\Gamma^s+\frac{\phi^u}{\phi^s}\eta_{us}\right)+\phi^u\left(\gamma^u+\theta\right)\left(\Gamma^u+\frac{\phi^s}{\phi^u}\eta_{su}\right)} \tag{A.28u}$$

that must be positive for $\theta \geq 0$. This can be simplified when $\theta = 0$ to

$$\frac{\phi^s\phi^u\left[\gamma^s\left(\Gamma^s+\frac{\phi^u}{\phi^s}\eta_{us}\right)\frac{MRS^u}{w^u l^u}+\gamma^u\left(\Gamma^u+\frac{\phi^s}{\phi^u}\eta_{su}\right)\frac{MRS^s}{w^s l^s}\right]}{\phi^s\gamma^s\left(\Gamma^s+\frac{\phi^u}{\phi^s}\eta_{us}\right)+\phi^u\gamma^u\left(\Gamma^u+\frac{\phi^s}{\phi^u}\eta_{su}\right)} > 0 \tag{A.28v}$$

Then for both objectives,

$$\left.\frac{d\tau_I}{dg}\right|_{S=\overline{S}}^{R} = \frac{1-\tau_I\left(1+\theta\right)\overline{w}_g - N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}^s_g-\hat{N}^u_g\right)}{1+\tau_I\left(1+\theta\right)\overline{w}_{\tau_I}-\theta\frac{\tau_I}{(1-\tau_I)}+N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}^s_{\tau_I}-\hat{N}^u_{\tau_I}\right)}. \tag{A.28w}$$

A.3.2 Proof of ***Proposition 2***

Comparisons of equilibrium for centralized and regional governments for 3 cases: general welfare maximization, rent maximization, and utility maximization,

General Welfare Maximization

$$\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R} >=< \left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{C} \iff \begin{array}{c} \frac{\overline{MRS}^C+\widetilde{\omega\hat{w}}^R_g}{(\omega^s\phi^s+\omega^u\phi^u)-\widetilde{\omega\hat{w}}^R_{\tau_I}} >=< \frac{\overline{MRS}^C}{\left[\omega^s\phi^s+\omega^u\phi^u\right]-\ \widetilde{\omega w}^C_{\tau_I}} \\ \left(\widetilde{\omega\hat{w}}^C_{\tau_I}-\widetilde{\omega\hat{w}}^R_{\tau_I}\right)\overline{MRS}^C >=< \left[\overline{MRS}^C+\widetilde{\omega\hat{w}}^R_g\right]\widetilde{\omega w}^C_{\tau_I} \end{array} \tag{A.15}$$

and

$$\left.\frac{d\tau_I}{dg}\right|_{S=\overline{S}}^{R} >=< \left.\frac{d\tau_I}{dg}\right|_{S=\overline{S}}^{C} \iff \begin{array}{c} \frac{1-\tau_I(1+\theta)\overline{w}^R_g-N^s(\tau_I w^s l^s-g)\left(\hat{N}^s_g-\hat{N}^u_g\right)}{1+\tau_I(1+\theta)\overline{w}^R_{\tau_I}-\theta\frac{\tau_I}{(1-\tau_I)}+N^s(\tau_I w^s l^s-g)\left(\hat{N}^s_{\tau_I}-\hat{N}^u_{\tau_I}\right)} >=< \frac{1}{1+\tau_I(1+\theta)\overline{w}^C_{\tau_I}-\ \theta\frac{\tau_I}{(1-\tau}} \\ -\left[\tau_I\left(1+\theta\right)\overline{w}^R_g+N^s\left(\tau_I w^s l^s-g\right)\left(\hat{N}^s_g-\hat{N}^u_g\right)\right] >=< \frac{\tau_I(1+\theta)\left(\overline{w}^R_{\tau_I}-\overline{w}^C_{\tau_I}\right)+N^s(\tau_I w}{1+\tau_I(1+\theta)\overline{w}^C_{\tau_I}-\ \theta} \end{array} \tag{A.16}$$

with $\theta = 0$

$$\left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{R} >=< \left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{C} \iff -\left[\tau_I \overline{w}_g^R + N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}_g^s - \hat{N}_g^u\right)\right] >=< \frac{\tau_I\left(\overline{w}_{\tau_I}^R - \overline{w}_{\tau_I}^C\right) + N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}_{\tau_I}^s - \hat{N}\right.}{1+\tau_I \overline{w}_{\tau_I}^C} \tag{A.17}$$

$$\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R_R} = \frac{\overline{MRS}^C - \tau_I \overline{w}_g}{1+\tau_I \overline{w}_{\tau_I}} > (=) < \frac{1-\tau_I\left(1+\theta\right)\overline{w}_g - \Delta S_g}{1+\tau_I\left(1+\theta\right)\overline{w}_{\tau_I} - \theta\frac{\tau_I}{(1-\tau_I)} + \Delta S_{\tau_I}} \tag{A.18}$$

From (4.2) with $\theta = 0$ we have gives

$$\left.\frac{d\tau_I}{dg}\right|_{\substack{W=\overline{W}\\ \{\tau_I^c, g^c\}}}^{R} = \frac{1-\tau_I \overline{w}_g}{1+\tau_I \overline{w}_{\tau_I}} > (=) < \frac{1 - \tau_I \overline{w}_g - N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}_g^s - \hat{N}_g^u\right)}{1+\tau_I \overline{w}_{\tau_I} + N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}_{\tau_I}^s - \hat{N}_{\tau_I}^u\right)} = \left.\frac{d\tau_I}{dg}\right|_{\substack{R=\overline{R}\\ \{\tau_I^c, g^c\}}}^{R} \tag{A.19a}$$

for $g^R > (=) < g^C$ when $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R}$ and $\left.\frac{d\tau_I}{dg}\right|_{R=\overline{R}}^{R}$ are evaluated at $\{\tau_I^C, g^C\}$ which we get from substituting 1 for $\phi^s \frac{MRS^s}{w^s l^s} + \phi^u \frac{MRS^u}{w^u l^u}$ in $\left.\frac{d\tau_I}{dg}\right|_{W=\overline{W}}^{R}$. Then we can express the inequality (A.19a) as

$$\left(1-\tau_I \overline{w}_g\right)\left(\hat{N}_{\tau_I}^s - \hat{N}_{\tau_I}^u\right) > (=) < -\left(1+\tau_I \overline{w}_{\tau_I}\right)\left(\hat{N}_g^s - \hat{N}_g^u\right) \tag{A.19b}$$

or equivalently,

$$\hat{N}_g^s - \hat{N}_g^u > (=) < -\left[\frac{1-\tau_I \overline{w}_g}{1+\tau_I \overline{w}_{\tau_I}}\right]\left(\hat{N}_{\tau_I}^s - \hat{N}_{\tau_I}^u\right). \tag{A.19c}$$

Approximating $\frac{1-\tau_I \overline{w}_g}{1+\tau_I \overline{w}_{\tau_I}} \approx 1$ we can express (A.19c) by

$$\hat{N}_{\tau_I}^s + \hat{N}_g^s - \left(\hat{N}_{\tau_I}^u + \hat{N}_g^u\right) > (=) < 0. \tag{A.19d}$$

Then using (A.12a) and (A.12d) in (A.5) gives

$$\begin{aligned}\hat{N}_{\tau_I}^s + \hat{N}_g^s - &= \gamma^s\left[\hat{w}_g^s + \tfrac{MRS^s}{w^s l^s(1-\tau_I)} + \hat{w}_{\tau_I}^s - \tfrac{1}{(1-\tau_I)}\right]\\ &= \tfrac{\gamma^s}{|A|(1-\tau_I)}\left[\left(\gamma^u - \eta_{uu}\right)\gamma^s\left(1 - \tfrac{MRS^s}{w^s l^s}\right) + |A|\left(\tfrac{MRS^s}{w^s l^s} - 1\right) + \gamma^s\gamma^u\eta_{su}\left(1 - \tfrac{MRS^u}{w^u l^u}\right)\right]\\ &= \tfrac{\gamma^s}{|A|(1-\tau_I)}\left[\left(-\eta_{ss}\right)\gamma^u + \left(\eta_{uu}\eta_{ss} - \eta_{su}\eta_{us}\right)\left(\tfrac{MRS^s}{w^s l^s} - 1\right) - \gamma^s\gamma^u\eta_{su}\left(\tfrac{MRS^u}{w^u l^u} - 1\right)\right]\end{aligned} \tag{A.19e}$$

Then subtracting an analogous expression to (A.19e for the unskilled workers gives

$$\hat{N}^s_{\tau_I} + \hat{N}^s_g - \left(\hat{N}^u_{\tau_I} + \hat{N}^u_g\right) = \frac{1}{|A|(1-\tau_I)} \left[\begin{array}{c} \gamma^s \left[(-\eta_{ss} + \eta_{us})\gamma^u + (\eta_{uu}\eta_{ss} - \eta_{su}\eta_{us})\left(\frac{MRS^s}{w^s l^s} - 1\right)\right] - \\ \gamma^u \left[(-\eta_{uu} + \eta_{su})\gamma^s + (\eta_{uu}\eta_{ss} - \eta_{su}\eta_{us})\left(\frac{MRS^u}{w^u l^u} - 1\right)\right] \end{array} \right] \tag{A.19f}$$

where $\gamma^i\left[(-\eta_{ii} + \eta_{ij})\gamma^j + (\eta_{uu}\eta_{ss} - \eta_{su}\eta_{us})\right] > 0,\ i,j = u,s;\ i \neq j$. Then at the efficient output if $\frac{MRS^s}{w^s l^s} > (<) 1$ then $\frac{MRS^s}{w^s l^s} < (>) 1$ making $\hat{N}^s_{\tau_I} + \hat{N}^s_g - \left(\hat{N}^u_{\tau_I} + \hat{N}^u_g\right) > (<)\ 0$.

Case 2: $\theta \neq 0$

At the central government policies $\left\{\tau_I^C, g^C\right\}$ we have

$$\left.\frac{d\tau_I}{dg}\right|^R_{W=\overline{W}} = \frac{1-\tau_I\overline{w}_g - \frac{\theta\tau_I\left(\overline{w}^c_{\tau_I} - \frac{1}{(1-\tau_I)}\right)}{\left(1+\tau_I\overline{w}^c_{\tau_I} + \theta\tau_I\left(\overline{w}^c_{\tau_I} - \frac{1}{(1-\tau_I)}\right)\right)}}{1+\tau_I\overline{w}_{\tau_I}}\ and\ . \tag{A.20}$$

$$\left.\frac{d\tau_I}{dg}\right|^R_{R=\overline{R}} = \frac{1-\tau_I\overline{w}_g - N^s(\tau_I w^s l^s - g)\left(\hat{N}^s_g - \hat{N}^u_g\right)}{1+\tau_I\overline{w}_{\tau_I} + \theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + N^s(\tau_I w^s l^s - g)\left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right)} \tag{A.21}$$

Then if $\theta > 0$, it must be the case that $\left.\frac{d\tau_I}{dg}\right|^R_{\substack{W=\overline{W}\\ \{\tau_I^C, g^C\}}} > \frac{1-\tau_I\overline{w}_g}{1+\tau_I\overline{w}_{\tau_I}}$. Then if $\left.\frac{d\tau_I}{dg}\right|^R_{\substack{R=\overline{R}\\ \{\tau_I^C, g^C\}}} < \frac{1-\tau_I\overline{w}_g}{1+\tau_I\overline{w}_{\tau_I}}$ it follows that $\left.\frac{d\tau_I}{dg}\right|^R_{\substack{W=\overline{W}\\ \{\tau_I^C, g^C\}}} > \frac{1-\tau_I\overline{w}_g}{1+\tau_I\overline{w}_{\tau_I}} > \left.\frac{d\tau_I}{dg}\right|^R_{\substack{R=\overline{R}\\ \{\tau_I^C, g^C\}}}$ and $g^R > g^C$.

$\left.\frac{d\tau_I}{dg}\right|^R_{\substack{R=\overline{R}\\ \{\tau_I^C, g^C\}}} = \frac{1-\tau_I\overline{w}_g - N^s(\tau_I w^s l^s - g)\left(\hat{N}^s_g - \hat{N}^u_g\right)}{1+\tau_I\overline{w}_{\tau_I} + \theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + N^s(\tau_I w^s l^s - g)\left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right)} < \frac{1-\tau_I\overline{w}_g}{1+\tau_I\overline{w}_{\tau_I}} \Rightarrow \hat{N}^s_g - \hat{N}^u_g > -\left[\frac{1-\tau_I\overline{w}_g}{1+\tau_I\overline{w}_{\tau_I}}\right]\left[\frac{\theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right)}{N^s(\tau_I w^s l^s - g)} + \left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right)\right]$ with the inequalities reversed when $\theta < 0$.

A.3.3 *Proof of Proposition 3*

From the first order conditions (4.4a) and (4.4b) evaluated at $\tau_k = 0$ we obtain

$$\left.\frac{d\tau_I}{d\tau_k}\right|_{W=\overline{W}} = \frac{\tau_I\overline{w}_{\tau_K} + K}{1+\tau_I\overline{w}_{\tau_I}}\ and \frac{\tau_I\overline{w}_{\tau_K} + K + N^s(\tau_I w^s l^s - g)\left(\hat{N}^s_{\tau_k} - \hat{N}^u_{\tau_k}\right) + \theta\tau_I\overline{w}_{\tau_k}}{\frac{1+\tau_I\overline{w}_{\tau_I}}{\lambda^L}} = \left.\frac{d\tau_I}{d\tau_k}\right|_{R=\overline{R}}\ . \tag{A.22}$$

where λ^L is the value of λ that satisfies (4.4a) and (4.4c) when $\tau_k = 0$. Then it follows from (A.22) that $\left.\frac{d\tau_I}{d\tau_k}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_k}\right|_{R=\overline{R}} > (=) < 0 \Longleftrightarrow$

$$\underset{(a)}{\left(\tfrac{1}{\lambda^L} - 1\right)\left[\tfrac{\tau_I \overline{w}_{\tau_K} + K}{1+\tau_I \overline{w}_{\tau_I}}\right]} > (=) < \underset{(b)}{N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}^s_{\tau_k} - \hat{N}^u_{\tau_k}\right)} \underset{(c)}{+\theta\tau_I \overline{w}_{\tau_k}} . \tag{A.23}$$

Then if $\theta > 0$ and $\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I} < 0$ then $\lambda^L > 1$ and term (a) is negative. Then term (c) is positive if $\overline{w}_{\tau_k} > 0$. Then if $\hat{N}^s_{\tau_k} > \hat{N}^u_{\tau_k}$ term (b) is positive making (b) + (c) is positive and $\left.\frac{d\tau_I}{d\tau_k}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_k}\right|_{R=\overline{R}} < 0$ and $\tau_k < 0$. If $\theta < 0$ and $\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I} > 0$ then $\lambda^L < 1$ term (a) is positive. Then term (c) is negative if $\overline{w}_{\tau_k} > 0$ and term (b) is negative if $\hat{N}^s_{\tau_k} - \hat{N}^u_{\tau_k} < 0$ then $\left.\frac{d\tau_I}{d\tau_k}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_k}\right|_{R=\overline{R}} > 0$ and $\tau_k > 0$.

Impact of Capital Tax on Public Service Provision and Income Tax

Treating τ_k as an exogenous parameter, consider the impacts on g,τ_I, *and* λ as defined by (4.4a) and (4.4c) and the budget constraint. Then totally differentiating this system with respect to τ_k yields

$$\underset{H}{\begin{bmatrix} 0 & R_{\tau_I} & R_g \\ R_{\tau_I} & L_{\tau_I\tau_I} & L_{\tau_I g} \\ R_g & L_{g\tau_I} & L_{gg} \end{bmatrix}} \begin{bmatrix} \frac{d\lambda}{d\tau_k} \\ \frac{d\tau_I}{d\tau_k} \\ \frac{dg}{d\tau_k} \end{bmatrix} = \begin{bmatrix} -R_{\tau_k} \\ -\lambda R_{\tau_I\tau_k} \\ -\lambda R_{g\tau_k} \end{bmatrix} \tag{A.24}$$

Then solving gives

$$\frac{d\tau_I}{d\tau_k} = \left[\underset{(+)}{R_{\tau_k}} \underset{(-)}{\left(\underset{(-)}{R_{\tau_I} L_{gg}} \; \underset{(+)}{-R_g L_{\tau_I g}} \right)} - \lambda \underset{(-)}{R_g} \underset{(-)}{\left(\underset{(+)}{R_{\tau_I} R_{g\tau_k}} \; \underset{(-)}{-R_g R_{\tau_I\tau_k}} \right)} \right] |H|^{-1} < 0 \tag{A.25}$$

and

$$\frac{dg}{d\tau_k} = \left[\underset{(-)}{-R_{\tau_k}} \underset{(-)}{\left(\underset{(-)}{R_{\tau_I} L_{g\tau_I}} \; \underset{(-)}{-R_g L_{\tau_I\tau_I}} \right)} - \lambda \underset{(+)}{R_{\tau_I}} \underset{(-)}{\left(\underset{(+)}{R_{\tau_I} R_{g\tau_k}} \; \underset{(-)}{-R_g R_{\tau_I\tau_k}} \right)} \right] |H|^{-1} < 0 \tag{A.26}$$

where $R_{\tau} = 1 + \tau_I \overline{w}_{\tau_I} + \theta\tau_I \left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + N^s\left(\tau_I w^s l^s - g\right)\hat{N}^s_{\tau_I} + N^u\left(\tau_I w^u l^u - g\right)\hat{N}^u_{\tau_I} +$ $\tau_k K\hat{K}_{\tau_I} > 0$, $R_{\tau_k} = K(1{+}\tau_k\hat{K}_{\tau_k}){>}0$ $\;R_g = \; -1 + \left[N^s\left(\tau_I w^s l^s - g\right)\hat{N}^s_g + N^s\left(\tau_I w^u l^u - g\right)\hat{N}^u_g + \tau_k K\hat{K}_g\right] + [\tau$ $R_{\tau_I\tau_k} = \tau_I\left[\frac{d[N^s w^s l^s]}{d\tau_k}\hat{N}^s_{\tau_I} + \frac{d[N^u w^u l^u]}{d\tau_k}\hat{N}^u_{\tau_I}\right] {+} K(1{+}\tau_k\hat{K}_{\tau_k})\hat{K}_{\tau_I}$, $R_{g\tau_k} = \tau_I\left[\frac{d[N^s w^s l^s]}{d\tau_k}\hat{N}^s_g + \frac{d[N^u w^u l^u]}{d\tau_k}\hat{N}^u_g\right]+$ $K(1{+}\tau_k\hat{K}_{\tau_k})\hat{K}_g$, $L_{\tau_I g} = R_{\tau_I g} = N^s\left(\tau_I w^s l^s - g\right)\hat{N}^s_g\hat{N}^s_{\tau_I} {+} N^u\left(\tau_I w^s l^s - g\right)\hat{N}^u_g\hat{N}^s_{\tau_I} {+} \tau_I(1{+}\theta)\overline{w}_g$

			$\frac{d\tau_I}{d\tau_k} < 0$	$\frac{dg}{d\tau_k} > 0$
$L_{\tau_I\tau_I}$	<0	SOC		
L_{gg}	<0	SOC		
$L_{g\tau_I}{=}L_{\tau_I g} = R_{\tau_I g} = R_{g\tau_I}$	?		<0	
$L_{\tau_I g}$	?		<0	
R_{τ_I}	>0			
R_g	<0			
R_{τ_k}	>0			
$R_{\tau_I\tau_k}$			<0	
$R_{g\tau_k}$			<0	
$R_{\tau_I}L_{gg} \quad -R_g L_{\tau_I g} \; = \frac{R_{\tau_I}}{R_g}\left(\frac{L_{gg}}{R_g} - \frac{L_{\tau_I g}}{R_{\tau_I}}\right)$			<0	
$R_{\tau_I}R_{g\tau_k} \quad -R_g R_{\tau_I\tau_k} \; = \frac{R_{\tau_I}}{R_g}\left(\frac{R_{g\tau_k}}{R_g} - \frac{R_{\tau_I\tau_k}}{R_{\tau_I}}\right)$			<0	
$R_{\tau_I}L_{g\tau_I} \quad -R_g L_{\tau_I\tau_I}$			<0	<0
$R_{\tau_I}L_{gg} \quad -R_g L_{\tau_I g}$			<0	

A.4 Derivations for *Section 4.3*

A.4.1 Proof of *Proposition 4*

$\frac{d\tau_I}{d\tau_s}\Big|_{W=\overline{W}} - \frac{d\tau_I}{d\tau_s}\Big|_{R=\overline{R}} =$

$$\frac{\phi^s B^s_1 + \phi^u B^u_1 + \tau_I \overline{w}_{\tau_s}}{1 + \tau_I \overline{w}_{\tau_I}} - \frac{\begin{bmatrix} \phi^s B^s_1 + \phi^u B^u_1 + (1+\theta)\tau_I\overline{w}_{\tau_s} + N^s\left(\tau_I w^s l^s - g\right)\hat{N}^s_{\tau_s} \\ +N^u\left(\tau_I w^u l^u - g\right)\hat{N}^u_{\tau_s} + \tau_k K\hat{K}_{\tau_s}\end{bmatrix}}{\begin{bmatrix}(1+\tau_I\overline{w}_{\tau_I}) + \theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + N^s\left(\tau_I w^s l^s - g\right)\hat{N}^s_{\tau_I} \\ +N^u\left(\tau_I w^u l^u - g\right)\hat{N}^u_{\tau_I} + \tau_k K\hat{K}_{\tau_I}\end{bmatrix}}. \tag{A.27}$$

One interpretation of (A.27) is as relative gains in revenue (and loss in resident utility and profits) absent any changes labor supply, capital, or the fiscal deficits or supply, $\frac{d\tau_I}{d\tau_s}\Big|_{W=\overline{W}}$, that affect the tax base and public service costs minus the relative gains in revenue allowing for changes in the tax base and service costs. Then we can express (A.27) by

$$\left.\frac{d\tau_I}{d\tau_s}\right|_{\substack{W=\overline{W}\\ \tau_s=0}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{\substack{R=\overline{R}\\ \tau_s=0}} > (=) < 0 \; if \; \frac{\overline{R}_{\tau_s}}{\overline{R}_{\tau_I}}\left(\frac{\Delta R_{\tau_I}}{\overline{R}_{\tau_I}} - \frac{\Delta R_{\tau_s}}{\overline{R}_{\tau_s}}\right) > (=) < 0. \quad \text{(A.28)}$$

where $\overline{R}_{\tau_s} = \phi^s B_1^s + \phi^u B_1^u + \tau_I \overline{w}_{\tau_s}$, $\overline{R}_{\tau_I} = 1 + \tau_I \overline{w}_{\tau_I}$, $\Delta R_{\tau_I} = \theta\tau_I\left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + N^s\left(\tau_I w^s l^s - g\right)\hat{N}^s_{\tau_I} + N^u\left(\tau_I w^u l^u - g\right)\hat{N}^u_{\tau_I} + \tau_k K\hat{K}_{\tau_I}$, and $\Delta R_{\tau_s} = \theta\tau_I\overline{w}_{\tau_s} + N^s\left(\tau_I w^s l^s - g\right)\hat{N}^s_{\tau_s} + N^u\left(\tau_I w^u l^u - g\right)\hat{N}^u_{\tau_s} + \tau_k K\hat{K}_{\tau_s}$.

Then if at $\tau_s = 0$, if $\left.\frac{d\tau_I}{d\tau_s}\right|_{\substack{W=\overline{W}\\ \tau_s=0}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{\substack{R=\overline{R}\\ \tau_s=0}} < 0$, meaning that the loss in net income tax revenue as a percentage of marginal income tax revenue $\left(\frac{\Delta R_{\tau_I}}{\overline{R}_{\tau_I}}\right)$ is greater (in absolute value) than the loss for the sales tax $\left(\frac{\Delta R_{\tau_s}}{\overline{R}_{\tau_s}}\right)$, then it is optimal to institute a positive sales tax on x_1. However, from inspection of (A.28) given the definitions of its terms, its sign is not obvious. To focus the impact that differential mobility has on the implementation of a sales tax assume that $\theta = 0$ and a capital tax is not employed ($\tau_k = 0$). Further assume that $\eta_{us} = \eta_{su} = 0$. Then if this is case the $\hat{w}^i_{\tau_s} = B_1^i \hat{w}^i_{\tau_I}$ and $\hat{N}^i_{\tau_s} = B_1^i \hat{N}^i_{\tau_I}$ and $\overline{w}_{\tau_s} = \phi^s B_1^s \hat{w}^s_{\tau_I} + \phi^u B_1^u \hat{w}^u_{\tau_I}$. Then making these substitutions and simplifying gives $\left.\frac{d\tau_I}{d\tau_s}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{R=\overline{R}} > (=) < 0 \; if$

$$\underset{(a)}{\left(\phi^s B_1^s + \phi^u B_1^u - B_1^s + \tau_I\left(\phi^u (B_1^u - B_1^s)\hat{w}^u_{\tau_I}\right)\right)} \; \hat{N}^s_{\tau_I} > (=) < \; \underset{(b)}{\left(\phi^s B_1^s + \phi^u B_1^u - B_1^u + \tau_I\left(\phi^s\left(B_1^s - B_1^u\right)\hat{w}^s_{\tau_I}\right)\right)} \; \hat{N}^u_{\tau_I}$$

(A.29)

Then if $B_1^u > B_1^s$, the unskilled workers spend greater share of their income on x_1 then term (a) of (A.29) is negative and term (b) is positive. Then as both $\hat{N}^s_{\tau_I}$ and $\hat{N}^u_{\tau_I}$ are negative, the produce of (a) and $\hat{N}^s_{\tau_I}$ is positive and the product of (b) and $\hat{N}^u_{\tau_I}$ is negative making $\left.\frac{d\tau_I}{d\tau_s}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{R=\overline{R}}$ positive and for it to be optimal to employ a sales tax.

$$\left.\frac{d\tau_I}{d\tau_s}\right|_{W=\overline{W}} = \frac{\phi^s B_1^s + \phi^u B_1^u + \tau_I \overline{w}_{\tau_s}}{1 + \tau_I \overline{w}_{\tau_I}} = \frac{\left[\; \phi^s B_1^s + \phi^u B_1^u + \tau_I \overline{w}_{\tau_s} + N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}^s_{\tau_s} - \hat{N}^u_{\tau_s}\right) \;\right]}{\left[\; \left(1 + \tau_I \overline{w}_{\tau_I}\right) + N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_s}\right) \;\right]} = \left.\frac{d\tau_I}{d\tau_s}\right|_{R=\overline{R}}. \quad \text{(A.30)}$$

To simplify comparisons, assume that $\eta_{us} = \eta_{su} = 0$ the $\overline{w}_{\tau_s} = \phi^s B_1^s \hat{w}^s_{\tau_I} + \phi^u B_1^u \hat{w}^u_{\tau_I}$ and $\hat{N}^i_{\tau_s} = B_1^i \hat{N}^i_{\tau_I}$ and evaluate at $\tau_s = 0$ we obtain the condition $\left.\frac{d\tau_I}{d\tau_s}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{R=\overline{R}} > (=) < 0 \; if$

$$\left(\phi^s B_1^s + \phi^u B_1^u + \tau_I \overline{w}_{\tau_s}\right)\left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right) > \left(1 + \tau_I \overline{w}_{\tau_I}\right)\left(\hat{N}^s_{\tau_s} - \hat{N}^u_{\tau_s}\right) \tag{A.31}$$

To simplify comparisons, assume that $\eta_{us} = \eta_{su} = 0$ the $\overline{w}_{\tau_s} = \phi^s B_1^s \hat{w}^s_{\tau_I} + \phi^u B_1^u \hat{w}^u_{\tau_I}$ and $\hat{N}^i_{\tau_s} = B_1^i \hat{N}^i_{\tau_I}$and evaluate at $\tau_s = 0$ we obtain the condition $\left.\frac{d\tau_I}{d\tau_s}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{R=\overline{R}} > (=) < 0$ if

$$\begin{array}{c} \left(\phi^s B_1^s + \phi^u B_1^u + \tau_I \left(\phi^s B_1^s \hat{w}^s_{\tau_I} + \phi^u B_1^u \hat{w}^u_{\tau_I}\right)\right)\left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right) > (=) < \\ \left(1 + \tau_I\right)\left(\phi^s \hat{w}^s_{\tau_I} + \phi^u \hat{w}^u_{\tau_I}\right)\left(B_1^s \hat{N}^s_{\tau_I} - B_1^u \hat{N}^u_{\tau_I}\right) \end{array}. \tag{A.32}$$

and $\left.\frac{d\tau_I}{d\tau_s}\right|_{W=\overline{W}} - \left.\frac{d\tau_I}{d\tau_s}\right|_{R=\overline{R}} > (=) < 0$ if

$$\begin{array}{c} \left(\phi^s B_1^s + \phi^u B_1^u - B_1^s + \tau_I \left(\phi^u (B_1^u - B_1^s) \hat{w}^u_{\tau_I}\right)\right) \hat{N}^s_{\tau_I} > (=) < \\ \left(\phi^s B_1^s + \phi^u B_1^u - B_1^u + \tau_I \left(\phi^s \left(B_1^s - B_1^u\right) \hat{w}^s_{\tau_I}\right)\right) \hat{N}^u_{\tau_I} \end{array}. \tag{A.33}$$

A.4.2 Uniform Taxation with Mixed Costs

As an extension consider fixed costs now entering the cost function for the public service with $c(N^s, N^u) = F + (N^s + N^u)\, g,\ F > 0$. With fixed costs present, the first order conditions with respect to τ_I and g can now be expressed as

$$\frac{\partial W}{\partial \tau_I} = \begin{array}{c} \left(\lambda\left(1 - \tau_I \frac{\theta}{(1-\tau_I)}\right) - 1\right) + \tau_I \left(\lambda\left(1+\theta\right) - 1\right) \overline{w}_{\tau_I} \\ +\lambda \left[\ N^s \left(\tau_I w^s l^s - g\right)\ \hat{N}^s_{\tau_I} +\ \ N^u \left(\tau_I w^u l^u - g\right)\ \hat{N}^u_{\tau_I}\right] \end{array} = 0 \tag{A.34a}$$

$$\frac{\partial W}{\partial g} = \begin{array}{c} \phi^s \frac{MRS^s}{w^s l^s} + \phi^u \frac{MRS^u}{w^u l^u} - \lambda + \tau_I \left(\lambda\left(1+\theta\right) - 1\right) \overline{w}_g \\ +\lambda \left[\ N^s \left(\tau_I w^s l^s - g\right)\ \hat{N}^s_g +\ \ N^u \left(\tau_I w^u l^u - g\right)\ \hat{N}^u_g\right] \end{array} = 0. \tag{A.34b}$$

With the fixed cost (F) it is no longer the case that $-N^s\left(\tau w^s l^s - g\right) = N^u\left(\tau w^u l^u - g\right)$ so that the first order conditions cannot be simplified to being a simple difference in the population changes, $\hat{N}^s_x - \hat{N}^s_x$. However, if wages are not equal for the two groups or there are differences in mobility between the two groups, the optimal tax and public service policy will be influence these differences.

Proposition 6. *b) If costs are given by* $C(N^s, N^u, g) = (N^s + N^u)\, g + F,\ F > 0$ *and*

$$\frac{-\left[\theta \tau_I^* \overline{w}_g + N^s\left(\tau_I^* w^s l^s - g\right)\hat{N}^s_g + + N^u\left(\tau_I^* w^u l^u - g\right)\hat{N}^u_g\right]}{1 - \tau_I^* \overline{w}_g} < (=) > \frac{\left[\theta \tau_I^* \left(\overline{w}_{\tau_I} - \frac{1}{(1-\tau_I)}\right) + N^s\left(\tau_I^* w^s l^s - g\right)\hat{N}^s_{\tau_I} + N^u\left(\tau_I^* w^u l^u - g\right)\hat{N}^u_{\tau_I}\right]}{1 + \tau_I^* \overline{w}_{\tau_I}} \tag{A.35}$$

then $g^R > (=) < g^*$.

B Voting Equilibria

My examination of the policy choices of regional governments has been predicated on the assumption that they are maximizing a social welfare function that is the sum of land rent and (monetized) utility of current residents. While it may be argued that governments may have incentives to choose policies reflecting these considerations (Henderson (1985)), there is no explicit relationship between social welfare maximization and the political determination of these policies. This being the case much of the recent literature on fiscal competition has focused on the determination of government policies using a simple characterization of the political process. Here my goal is to show that the basic insights obtained in the preceding analysis, that differential mobility responses between skilled and unskilled workers to different government policies, influences the mix of tax policies and level of public services, play a role in the determination of policies in a simple voting model.

To keep the analysis tractable and focused on the impacts of mobility on policy determination, I assume that labor is inelastically supplied by the individual ($\theta = 0$). Then, as it is assumed that there is a majority of unskilled workers in the region, the median voter is unskilled and policies are chosen to maximize her utility. However, the budget constraint is unchanged and still described by (2.2). I restrict the discussion here to consideration of the level of a single public service with uniform taxation and the use of a capital taxation and ignore other extensions considered in *Section 4*.

B.1 Public Service Provision with Uniform Income Taxation

As mentioned, I assumed that the policy chosen by the regional government maximizes the utility of current unskilled residents. In this case, with the assumption that $\theta = 0$ and that the there are no fixed costs of providing the public service, the first order condition with respect to the income tax rate can be expressed as

$$\phi^u \left(-1 + (1-\tau_I)\hat{w}^u_{\tau_I}\right) + \lambda \left(1 + \tau_I \overline{w}_{\tau_I} + N^s \left(\tau_I w^s l^s - g\right) \left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right)\right) = 0 \qquad \text{(B.1a)}$$

and with respect to the public service by

$$\phi^u \left(\frac{MRS^u}{w^u l^u} + (1-\tau_I)\hat{w}^u_g\right) + \lambda \left(-1 + \tau_I \overline{w}_g + N^s \left(\tau_I w^s l^s - g\right) \left(\hat{N}^s_g - \hat{N}^u_g\right)\right) = 0 \qquad \text{(B.1b)}$$

where, as before, the assumption that $N^s + N^u = N^s w^s l^s + N^u w^u l^u = 1$ is applied to simplify (B.1a) and (B.1b).

We can obtain the policy of the central government from (B.1a) and (B.1b) by using the fact that when $\theta = 0$ with a central government and, therefore, no labor mobility, $\hat{w}^u_{\tau_I} = \overline{w}_{\tau_I} = \hat{N}^s_{\tau_I} = \hat{w}^u_g = \overline{w}_g = \hat{N}^s_g = 0$. Then from (B.1a), $\lambda = \phi^u$ and using λ in (B.1b), the central government will provide the public service at the level at which $MRS^u = 1$. Thus while the unskilled only pay a share of the tax cost of the public service as the cost of providing the public service is based on the entire population they have no incentive to increase the public service beyond the point at which the marginal benefit is equal to their tax cost.

Then, analogous to the analysis of social welfare-maximizing policies, to consider how the public service regional governments might provide a different level of public service and what its determinants are, using (B.1a) and (B.1b), the marginal rate of substitutions between the income tax and the public service for both the utility of unskilled workers and in revenue can be obtained. Then when evaluating at the central government policies ($g = g^c$) if $\left.\frac{d\tau_I}{dg}\right|_{U^u = \overline{U}^u} > (=) < \left.\frac{d\tau_I}{dg}\right|_{R = \overline{R}}$ then $g^R > (=) < g^C$where g^R is the regional provision of the public service. Then $\left.\frac{d\tau_I}{dg}\right|_{U^u = \overline{U}^u} > (=) < \left.\frac{d\tau_I}{dg}\right|_{R = \overline{R}}$ if

$$\frac{\tau_I \phi^s \hat{w}^s_g + N^s(\tau_I w^s l^s - g^c)\left(\hat{N}^s_g - \hat{N}^u_g\right)}{1+(1-\tau_I)\hat{w}^u_g} > (=) < \frac{-\left[\tau_I \phi^s \hat{w}^s_{\tau_I} + N^s(\tau_I w^s l^s - g)\left(\hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}\right)\right]}{1-(1-\tau_I)\hat{w}^u_{\tau_I}}. \tag{B.2}$$

The interpretation of (B.2) differs slightly from the analogous condition with welfare maximization, (4.3a). The numerator of each term can be considered the additional revenue from the impacts of the respective policy on mobility of the two groups that does not directly affect the utility of the unskilled, hence $\tau_I \phi^s \hat{w}^s_g$ and $\tau_I \phi^s \hat{w}^s_{\tau_I}$ appear but not $\tau_I \phi^u \hat{w}^u_g$ and $\tau_I \phi^u \hat{w}^u_{\tau_I}$. The denominator is the impact of the policy on the utility of the residents where, at $g^c$with $MRS^u = 1$. Then from (B.2) there is a trade off that occurs in increasing the public service or the tax rate – if the skilled workers have a stronger demand for the public service than unskilled workers at g^c then a balanced-budget increase in the public service will increase net revenue by increasing the percentage of residents who are skilled. However, the decrease in the wage for skilled workers will decrease in this case, reducing revenue. If the case in which the elasticity of substitution between the two types of labor is zero ($\eta_{su} = 0$) the condition (B.2) is equivalent to the condition

$$\left(\frac{MRS^s}{w^s l^s} - 1\right)\left(1 + \gamma^s\left(1 - \frac{g^c}{w^s l^s}\right)\left(-\hat{w}^s_{\tau_I}\right) + \frac{\left(1 - \frac{g^c}{w^s l^s}\right)}{(1-\tau_I)}\right). \tag{B.3}$$

As the second term is positive the sign of (B.3) is the sign of $\frac{MRS^s}{w^s l^s} - 1$ and if the preferred level of the public service for the skilled workers exceeds g^c then it will be optimal for the unskilled voters to favor a higher level of public service than g^c; if the skilled workers prefer a lower value of public service, the unskilled workers favor a level below g^c.

B.2 Capital and Income Taxation with Voting

Critical to the determination of whether capital should be taxed or subsidized was the changes in the mix of skilled and unskilled workers from a change in the capital tax relative to the change from a change in the income tax with capital taxed if increases in it lead to smaller reductions in skilled workers relative to unskilled workers than did increases in the income tax. For the voting equilibria, similar results are obtained. As my interest is only in determining whether capital should be taxed or subsidized, consider the impact on unskilled utility of instituting a capital tax. This can be expressed by

$$\phi^u(1-\tau_I)\hat{w}^u_{\tau_k} + \lambda\left(K + \tau_I \overline{w}_{\tau_k} + N^s\left(\tau_I w^s l^s - g\right)\left(\hat{N}^s_{\tau_k} - \hat{N}^u_{\tau_k}\right)\right) \tag{B.4}$$

Then using (B.4) and (B.1a) if at $\tau_k = 0$ $\left.\frac{d\tau_I}{d\tau_k}\right|_{U^u = \overline{U}^u} < (=) > \left.\frac{d\tau_I}{d\tau_k}\right|_{R = \overline{R}}$ then

$$\frac{K+\tau_I\phi^s\hat{w}^s_{\tau_k}+N^s(\tau_I w^s l^s-g)\left(\hat{N}^s_{\tau_k}-\hat{N}^u_{\tau_k}\right)}{(1-\tau_I)(-\hat{w}^u_{\tau_k})} > (=) < \frac{1+\tau_I\phi^s\hat{w}^s_{\tau_k}+N^s(\tau_I w^s l^s-g)\left(\hat{N}^s_{\tau_I}-\hat{N}^u_{\tau_I}\right)}{1-(1-\tau_I)\hat{w}^u_{\tau_I}} \; then \; \tau_k > (=) < 0. \tag{B.5}$$

Assuming that capital and unskilled labor are complements, then from (B.5) capital will be taxed if the marginal tax revenue from it relative to the associated utility loss for unskilled workers exceeds the marginal tax revenue relative to utility-loss from an increase in the income tax. As (B.5) the difference in marginal tax revenue between the two taxes depends on the impact the two taxes have on wage rate of skilled workers as that affects tax revenue and the impacts on changes in the two taxes have on the change in the population and through them, fiscal surpluses and deficits. Then, as outlined in detail in *Section 4.2)* whether $\hat{N}^s_{\tau_k} - \hat{N}^u_{\tau_k} >< \hat{N}^s_{\tau_I} - \hat{N}^u_{\tau_I}$ depends on the relative cross-price elasticities with capital for skilled and unskilled labor; the strong the substitute skilled labor is for capital the more likely it will be optimal to tax capital.

Table 1: State and Local Revenue Sources, U.S. Total 2009

	State and Local		Local		State	
Total	1,271,355	%	555,859	%	715,496	%
Individual Income	270,355	21.3	24,636	4.4	245,880	34.4
Corporate Net Income	45,979	3.6	6,702	1.2	39,277	5.5
Property	424,014	33.4	411,049	73.9	12,964	1.8
General Sales	291,045	22.9	62,316	11.2	228,728	32.0
Select Sales	142,510	11.2	26,671	4.8	115,839	16.2
Motor Vehicle	21,296	1.7	1,669	0.3	19,626	2.7
All Other	130,162	0.6	22,812	4.1	53,179	7.4

Amounts are in $1,000,000.
Source: Quarterly Summary of State and Local Government Tax Revenue

Figure 1: Public Service Provision with Uniform Income Tax

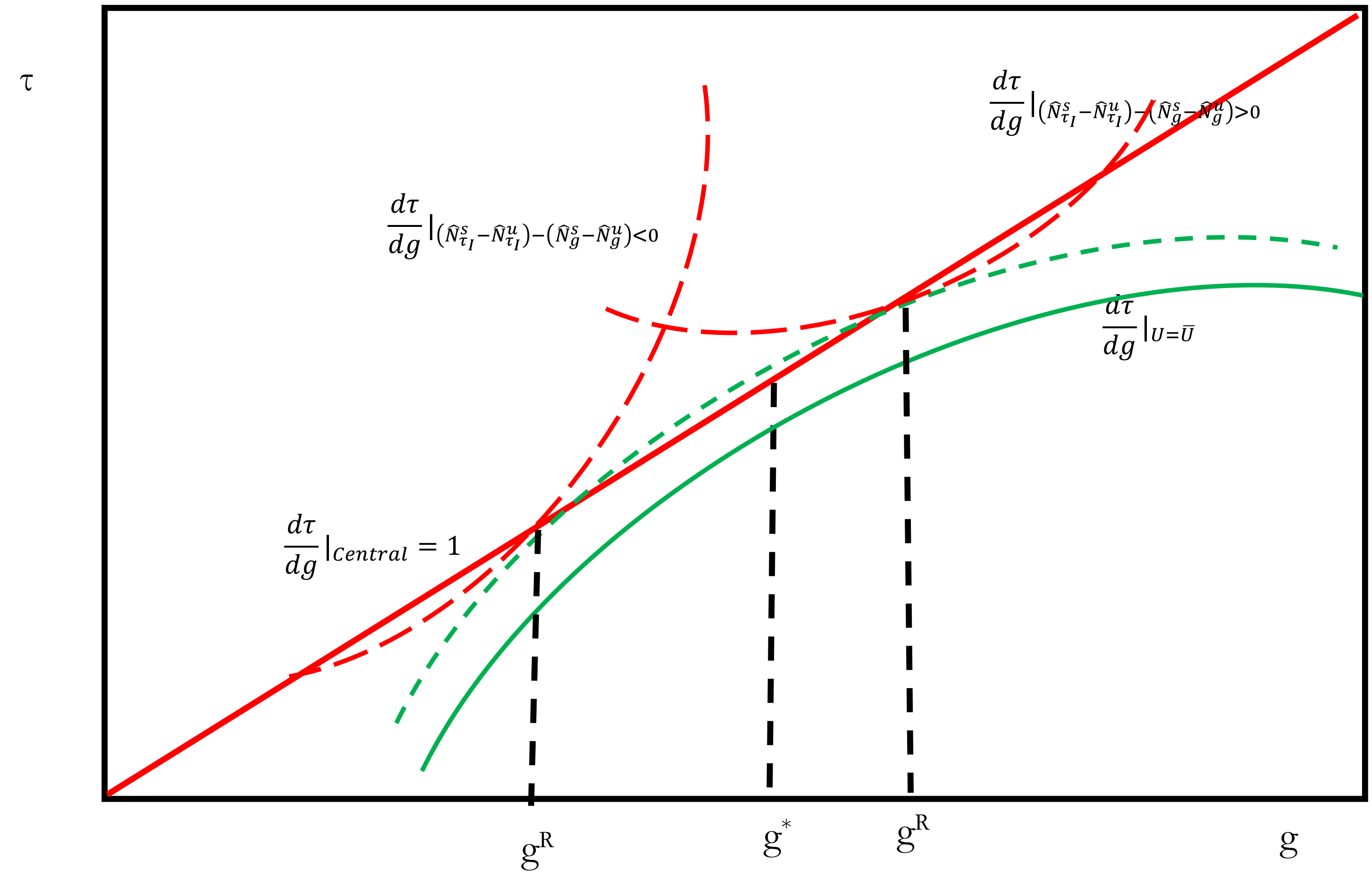

Figure 2: Capital Taxation or Subsidy with Income Tax

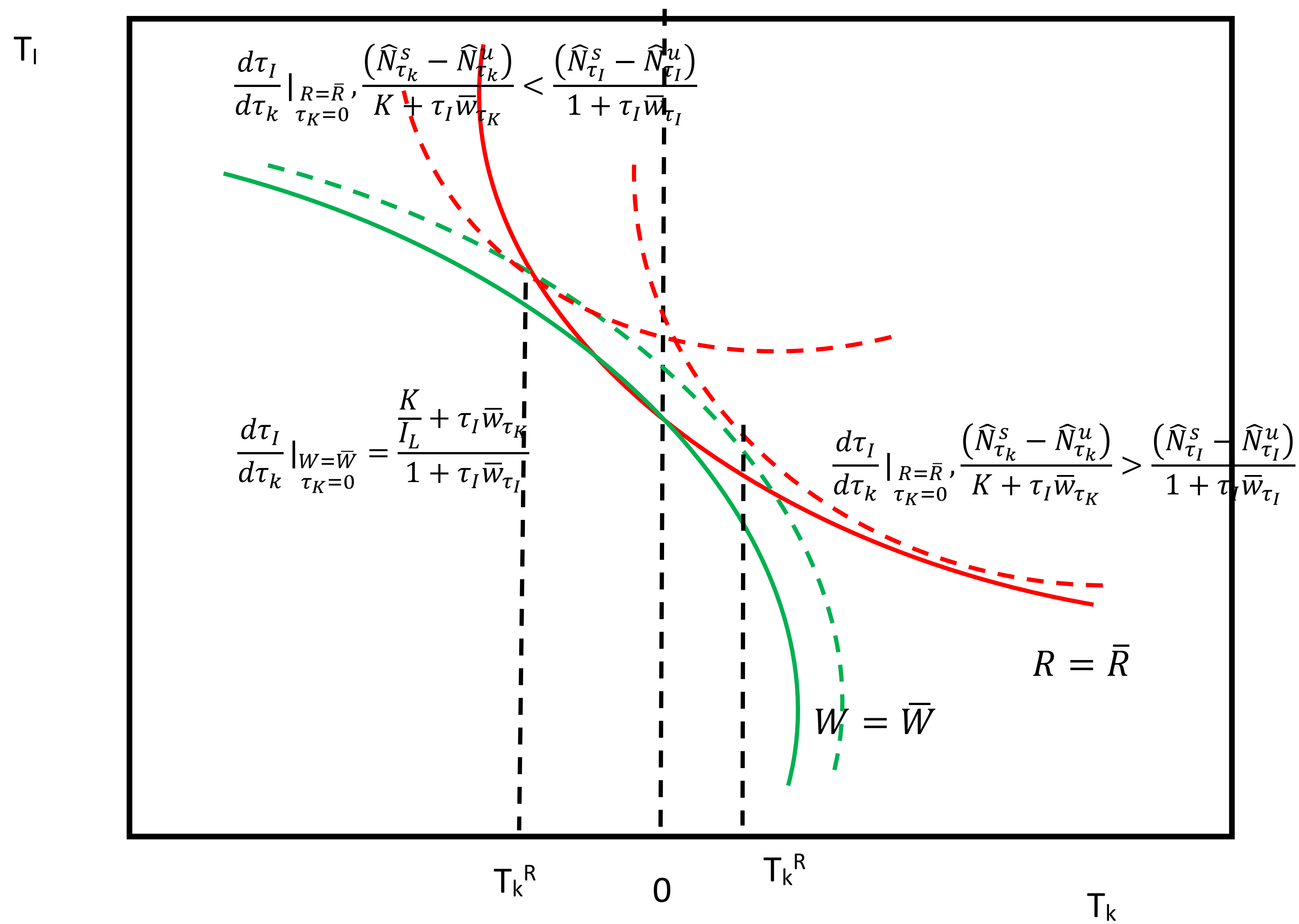

Table 2: U.S. State Income Tax Rates, 2017

State	Marginal Tax Rate		Brackets		
	Low	High	Number	Lowest	Highest
ALABAMA	2.0	5.0	3	500	3,001
ARIZONA	2.59	4.54	5	10,179	152,668
ARKANSAS	0.9	6.9	6	4,299	35,100
CALIFORNIA	1.0	12.3	9	8,015	537,498
COLORADO	4.63		1		
CONNECTICUT	3.0	6.99	7	10,000	500,000
DELAWARE	0.0	6.6	7	2,000	60,001
GEORGIA	1.0	6.0	6	750	7,001
HAWAII	1.4	8.25	9	2,400	48,000
IDAHO	1.6	7.4	7	1,454	10,905
ILLINOIS	3.75		1		
INDIANA	3.23		1		
IOWA	0.36	8.98	9	1,573	70,785
KANSAS	2.7	4.6	2		
KENTUCKY	2.0	6.0	6	3,000	75,001
LOUISIANA	2.0	6.0	3	12,500	50,001
MAINE	5.8	10.15	4	21,100	200,000
MARYLAND	2.0	5.75	8	1,000	250,000
MASSACHUSETTS	5.10		1		
MICHIGAN	4.25		1		
MINNESOTA	5.35	9.85	4	25,390	156,911
MISSISSIPPI	3.0	5.0	3	5,000	10,001
MISSOURI	1.5	6.0	10	1,000	9,001
MONTANA	1.0	6.9	7	2,900	17,600
NEBRASKA	2.46	6.84	4	3,090	29,830
NEW JERSEY	1.4	8.97	6	20,000	500,000
NEW MEXICO	1.7	4.9	4	5,500	16,001
NEW YORK	4.0	8.82	8	8,500	1,077,550
NORTH CAROLINA	5.499		1		
NORTH DAKOTA	1.10	2.90	5	37,950	416,700
OHIO	0.495	4.997	9	5,250	210,600
OKLAHOMA	0.5	5.0	6	1,000	7,200
OREGON	5.0	9.9	4	3,400	125,000
PENNSYLVANIA	3.07		1		
RHODE ISLAND	3.75	5.99	3	61,300	139,400
SOUTH CAROLINA	0.0	7.0	6	2,930	14,650
UTAH	5.0		1		
VERMONT	3.55	8.95	5	37,950	416,700
VIRGINIA	2.0	5.75	4	3,000	17,001
WEST VIRGINIA	3.0	6.5	5	10,000	60,000
WISCONSIN	4.0	7.65	4	11,230	247,350
From the *Federation of Tax Administrators*					

Table 3: U.S. One-Year Mobility Rates, 2015 to 2016, By Age, Educational Attainment, Nativity, Tenure, and Poverty Status

United States	Non-mover	Same county	Different county, same state	Different State or Abroad	Abroad
TOTAL 1+ years	88.8	6.9	2.4	1.9	0.4
20 to 24 years	77.0	14.3	5.1	3.6	0.8
25 to 29 years	76.0	14.4	5.1	4.5	1.1
30 to 34 years	84.0	9.4	3.3	3.2	0.8
35 to 39 years	87.3	7.9	2.5	2.2	0.5
40 to 44 years	90.4	5.7	2.1	1.8	0.4
45 to 49 years	91.6	5.2	1.8	1.5	0.3
50 to 54 years	93.2	3.9	1.6	1.3	0.2
55 to 59 years	94.3	3.1	1.5	1.0	0.1
EDUCATIONAL ATTAINMENT					
Not a high school graduate	90.6	6.5	1.6	1.2	0.3
High school graduate	91.2	5.5	2.1	1.2	0.2
Some college or AA degree	90.6	5.8	2.0	1.5	0.2
Bachelor's degree	89.8	5.5	2.3	2.3	0.6
Prof. or graduate degree	90.3	4.9	2.1	2.7	0.7
NATIVITY					
Native	89.1	6.8	2.4	1.7	0.1
Foreign born	87.5	7.0	2.0	3.5	2.1
Not a U.S. citizen	83.2	9.1	2.2	5.5	3.6
TENURE					
.In an owner-occupied housing unit	95.0	3.0	1.3	0.7	0.1
.In a renter-occupied housing unit	77.1	14.1	4.5	4.2	1.0
POVERTY STATUS					
.Below 100% of poverty	81.4	12.1	3.4	3.1	1.1
100% to 149% of poverty	86.2	8.9	2.8	2.1	0.4
150% of poverty and above	90.4	5.7	2.1	1.7	0.3
From U.S. Census Geographical Mobility: 2015 to 2016					

Table 4: Income Taxation

Tastes for Public Service															
	(a)			(b)			(c)			(d)			(e)_		
a^s, a^u	0.2, 0.2			0.4,0			0,0.4			0.3, 0.1			0.25, 0.15		
Mobility															
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
γ^s	3	5.5	0.5	3	5.5	0.5	3	5.5	0.5	3	5.5	0.5	3	5.5	0.5
γ^u	3	0.5	5.5	3	0.5	5.5	3	0.5	5.5	3	0.5	5.5	3	0.5	5.5
Decentralized Policies															
τ	0.131	0.132	0.131	0.137	0.136	0.137	0.126	0.128	0.127	0.1343	0.1340	0.1342	0.1329	0.1330	0.1329
g	0.1972	0.1981	0.1975	0.2060	0.2039	0.2053	0.1892	0.1927	0.1903	0.2015	0.2010	0.2013	0.1993	0.1995	0.1994
λ	1	1.0119	0.9881	1.0000	1.0123	0.9876	1.0000	1.0116	0.9886	1.0000	1.0121	0.9879	1.0000	1.0120	0.9880
Decentralized Polices relative to Centralized Policies															
Relative Utility, Skilled $\left(\frac{U^{S^R}}{U^{S^C}}\right)$	1.0007	1.0004	1.0006	1.0035	1.0023	1.0031	1.0083	1.0056	1.0075	1.0002	1.0001	1.0002	1.00004	1.00003	1.00004
Relative Utility, Unskilled $\left(\frac{U^{u^R}}{U^{u^C}}\right)$	0.9982	0.9988	0.9984	0.9954	0.9970	0.9959	0.9325	0.9550	0.9398	0.9997	0.9998	0.9997	0.99990	0.99994	0.99991
Relative Welfare $\left(\frac{W^R}{W^C}\right)$	1.0000	1.0000	1.0000	0.9999	1.0000	0.9999	0.9998	0.9999	0.9998	1.0000	1.0000	1.0000	1.00000	1.00000	1.00000
Gradients															
$\hat{N}^S_{\tau_I}/\hat{N}^u_{\tau_I}$	1.000	2.538	0.394	1.000	2.538	0.394	1.000	2.538	0.394	1.000	2.538	0.394	1.000	2.539	0.394
$\hat{N}^S_g/\hat{N}^u_g$	0.5	1.2692	0.1970	---	---	---	0.0000	0.0000	0.0000	1.5000	2.8077	0.5909	0.833	2.115	0.328

Table 5: Capital and Income Taxation with Identical Preferences for the Public Service

a^s, a^u	0.2, 0.2											
	(a)			(b)			(c)			(d)		
η_{sk}, η_{uk}	0.5, 0			-0.5, 0			0, 0.5			0, -0.5		
γ^s, γ^u	3, 3	5.5, 0.5	0.5, 5.5	3, 3	5.5, 0.5	0.5, 5.5	3, 3	5.5, 0.5	0.5, 5.5	3, 3	5.5, 0.5	0.5, 5.5
Capital Tax Rate (τ_k)	0.0145	0.0311	-0.0132	-0.0684	-0.0255	-0.0842	-0.0171	0.0116	-0.0430	0.0381	0.0458	0.0143
λ	0.9889	0.9885	0.9944	1.0182	1.0166	1.0090	1.0135	1.0051	1.0174	0.9903	0.9958	0.9873
g^{LK}	0.1998	0.2031	0.1957	0.1919	0.1972	0.1904	0.1939	0.1995	0.1900	0.1999	0.2014	0.1980
Relative Income Tax Rate (τ_I^{LK}/τ_I^{L})	0.9399	0.8680	1.0582	1.3205	1.1238	1.3906	1.0705	0.9486	1.1796	0.8203	0.7854	0.9306
Relative Public Service, (g^{LK}/g^{L})	1.0136	1.0251	0.9912	0.9734	0.9953	0.9640	0.9837	1.0072	0.9620	1.0137	1.0167	1.0029
Relative Utility, Skilled $\left(U^{s^{LK}}/U^{s^{L}}\right)$	1.0131	1.0282	0.9879	0.9365	0.9762	0.9220	0.9846	1.0106	0.9610	1.0354	1.0425	1.0133
Relative Utility, Unskilled $\left(U^{u^{LK}}/U^{u^{L}}\right)$	1.0195	1.0412	0.9826	0.9126	0.9682	0.8920	0.9769	1.0151	0.9423	1.0485	1.0582	1.0178
Relative Welfare, (W^{LK}/W^{L})	1.00001	0.99999	0.99998	0.99993	0.99999	0.99989	0.99996	1.00000	0.99988	1.00001	1.00000	1.00000
$\hat{N}^S_{\tau_I}/\hat{N}^u_{\tau_I}$	0.3696	0.4103	0.1689	-0.4025	-0.4341	-0.1820	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{N}^S_g/\hat{N}^u_g$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3815	0.1648	0.4421	-0.3612	-0.1594	-0.4171

Table 6: Income and Capital Taxation, Demand for Public Service only by Skilled Workers

a^s, a^u	0.4, 0											
	(a)			(b)			(c)			(d)		
η_{sk}, η_{uk}	0.5, 0			-0.5, 0			0, 0.5			0, -0.5		
γ^s, γ^u	3, 3	5.5, 0.5	0.5, 5.5	3, 3	5.5, 0.5	0.5, 5.5	3, 3	5.5, 0.5	0.5, 5.5	3, 3	5.5, 0.5	0.5, 5.5
Capital Tax Rate (τ_k)	0.0151	0.0319	-0.0139	-0.0731	-0.0262	-0.0896	-0.0179	0.0120	-0.0450	0.0394	0.0468	0.0145
λ	0.9884	0.9882	0.9942	1.0195	1.0171	1.0097	1.0141	1.0052	1.0183	0.9900	0.9957	0.9869
g^{LK}	0.2081	0.2083	0.2041	0.2040	0.2038	0.2020	0.2033	0.2051	0.1994	0.2069	0.2060	0.2052
Relative Income Tax Rate $\left(\tau_I^{LK}/\tau_I^L\right)$	0.9372	0.8651	1.0616	1.3455	1.1279	1.4201	1.0742	0.9471	1.1909	0.8132	0.7804	0.9290
Relative Public Service, $\left(g^{LK}/g^L\right)$	1.0105	1.0216	0.9941	0.9905	0.9994	0.9838	0.9872	1.0058	0.9715	1.0047	1.0101	0.9997
Relative Utility, Skilled $\left(U^{s^{LK}}/U^{s^L}\right)$	1.0196	1.0414	0.9824	0.9097	0.9679	0.8888	0.9767	1.0153	0.9416	1.0486	1.0584	1.0177
Relative Utility, Unskilled $\left(U^{u^{LK}}/U^{u^L}\right)$	1.0100	1.0212	0.9902	0.9450	0.9799	0.9334	0.9882	1.0083	0.9697	1.0297	1.0346	1.0113
Relative Welfare, $\left(W^{LK}/W^L\right)$	0.99996	0.99993	1.00001	1.00002	1.00000	1.00003	1.00003	0.99999	1.00003	0.99998	0.99997	1.00000
$\hat{N}^S_{\tau_I}/\hat{N}^u_{\tau_I}$	0.3694	0.4100	0.1690	-0.4046	-0.4345	-0.1831	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{N}^S_g/\hat{N}^u_g$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3818	0.1647	0.4430	-0.3608	-0.1592	-0.4170

Table 7: Income and Capital Taxation, Demand for Public Service only by Unskilled Worker

a^s, a^u	0, 0.4											
	0.5, 0			-0.5, 0			0, 0.5			0, -0.5		
η_{sk}, η_{uk}	3, 3			5.5, 0.5			0.5, 5.5			3, 3		
γ^s, γ^u	3, 3	5.5, 0.5	0.5, 5.5	3, 3	5.5, 0.5	0.5, 5.5	3, 3	5.5, 0.5	0.5, 5.5	3, 3	5.5, 0.5	0.5, 5.5
Capital Tax Rate (τ_k)	0.0140	0.0304	-0.0127	-0.0644	-0.0247	-0.0796	-0.0164	0.0113	-0.0411	0.0370	0.0449	0.0140
λ	0.9893	0.9887	0.9946	1.0171	1.0160	1.0084	1.0129	1.0050	1.0165	0.9906	0.9959	0.9878
g^{LK}	0.1923	0.1982	0.1881	0.1814	0.1911	0.1803	0.1855	0.1943	0.1815	0.1933	0.1971	0.1914
Relative Income Tax Rate (τ_I^{LK}/τ_I^L)		0.8707	1.0553	1.2996	1.1200	1.3659	1.0672	0.9499	1.1699	0.8266	0.7901	0.9322
Relative Public Service, (g^{LK}/g^L)	1.0164	1.0285	0.9886	0.9591	0.9916	0.9474	0.9805	1.0086	0.9538	1.0220	1.0229	1.0058
Relative Utility, Skilled $\left(U^{s^{LK}}/U^{s^L}\right)$	1.0083	1.0191	0.9920	0.9568	0.9823	0.9468	0.9903	1.0074	0.9753	1.0250	1.0309	1.0099
Relative Utility, Unskilled $\left(U^{u^{LK}}/U^{u^L}\right)$	1.0662	1.1308	0.9447	0.7379	0.9118	0.6752	0.9214	1.0463	0.8068	1.1470	1.1692	1.0522
Relative Welfare, (W^{LK}/W^L)	1.00008	1.00007	0.99994	0.99969	0.99997	0.99960	0.99987	1.00003	0.99966	1.00010	1.00006	1.00003
$\hat{N}^S_{\tau_I}/\hat{N}^u_{\tau_I}$	0.3698	0.4106	0.1688	-0.4008	-0.4338	-0.1811	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{N}^S_g/\hat{N}^u_g$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3812	0.1648	0.4412	-0.3616	-0.1595	-0.4172