Quizzes 1-6 and Final Exam questions/answer keys for ECO 479, FALL 2001

Economics 479
Professor Yelowitz
Answer Key - Quiz 1

This quiz is worth 15 points. You have 50 minutes.

1. (4 points) Consider a consumer who has preferences over food and clothing, represented by the utility function $u = F^{\frac{1}{2}}C^{\frac{1}{2}}$, where F and C represent the quantity of food and clothing respectively. She faces prices P_F and P_C for food and clothing and has income equal to I.

Write down the demand curve for food, expressed in terms of prices and income.

By setting the MRS equal to the price ratio, we get:

$\frac{MU_F}{MU_C} = \frac{\frac{1}{2}F^{-\frac{1}{2}}C^{\frac{1}{2}}}{\frac{1}{2}F^{\frac{1}{2}}C^{-\frac{1}{2}}} = \frac{C}{F} = \frac{P_F}{P_C}$ **which simplifies to** $P_F F = P_C C$**. If we substitute this into the budget constraint,** $P_F F + P_C C = I$ **, we get** $P_F F + P_F F = I$ **, or** $F = \frac{1}{2}\frac{I}{P_F}$**.**

4 points awarded for the demand curve at the end; showing the steps is not required.

2. (4 points) For the utility function given above, suppose that $P_F = \$5$, $P_C = \$3$, and $I = \$500$. What quantity of food would this consumer demand?

Substituting into the demand curve above, we get $F = \frac{1}{2}\frac{I}{P_F} = \frac{1}{2}\frac{500}{5} = 50.$

4 points awarded for the correct answer of 50.

3. (4 points) Suppose, instead that the utility function was $u = \min\left\{\frac{1}{3}F, 3C\right\}$, and that $P_F = \$5$, $P_C = \$3$, and $I = \$500$. What quantity of food would this consumer demand?

To maximize utility, the consumer must consume these goods in fixed proportions: $\frac{1}{3}F = 3C$, which can be rewritten as $\frac{1}{9}F = C$. If we substitute this into the budget constraint, $5F + 3C = 500$, we get $5F + \frac{1}{3}F = 500$, or $\frac{16}{3}F = 500$, or $F = \frac{1500}{16} = 93.75$.

4 points awarded for the correct answer of 93.75 (or the fraction $\frac{1500}{16}$).

4. (3 points) Consider an exchange economy (e.g., the edgeworth box model we reviewed in class). Evaluate the following statement: "If both people in the exchange economy start off with identical endowments, then there is no possibility that they will trade with each other."

The statement is false - even with identical endowments, the two people will trade if their marginal rates of substitution are not equal to each other. Put differently, even for an endowment at the center of the edgeworth box, a "lens-shaped area" could exist that gives each consumer higher utility.

3 points for *explaining* why there might be a possibility of trade. No credit simply for saying true or false.

Economics 479
Professor Yelowitz
Quiz 2 Solution Key

This quiz is worth 15 points. You have 50 minutes.

Assume that fireworks are a public good for parts 1. and 2. Allison, Billy, and Carlos have the following individual demand curves for fireworks. $P_A = 200 - Q_A$, $P_B = 100 - Q_B$, $P_C = 100 - Q_C$, where Q_A, Q_B, and Q_C represent the amount of fireworks consumed by Allison, Billy, and Carlos respectively. The marginal cost of producing another unit of fireworks is given by: $MC = 200 + Q$.

1. (4 points) Calculate the socially optimal quantity of fireworks.

Adding these curves up, the aggregate demand curve is
$P = P_A + P_B + P_C = 200 - Q + 100 - Q + 100 - Q = 400 - 3Q$ if $Q \leq 100$.
$P = P_A = 200 - Q$ if $Q > 100$.
Setting $P = MC$ along the first segment gives
$400 - 3Q = 200 + Q \Rightarrow 200 = 4Q \Rightarrow Q = 50$, which is in the relevant range.
4 points for getting correct quantity of 50; partial credit of 2 points (1 point each) for getting both segments of demand curve.

2. (4 points) If Billy and Carlos did not contribute at all for the fireworks, and Allison provided her privately optimal quantity, what quantity would Allison provide and what would be the deadweight loss to society?

Since Allison's demand curve is everywhere below the marginal cost curve, Allison would provide zero. The deadweight loss uses the familiar triangle formula: $\frac{1}{2} base * height$, where the base is equal to 50 (that is, $Q_{SOCIAL} - Q_A$) and the height is equal to 200 (that is, the $\left(MB_{SOCIAL} - MC_{SOCIAL}\right)\Big|_{Q_A}$). Thus, the DWL is equal to $\frac{1}{2} 50 * 200 = 5000$.
4 points for the correct answer, zero partial credit.

3. (3 points) Why will the fundamental welfare theorem be violated in the case of public goods? Why might government intervention lead to a more efficient outcome?

The fundamental welfare might be violated because consumers can free-ride – that is, they can enjoy the benefits of a public good without potentially contributing for it. Thus, the public good might be underprovided (as in example one, above). The government can coerce people to pay for the public good through taxation - an ability that the private market does not have.

2 points for clearly discussing the free-rider problem, 1 point for discussing coercion.

4. (2 points) Evaluate the following statement: "Since the marginal cost curve presented at the beginning of the question, MC=200+Q, is always greater than 0 (for Q>0), then fireworks cannot possibly be a public good."

Public goods are non-rival – meaning that the marginal cost of another person is zero. Even with public goods, however, the marginal cost of another unit is always positive. Thus, the marginal cost curve is perfectly consistent with the good being a public good.

2 points total - 1 point for discussing the MC of another person consuming the good, 1 point for discussing the MC of another unit.

5. (2 points) Evaluate the following statement: "Public libraries are considered public goods, and will likely be underprovided by the private market."

Public libraries (or the services they provide) are considered rival and excludable, so they would not be considered public goods. Libraries are rival since books can only be checked out to one person at a time. Libraries are excludable since it is easy to decide whether to let people in or not, and whom to let check out books. Simply having the word "public" in front of a word does not make the good in question a public good.

2 points total - 1 point for discussing why libraries are rival, 1 point for discussing why they are excludable.

Economics 479
Professor Yelowitz
Quiz 3 Solution Key

This quiz is worth 15 points. You have 50 minutes. To receive full credit, you must give the correct numerical answer. Correct graphical answers alone will result in partial credit only.

Amanda bakes blueberry pies which she sells in a perfectly competitive market facing an inverse demand curve: $P = 40 - Q$. Her marginal costs of producing pies are given by: $MC = 4Q$. Amanda has two next door neighbors. Her neighbor, Kyle, likes the smell of the pies. His marginal external benefit is given by: $MEB = 50 - \frac{5}{4}Q$. Her other neighbor, Jane, does not like the smell of the pies. Jane's marginal external cost is given by: $MEC = 2Q$.

a. (3 points) What quantity of pies would Amanda produce on her own, what quantity would Kyle prefer the most, what quantity would Jane prefer the most?

Amanda: sets $P = MC \Rightarrow 40 - Q = 4Q \Rightarrow Q_{PRIVATE} = 8$

Kyle: sets $MEB = 0 \Rightarrow 50 - \frac{5}{4}Q = 0 \Rightarrow Q = 40$.

Jane sets $MEC = 0 \Rightarrow 2Q = 0 \Rightarrow Q = 0$.

3 points total - 1 point for each correct numerical answer, no partial credit.

b. (2 points) Calculate the deadweight loss from the private market providing pies.

To compute DWL, we need to know the socially optimal quantity.

$$Q_{SOCIAL} \Rightarrow MSB = MSC \Rightarrow MB + MEB = MC + MEC$$

$$\Rightarrow (40 - Q) + \left(50 - \frac{5}{4}Q\right) = (4Q) + (2Q) \Rightarrow Q_{SOCIAL} \approx 10.91$$

Deadweight loss is computed as:

$$DWL = \frac{1}{2}\left(Q_{SOCIAL} - Q_{PRIVATE}\right)\left(MSB\Big|_{Q_{PRIVATE}} - MSC\Big|_{Q_{PRIVATE}}\right)$$

$$= \frac{1}{2}(10.91 - 8)(72 - 48) \approx 34.92$$

2 points for correct numerical answer, 1 point partial credit for correct graphical presentation.

c. (2 points) What per-unit tax or subsidy on Amanda's production would lead to the socially optimal level?

Since Amanda privately produces less than the socially optimal amount, we need a subsidy. Amanda now faces the following situation: $P = MC - subsidy \Rightarrow 40 - Q = 4Q - subsidy$. **We evaluate this expression at Q_{SOCIAL}, which gives**

$40 - 10.91 = 4(10.91) - subsidy \Rightarrow subsidy = \$14.55$.

2 points for correct numerical answer, 1 point partial credit for correctly setting up the equation.

d. (2 points) What is the maximum amount of money that Kyle would be willing to pay for Amanda to move from $Q_{PRIVATE}$ to Q_{SOCIAL}? What is the maximum amount of money that Jane would be willing to pay for Amanda to move from $Q_{PRIVATE}$ to Q_{SOCIAL}?

Since Kyle enjoys the production, he gains by moving from $Q_{PRIVATE}$ to Q_{SOCIAL} by an amount under his marginal external benefit curve. Using the trapezoid formula, his maximum bribe is equal to

$$\frac{\left(MEB\big|_{Q_{SOCIAL}} + MEB\big|_{Q_{PRIVATE}}\right)}{2}\left(Q_{SOCIAL} - Q_{PRIVATE}\right) = \frac{(36.3625 + 40)}{2}(2.91) \approx 111.11$$

Since Jane does not enjoy production, she would not want Amanda to increase production. Hence, she would pay zero to move from $Q_{PRIVATE}$ to Q_{SOCIAL}.

2 points for correct numerical answer - 1 point for each part.

e. (2 points) What is the loss in surplus to Amanda and her customers by moving from $Q_{PRIVATE}$ to Q_{SOCIAL}?

Amanda's loss in surplus by producing Q_{SOCIAL} is the area between the MC and MB curves – that is,

$$\frac{1}{2}\left(Q_{SOCIAL} - Q_{PRIVATE}\right)\left(MC\big|_{Q_{SOCIAL}} - MB\big|_{Q_{SOCIAL}}\right)$$

$$= \frac{1}{2}(10.91 - 8)\left(4(10.91) - (40 - 10.91)\right) \approx 21.17$$

2 points for correct numerical answer, 1 point partial credit for correctly setting up the equation.

f. (2 points) What per-unit tax or subsidy on Amanda's production would a regulator choose in order to get Q=6?

Since Amanda would produces more than six units on her own, a tax is necessary to reduce her production. Thus, Amanda faces: $P = MC + \tau \Rightarrow 40 - Q = 4Q + \tau$. **We evaluate this expression at Q=6, which gives:** $40 - 6 = 4(6) + \tau$, **or** $\tau = 10$.

2 points for correct numerical answer, 1 point partial credit for correctly setting up the equation.

g. (2 points) If Amanda were a monopolist in the situation described previously, would her optimal quantity produced result in less deadweight loss than if she were in a perfectly competitive industry?

If Amanda were a monopolist, her total revenue curve would be
$TR = PQ = (40 - Q)Q = 40Q - Q^2$ **, and her marginal revenue would be:**
$MR = \frac{dTR}{dQ} = 40 - 2Q$**. Setting** $MR = MC \Rightarrow 40 - 2Q = 4Q \Rightarrow Q_M \approx 6.67$**. The DWL at the monopolist quantity is therefore** $\frac{1}{2}(10.91 - 6.67)(75 - 40) = \$74.20$**. This DWL exceeds the DWL from perfect competition.**
2 points for correct numerical answer, 1 point partial credit for correctly setting up the equation.

Economics 479
Professor Yelowitz
Quiz 4 Solution Key
Chandler has preferences over food and coffee given by the utility function
$U(F,C) = F^{\frac{1}{3}}C^{\frac{2}{3}}$. The price of food is $5, while the price of coffee is $4. His income is originally $200. The government now gives Chandler a transfer, and is debating two different transfer schemes:
Transfer #1: $100 in cash
Transfer #2: 12 units of food and 10 units of coffee.

Chandler still keeps his original $200, in addition to the transfer.

a. On separate graphs, clearly illustrate Chandler's budget constraint before the transfer, and how each of the transfers changes the budget constraint. Make sure to label all relevant kink points and intercepts.

Original budget constraint correctly (and labeling), the new budget constraint with cash, the budget constraint with in-kind benefits.

<SEE FIGURE 1>

1 point for correctly drawing and labeling the original budget constraint, 1 point for drawing and labeling Transfer #1, and 1 point for drawing and labeling Transfer #2.

b. In this particular example, is Chandler happier with Transfer #1 or Transfer #2?

Under a cash grant, his favorite bundle is
$F = \frac{1}{3}\left(\frac{I}{P_F}\right) = \frac{1}{3}\left(\frac{300}{5}\right) = 20, C = \frac{2}{3}\left(\frac{I}{P_C}\right) = \frac{2}{3}\left(\frac{300}{4}\right) = 50$. **Since he can still choose this bundle with the in-kind grant, his utility is equal under both transfers.**

3 points for demonstrating that Chandler is equally happy – must compute (F,C) with income of $300, and then show that the same bundle is available under the in-kind benefit.

Amanda has a time endowment of 100 hours per month. When she works, she receives a wage of \$5 per hour. She has preferences over leisure and consumption that are represented by the following utility function: $U(L,C) = L^{\frac{1}{3}}C^{\frac{1}{3}}$. The price of consumption goods is \$1.00 per unit.

c. How many hours will Amanda work?

Her demand for leisure/work is $L = \frac{1}{2}\left(\frac{I}{w}\right) = \frac{1}{2}\left(\frac{wT}{w}\right) = \frac{1}{2}T = 50 \Rightarrow H = 50$**.**

3 points for correct answer.

Suppose that the government introduced a welfare system with a grant is \$200, and this grant is reduced dollar-for-dollar with earnings.

d. Draw Amanda's new budget constraint, carefully labeling the axes, intercepts, and all kink points.

Her budget constraint looks like this:

<FIGURE 2, ignore the indifference curves - they don't apply to this question>

X-axis with the time endowment of 100, Y-axis with consumption of 500, tax rate of 100%.
Welfare grant bundle, ($L = 100, C = 200$ **)**
Bundle where welfare eligibility ends, ($L = 60, C = 200$ **)**

3 points total – 1 point for getting the welfare grant bundle, 1 point for labeling the bundle where welfare eligibility ends, and 1 point for getting the x-axis, y-axis, and the general shape of the budget constraint.

e. Since the tax rate is 100%, the most preferred bundle on welfare is ($L = 100, C = 200$), and the only bundle she would possibly choose on welfare. Compare this with the bundle when working, ($L = 50, C = 250$). The utility function implies that $U(C,L)\big|_{C=200,L=100} = 200^{\frac{1}{3}}100^{\frac{1}{3}}$ is on a higher indifference curve than $U(C,L)\big|_{C=250,L=50} = 250^{\frac{1}{3}}50^{\frac{1}{3}}$. Thus, she now works zero hours.

Must make explicit comparison of utilities and recognize that the only bundle that must be compared is ($L = 100, C = 200$ **) because the tax rate is 100%. 3 points for correct answer.**

Figure 1

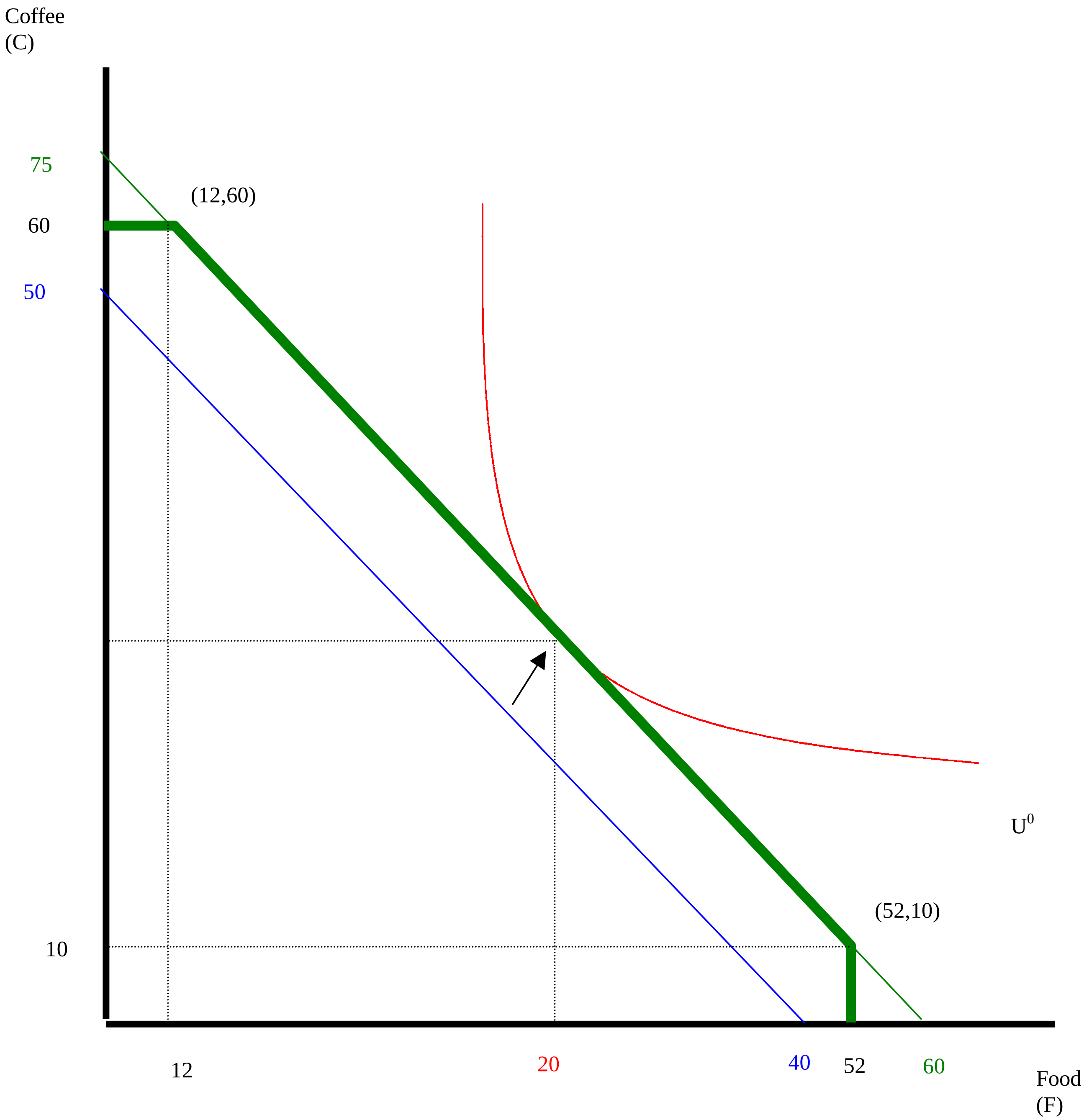
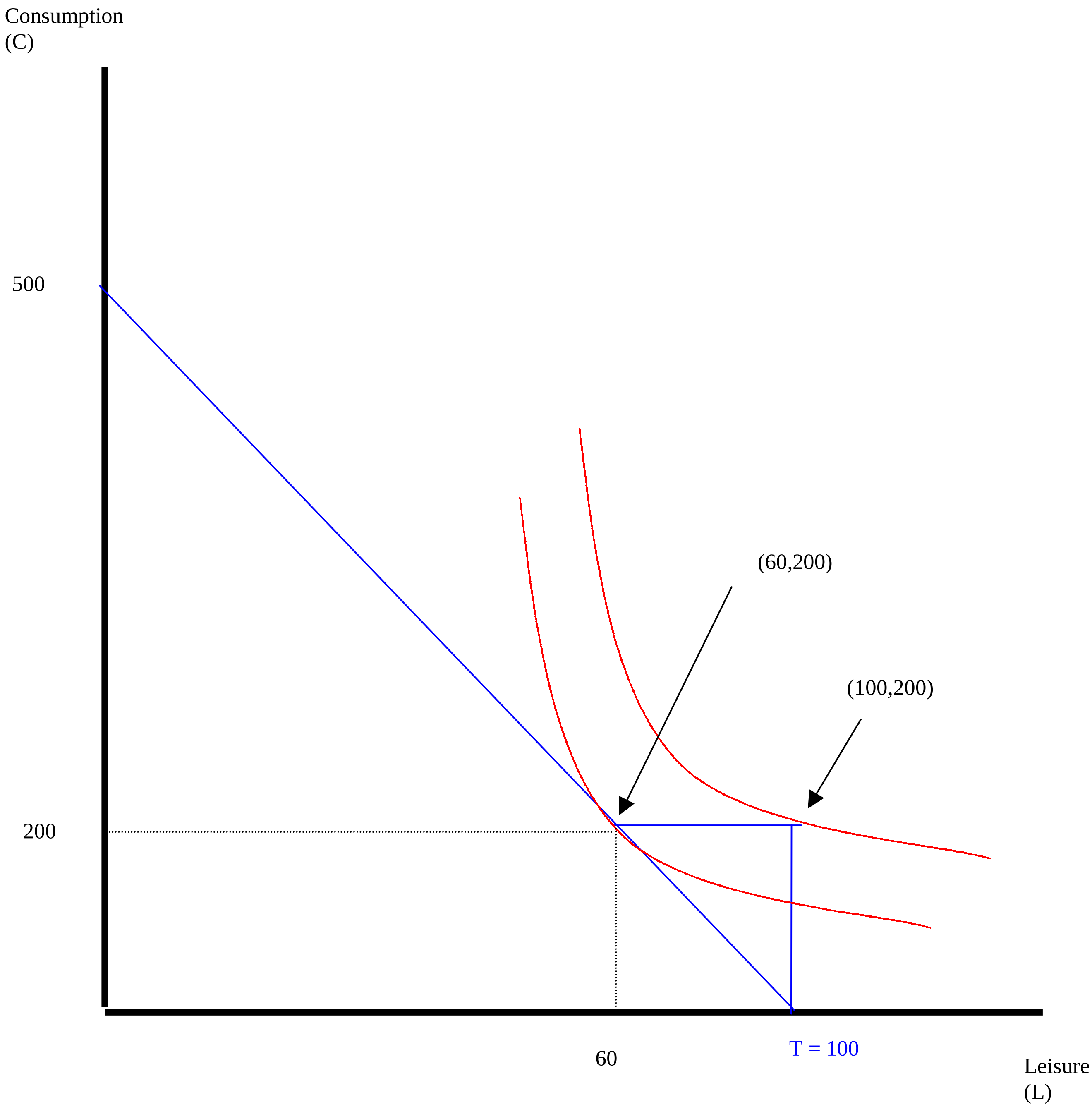


Figure 2 - ignore the indifference curves



Economics 479
Professor Yelowitz
Quiz 5 Solution

This quiz is worth 15 points. You have 50 minutes.

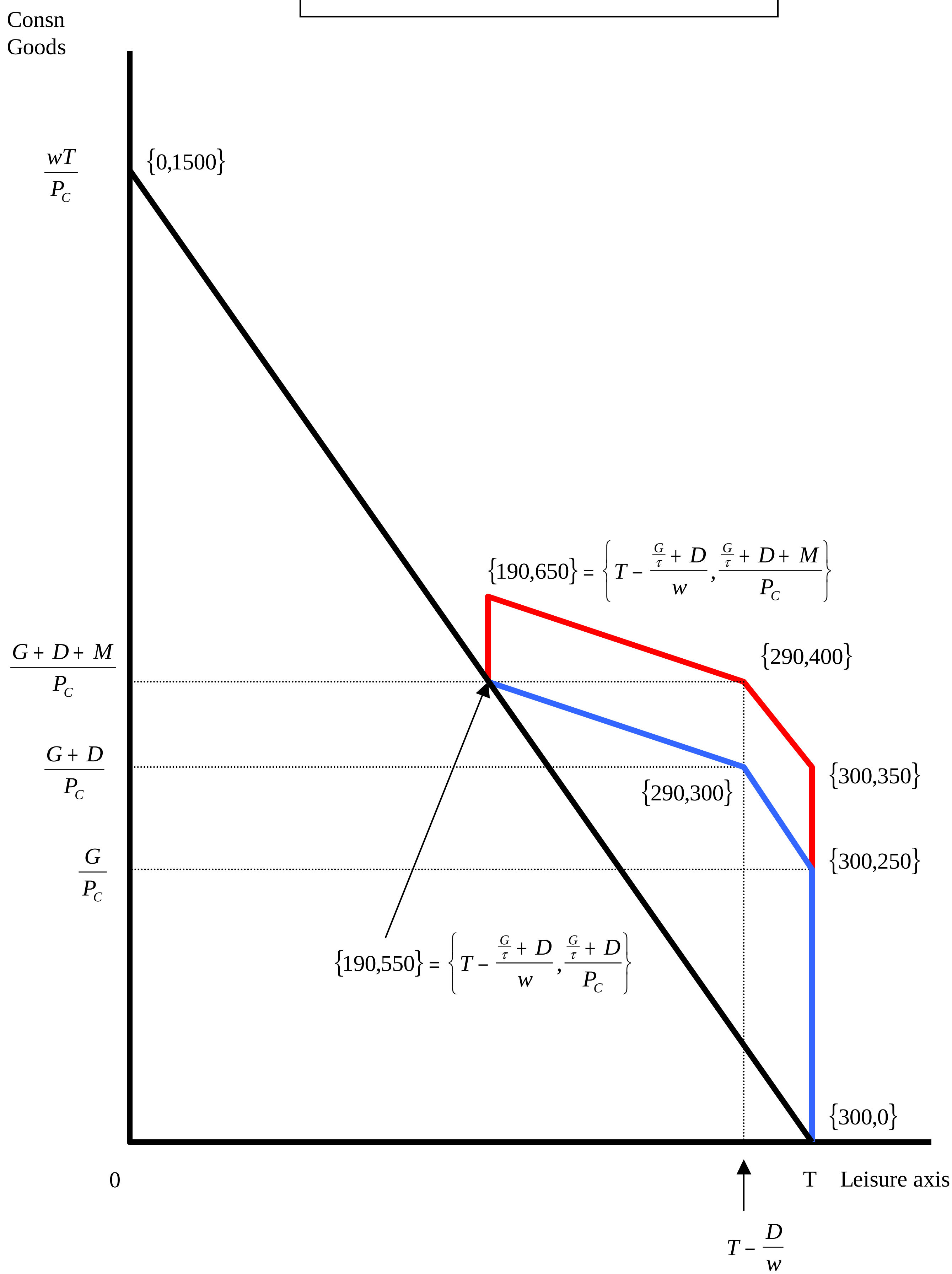
Jane lives in a state that offers a maximum cash benefit of \$500. Earnings of \$100 is allowed through a deduction before cash benefits are reduced at J=50%. Jane also receives Medicaid, which she values at \$200. Medicaid is lost when cash welfare eligibility is lost. Jane is endowed with 300 hours of leisure. The price of consumption goods is \$2. Jane's wage rate is \$10 per hour.

a) (4 points) Draw the budget constraint facing Jane, clearly labeling the axes, intercepts and any kink points. Is there any region of hours which Jane will definitely not work? If so, what region? At what level of earnings is Medicaid eligibility lost?

See Figure 1 for an illustration of the budget constraint, and the relevant formulas for computing the intercepts and kinks. Jane loses Medicaid if she works more than 110 hours (corresponding to 190 hours of leisure). This is determined by the cash welfare eligibility limit, which is determined by the formula $\frac{G}{\tau} + D$ which corresponds to $\frac{500}{0.5} + 100 = \$1100$ of earnings. She loses Medicaid valued at \$200, and it takes her 20 additional hours of work to recoup this loss of Medicaid. Thus, work of between 110-130 hours (corresponding to leisure of 170-190 hours) will definitely not be chosen.

1 point for drawing the general shape of the budget constraint correctly (see Figure 1).
1 point for labeling all kink points and intercepts on the budget constraint (see Figure 1).
1 point for getting the correct earnings level for where Medicaid is lost, \$1100.
1 point for figure out the range of hours of work (or leisure) where Jane will not work.

QUIZ 5, FIGURE 1



Suppose the government doubles the services that Medicaid provides (all the services are valued equally). Thus, Medicaid is now valued at $400.

b) (4 points) Compared to the initial case in part a), will government Medicaid spending exactly double, more than double, or less than double? Why?

See Figure 2 for an illustration of how the budget constraint changes when Medicaid's value is doubled. When Medicaid's value is doubled, and nothing else is changed about the welfare system, then:

- Everyone who is on cash welfare (and on Medicaid) will stay on, although they may adjust their hours of work/leisure and consumption. Hence, their Medicaid expenditure exactly doubles, going from $200 to $400.

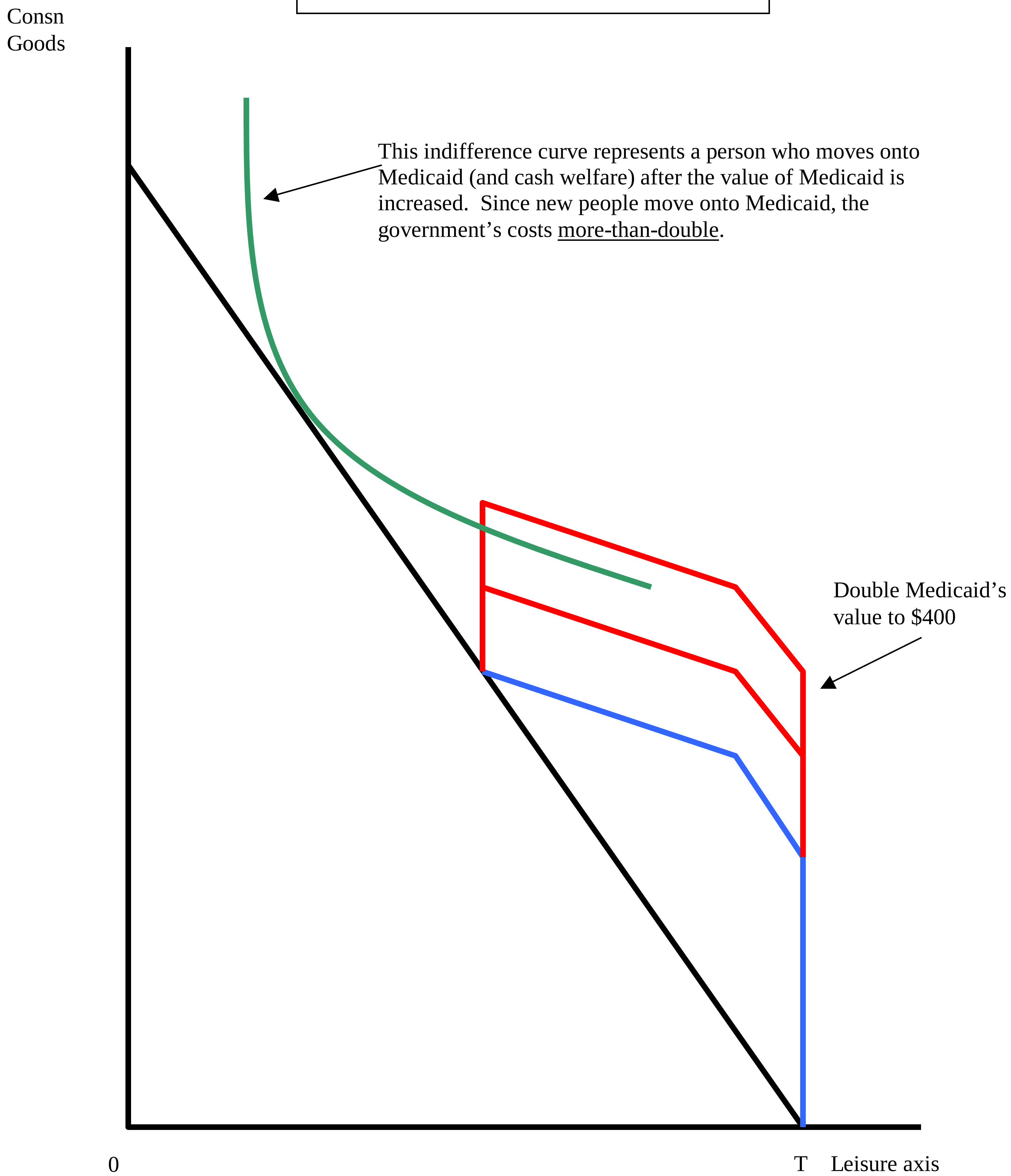
- Some people who were initially off cash welfare (and off Medicaid) will now move onto cash welfare (and Medicaid) by reducing their hours of work. The green indifference curve in Figure 2 shows this possibility. These people were initially not collecting Medicaid, so Medicaid expenditure more-than-doubles for them, going from $0 to $400.

- Some people who were initially off cash welfare (and off Medicaid) continue to stay off, and stay at the initial point that they chose. Medicaid expenditure remains constant, at $0 for them.

Altogether, government costs <u>more-than-double</u> because no one who was initially on Medicaid leaves, and some people who were initially off enter.

1 point for drawing the budget constraint.
1 point for arguing why costs double for those who were on Medicaid initially.
2 points for arguing why costs more-than-double for those who were off Medicaid initially.

QUIZ 5, FIGURE 2



Suppose the government charges Jane for Medicaid: the charge is equal to 5% of any earnings above the first $100 that Jane receives while on cash welfare. Medicaid eligibility is still contingent on cash welfare eligibility.

c) (4 points) Draw the new budget constraint facing Jane, clearly labeling the axes, intercepts and any kink points. Compared with part a), clearly explain (and justify) what predictions can be made about hours of work and cash welfare participation.

See Figure 3 for the new budget constraint after the charge is introduced. The charge operates as a tax rate – now each dollar earned on welfare above the first $100 is taxed at a cumulative rate of 55% rather than 50%. At $1100 of earnings, where the recipient would become ineligible for cash welfare and Medicaid, the charge for Medicaid is equal to

$\tau_{MEDICAID}\left(earnings - D_{MEDICAID}\right) = .05(1100 - 100) = \$50$. Thus, the total income at the cash welfare eligibility limit (corresponding to 110 hours of work or 190 hours of leisure) is $1100 of earnings plus $200 of Medicaid's value minus the $50 charge, or $1250 of total income. This translates into 625 units of the consumption good.

As shown in Figure 3, the yellow portion of the budget constraint is taken away after the charge on Medicaid is implemented. Thus,

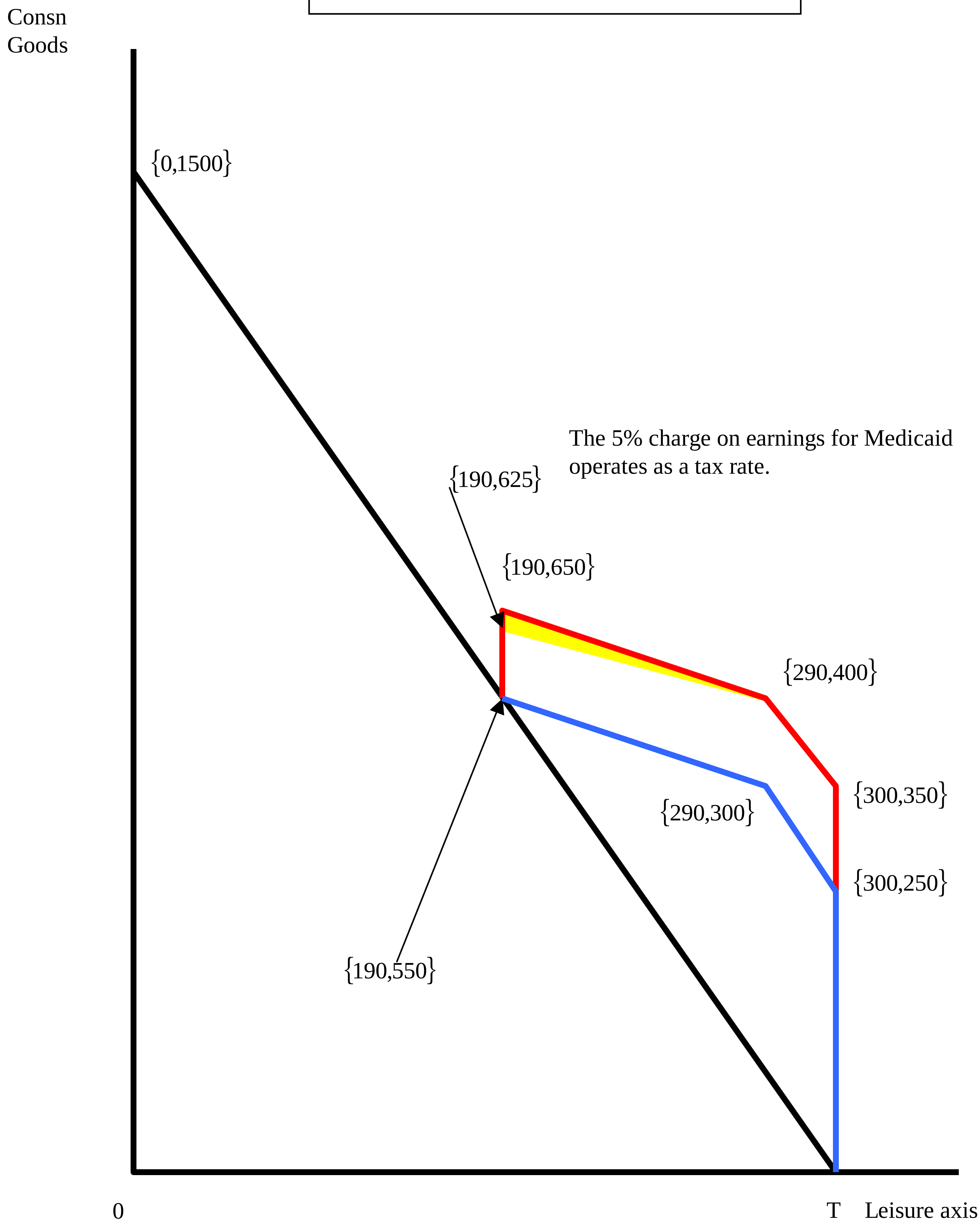
- Anyone who was initially off welfare will stay off, and will not change their behavior at all. Those people preferred their initial point to the welfare system, and the only thing the charge on Medicaid does is make the welfare system even less desirable.

- Anyone who was initially on the part of the budget constraint where the welfare standard deduction is operating will continue to stay there. They preferred that bundle to the bundles that were taken away.

- Those who were initially located in the yellow area on the budget constraint (where the tax rate was initially 50%) must change their behavior, because those bundles are no longer feasible. All of those bundles were where the person was in the labor force, working between 10-110 hours (consuming leisure between 190 and 290 hours), and was on cash welfare. Depending on the person's preferences, she could either increase or decrease hours of work, so the prediction there is ambiguous. On the other hand, the person either remains on cash welfare (resulting in no change) or leaves cash welfare (resulting in a decrease). No one who was initially off cash welfare goes on. So we can predict that for the population as a whole, cash welfare participation falls.

1 point for correctly computing the point (190,625).
1 point for correctly drawing the modified budget constraint.
1 point for correctly arguing that the effects on hours of work are ambiguous.
1 point for correctly arguing that cash welfare participation falls.

QUIZ 5, FIGURE 3



Return to the original case. The government now modifies the deduction (that was previously $100 per family) so that a welfare recipient can deduct $100 per child.

d) (3 points) What earnings level will cash eligibility be lost at for Jane, who has one child? What earnings level will cash eligibility be lost at for Amanda, who has three children?

The breakeven formula for Jane is now: $\frac{G}{\tau} + D_{CHILD}$, and for Amanda is $\frac{G}{\tau} + 3D_{CHILD}$. These correspond to $1100 and $1300 of earnings, respectively.

1 point for getting Jane's level.
2 points for getting Amanda's level.

Economics 479
Professor Yelowitz
Quiz 6 Solution

Suppose the cigarette market, which is assumed to be perfectly competitive, is characterized by the following demand and supply curves: $Q^D = 200 - 4P$ and $Q^S = \frac{1}{5}P - 4$

a. Suppose the government imposes a $25 *per-unit* tax on the demanders of cigarettes. What price do demanders pay after the tax? How much money does the government raise?

The per-unit tax leads to $P^D - 25 = P^S$. Thus,

$$200 - 4P^D = \frac{1}{5}\left(P^D - 25\right) - 4 \Rightarrow 209 = \frac{21}{5}P^D \Rightarrow P^D \approx 49.76.$$

Substituting this price into the demand curve gives $Q^D = 200 - 4(49.76) \approx 0.95$, and therefore revenue equals $25 * 0.95 \approx 23.75$.

3 points total - 2 for getting the price demanders pay, 1 for getting government revenue.

b. What is the statutory incidence of the tax in part a.? What is its economic incidence?

The statutory incidence falls 100% on the consumers.

The economic incidence falls 4.7% on consumers, and 95.3% on producers. Before taxes the price was $48.57. After, consumers pay approximately $1.19 more than before, and

$$\frac{\$1.19}{\$25} = 4.7\% .$$

3 points total - 2 for getting the economic incidence, 1 for getting the statutory incidence.

c. Evaluate the following statement: "If the government imposed a $15 per unit tax on demanders and a $10 per unit tax on suppliers, this combination would produce exactly the same deadweight loss as the tax proposed in part a."

True, both lead to the same deadweight loss because with the equation $P^D - \tau^D = P^S + \tau^S$, setting $J^S=10$ and $J^D=15$ gives the same answer as if $J^S=0$ and $J^D=25$.

3 points total for correct answer. Must explain reasoning.

d. Instead, suppose the government imposes a 20% *ad-valorem* tax on the demanders of cigarettes. What price do suppliers receive after this tax? What price do demanders pay after this tax?

Since $(1-\tau^D)P^D = P^S \Rightarrow \frac{4}{5}P^D = P^S$, then substituting into the previous equations gives

$$200 - 4P^D = \frac{1}{5}\left(\frac{4}{5}P^D\right) - 4 \Rightarrow P^D \approx 49.03, P^S \approx 39.23$$

3 points total - 2 points for supplier's price, 1 point for demander's price.

Instead of a perfectly competitive structure, suppose that there was a monopolist who faced the demand curve for cigarettes of $Q^D = 200 - 4P$ and had a marginal cost curve of $MC = 20 + 5Q$.

e. Again, suppose the government imposes a 20% *ad-valorem* tax on the demanders of cigarettes. What quantity of cigarettes is now provided? What price to demanders pay after the tax is imposed?

The original demand curve can be rewritten as $P = 50 - \frac{1}{4}Q$. After the 20% tax is imposed, the demand curve rotates downward, so it is now $P = 40 - \frac{1}{5}Q$. Thus, total revenue equals $40Q - \frac{1}{5}Q^2$. Therefore marginal revenue equals $MR = 40 - \frac{2}{5}Q$, and $MC = 20 + 5Q$. Hence the monopolist produces $MR = MC \Rightarrow 40 - \frac{2}{5}Q = 20 + 5Q \Rightarrow Q \approx 3.7$. Substituting this quantity back into the original demand curves gives $P^D \approx 49.08$.

3 points total - 2 points for quantity, 1 point for price.

Economics 479
Professor Yelowitz
Final Exam Key

This final is worth 25 points. Unless otherwise noted, each question is worth one point.

1. Consider a consumer who has preferences over food and clothing, represented by the utility function $u = F^{\frac{1}{2}}C^{\frac{1}{2}}$, where F and C represent the quantity of food and clothing respectively. She faces prices P_F and P_C for food and clothing and has income equal to I.
Write down the demand curve for food, expressed in terms of prices and income.

By setting the MRS equal to the price ratio, we get:

$\frac{MU_F}{MU_C} = \frac{\frac{1}{2}F^{-\frac{1}{2}}C^{\frac{1}{2}}}{\frac{1}{2}F^{\frac{1}{2}}C^{-\frac{1}{2}}} = \frac{C}{F} = \frac{P_F}{P_C}$ **which simplifies to** $P_F F = P_C C$**. If we substitute this into the budget constraint,** $P_F F + P_C C = I$**, we get** $P_F F + P_F F = I$**, or** $F = \frac{1}{2}\left(\frac{I}{P_F}\right)$**.**

1 point awarded for the demand curve.

2. **(2 points)** Suppose, instead that the utility function was $u = \min\left\{\frac{1}{3}F, 3C\right\}$, and that $P_F = \$5$, $P_C = \$3$, and $I = \$500$. What quantity of food would this consumer demand?

To maximize utility, the consumer must consume these goods in fixed proportions: $\frac{1}{3}F = 3C$**, which can be rewritten as** $\frac{1}{9}F = C$**. If we substitute this into the budget constraint,** $5F + 3C = 500$**, we get** $5F + \frac{1}{3}F = 500$**, or** $\frac{16}{3}F = 500$**, or** $F = \frac{1500}{16} = 93.75$**.**

2 points awarded for the correct answer of 93.75 (or the fraction $\frac{1500}{16}$**).**

Assume that fireworks are a public good for questions 3 and 4. Allison, Billy, and Carlos have the following individual demand curves for fireworks. $P_A = 200 - Q_A$, $P_B = 100 - Q_B$, $P_C = 100 - Q_C$, where Q_A, Q_B, and Q_C represent the amount of fireworks consumed by Allison, Billy, and Carlos respectively. The marginal cost of producing another unit of fireworks is given by: $MC = 200 + Q$.

3. Calculate the socially optimal quantity of fireworks.

Adding these curves up, the aggregate demand curve is
$P = P_A + P_B + P_C = 200 - Q + 100 - Q + 100 - Q = 400 - 3Q$ **if** $Q \leq 100$**.**
$P = P_A = 200 - Q$ **if** $Q > 100$**.**
Setting $P = MC$ **along the first segment gives:**
$400 - 3Q = 200 + Q \Rightarrow 200 = 4Q \Rightarrow Q = 50$ **which is in the relevant range.**
1 point for correct answer.

4. If Billy and Carlos did not contribute at all for the fireworks, and Allison provided her privately optimal quantity, what quantity would Allison provide and what would be the deadweight loss to society?

Since Allison's demand curve is everywhere below the marginal cost curve, Allison would provide zero. The deadweight loss uses the familiar triangle formula $\frac{1}{2} base * height$**, where the base is equal to 50 (that is** $Q_{SOCIAL} - Q_A$**) and the height is equal to 200 (that is, the** $\left(MB_{SOCIAL} - MC_{SOCIAL}\right)\Big|_{Q_A}$ **). Thus, the DWL is equal to** $\frac{1}{2} 50 * 200 = 5000$**.**
1 point for correct answer.

Amanda bakes blueberry pies which she sells in a perfectly competitive market facing an inverse demand curve: $P = 40 - Q$. Her marginal costs of producing pies are given by: $MC = 4Q$. Amanda has two next door neighbors. Her neighbor, Kyle, likes the smell of the pies. His marginal external benefit is given by: $MEB = 50 - \frac{5}{4}Q$. Her other neighbor, Jane, does not like the smell of the pies. Jane's marginal external cost is given by: $MEC = 2Q$.

5. **(2 points)** What quantity of pies would Amanda produce on her own and what quantity would Kyle prefer the most?

Amanda: sets $P = MC \Rightarrow 40 - Q = 4Q \Rightarrow Q_{PRIVATE} = 8$

Kyle: sets $MEB = 0 \Rightarrow 50 - \frac{5}{4}Q = 0 \Rightarrow Q = 40$.

2 points total - 1 point for each correct numerical answer, no partial credit.

6. **(2 points)** Calculate the deadweight loss from the private market providing pies.

To compute DWL, we need to know the socially optimal quantity.

$Q_{SOCIAL} \Rightarrow MSB = MSC \Rightarrow MB + MEB = MC + MEC$

$\Rightarrow (40 - Q) + \left(50 - \frac{5}{4}Q\right) = (4Q) + (2Q) \Rightarrow Q_{SOCIAL} \approx 10.91$

Deadweight loss is computed as:

$$DWL = \frac{1}{2}\left(Q_{SOCIAL} - Q_{PRIVATE}\right)\left(MSB\big|_{Q_{PRIVATE}} - MSC\big|_{Q_{PRIVATE}}\right)$$
$$= \frac{1}{2}(10.91 - 8)(72 - 48) \approx 34.92$$

2 points for correct numerical answer.

7. What per-unit tax or subsidy on Amanda's production would lead to the socially optimal level?

Since Amanda privately produces less than the socially optimal amount, we need a subsidy. Amanda now faces the following situation: $P = MC - subsidy \Rightarrow 40 - Q = 4Q - subsidy$ **.**

We evaluate this expression at Q_{SOCIAL}, which gives $40 - 10.91 = 4(10.91) - subsidy \Rightarrow subsidy = \$14.55$ **.**

1 point for correct numerical answer.

8. What is the maximum amount of money that Kyle would be willing to pay for Amanda to move from $Q_{PRIVATE}$ to Q_{SOCIAL}?

Since Kyle enjoys the production, he gains by moving from $Q_{PRIVATE}$ to Q_{SOCIAL} by an amount under his marginal external benefit curve. Using the trapezoid formula, his maximum bribe is equal to

$$\frac{\left(MEB\big|_{Q_{SOCIAL}} + MEB\big|_{Q_{PRIVATE}}\right)}{2}\left(Q_{SOCIAL} - Q_{PRIVATE}\right) = \frac{(36.3625 + 40)}{2}(2.91) \approx 111.11$$

1 point for correct numerical answer.

9. What is the loss in surplus to Amanda and her customers by moving from $Q_{PRIVATE}$ to Q_{SOCIAL}?

Amanda's loss in surplus by producing Q_{SOCIAL} is the area between the MC and MB curves – that is,

$$\frac{1}{2}\left(Q_{SOCIAL} - Q_{PRIVATE}\right)\left(MC\big|_{Q_{SOCIAL}} - MB\big|_{Q_{SOCIAL}}\right)$$
$$= \frac{1}{2}(10.91 - 8)(4(10.91) - (40 - 10.91)) \approx 21.17$$

1 point for correct numerical answer.

10. What per-unit tax or subsidy on Amanda's production would a regulator choose in order to get Q=6?

Since Amanda would produces more than six units on her own, a tax is necessary to reduce her production. Thus, Amanda faces: $P = MC + \tau \Rightarrow 40 - Q = 4Q + \tau$ **. We evaluate this expression at Q=6, which gives:** $40 - 6 = 4(6) + \tau$**, or** $\tau = 10$**.**
1 point for correct numerical answer.

Chandler has preferences over food and coffee given by the utility function $U(F,C) = F^{\frac{1}{3}}C^{\frac{2}{3}}$.
The price of food is $5, while the price of coffee is $4. His income is originally $200. The government now gives Chandler a transfer, and is debating two different transfer schemes:
Transfer #1: $100 in cash
Transfer #2: 12 units of food and 10 units of coffee.
Chandler still keeps his original $200, in addition to the transfer.

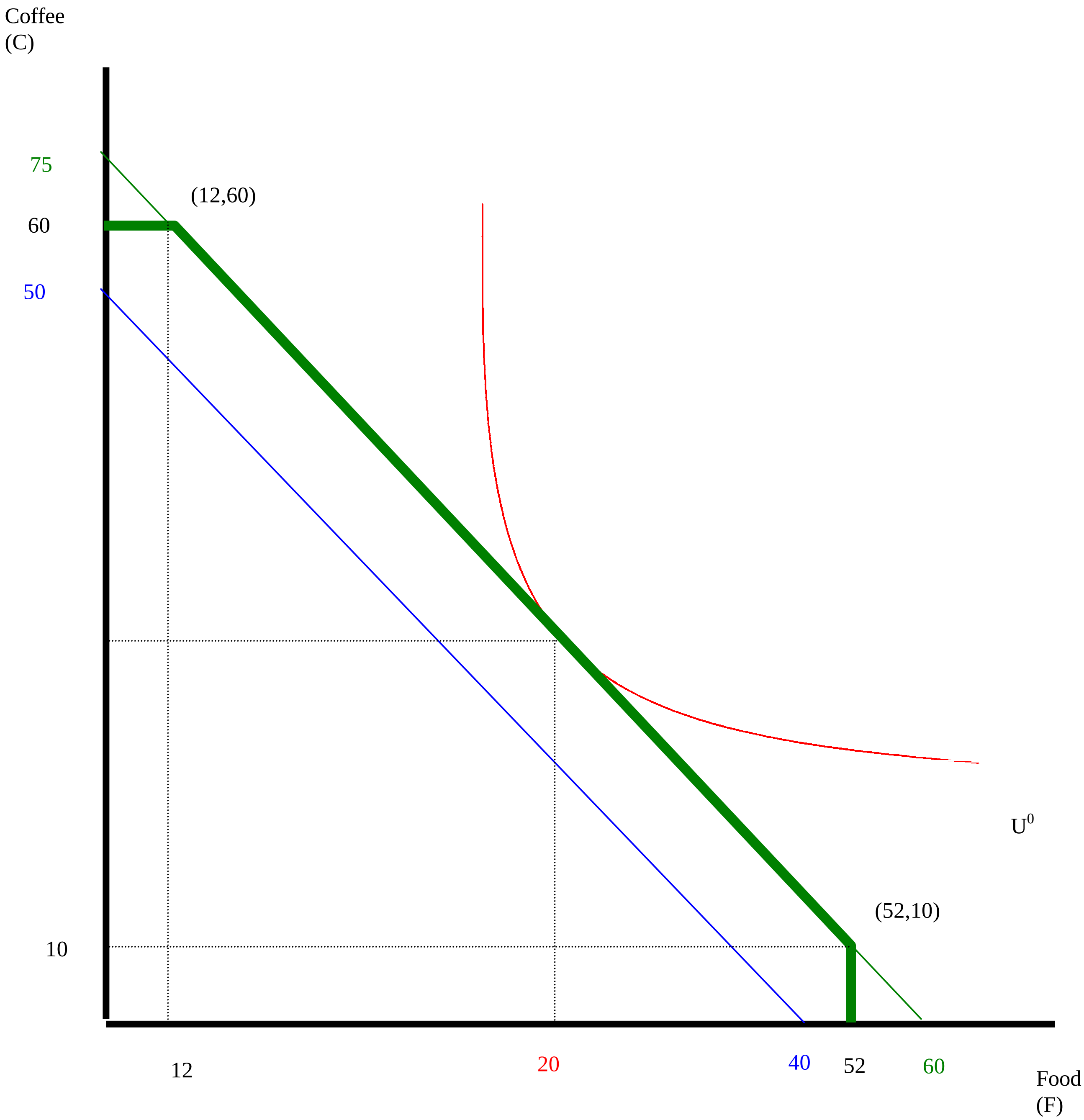
11. **(2 points)** <u>On separate graphs</u>, clearly illustrate Chandler's budget constraint before the transfer, and how each of the transfers changes the budget constraint. Make sure to label all relevant kink points and intercepts.

Original budget constraint correctly (and labeling), the new budget constraint with cash, the budget constraint with in-kind benefits.

<SEE FIGURE 1, ALL ON SAME GRAPH>

2 points total – 1 point for correctly drawing and labeling the original budget constraint, 1 point for drawing and labeling Transfer #1 <u>and</u> drawing and labeling Transfer #2.

Figure 1



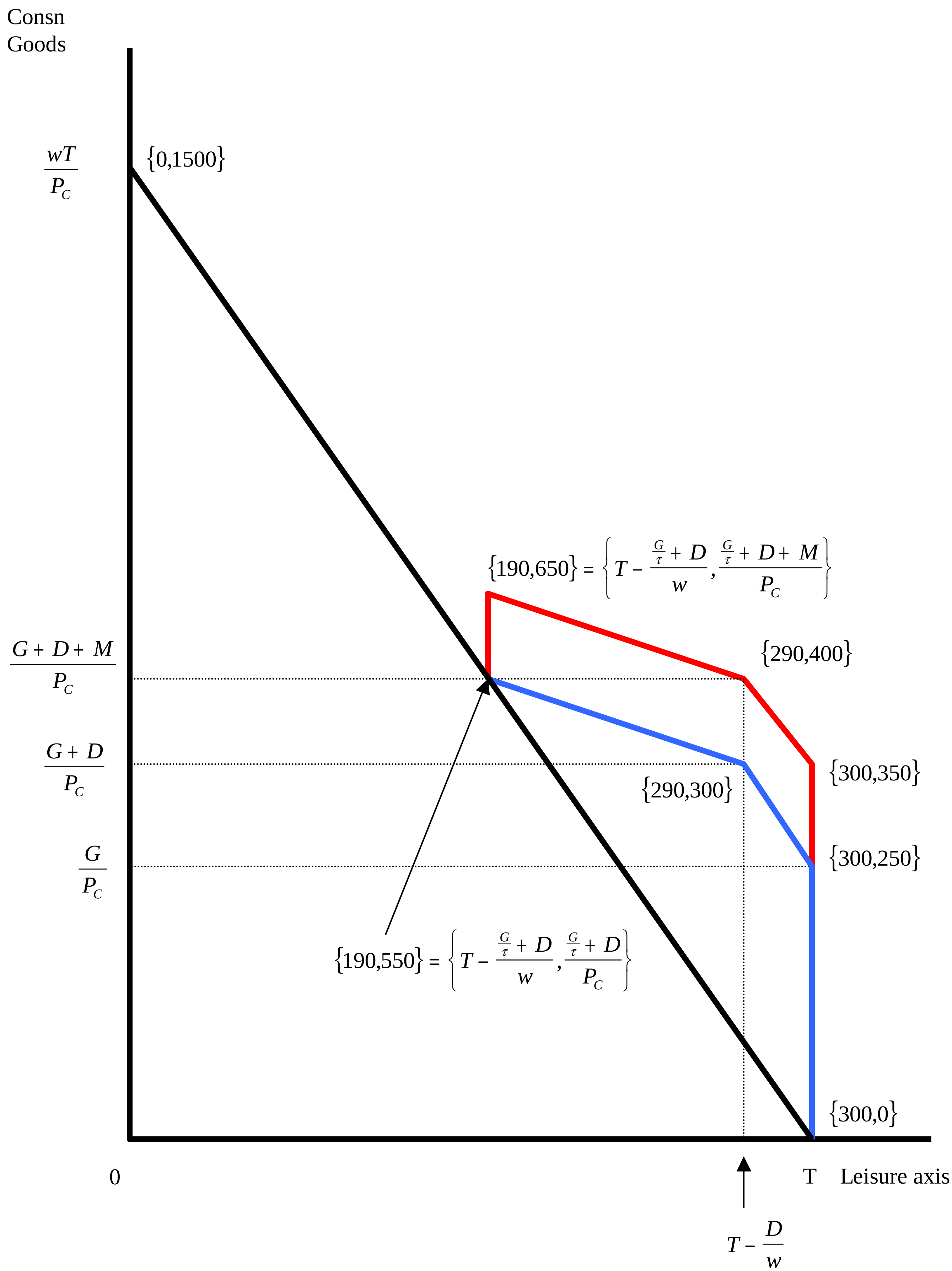
Jane lives in a state that offers a maximum cash benefit of $500. Earnings of $100 is allowed through a deduction before cash benefits are reduced at J=50%. Jane also receives Medicaid, which she values at $200. Medicaid is lost when cash welfare eligibility is lost. Jane is endowed with 300 hours of leisure. The price of consumption goods is $2. Jane's wage rate is $10 per hour.

12. **(2 points)** Draw the budget constraint facing Jane, clearly labeling the axes, intercepts and any kink points. Is there any region of hours which Jane will definitely not work? If so, what region? At what level of earnings is Medicaid eligibility lost?

See FIGURE 2 for an illustration of the budget constraint, and the relevant formulas for computing the intercepts and kinks. Jane loses Medicaid if she works more than 110 hours (corresponding to 190 hours of leisure). This is determined by the cash welfare eligibility limit, which is determined by the formula $\frac{G}{\tau} + D$ **which corresponds to** $\frac{500}{0.5} + 100 = \$1100$ **of earnings. She loses Medicaid valued at $200, and it takes her 20 additional hours of work to recoup this loss of Medicaid. Thus, work of between 110-130 hours (corresponding to leisure of 170-190 hours) will definitely not be chosen.**

1 point for drawing the general shape of the budget constraint correctly (see Figure 1) <u>AND</u> for labeling <u>all</u> kink points and intercepts on the budget constraint (see Figure 1).
1 point for getting the correct earnings level for where Medicaid is lost, $1100 AND for figuring out the range of hours of work (or leisure) where Jane will not work.

FIGURE 2



Consn
Goods
$\frac{wT}{P_C}$
$\{0,1500\}$
$\{190,650\} = \left\{T - \frac{\frac{G}{\tau} + D}{w}, \frac{\frac{G}{\tau} + D + M}{P_C}\right\}$
$\frac{G+D+M}{P_C}$
$\{290,400\}$
$\frac{G+D}{P_C}$
$\{300,350\}$
$\{290,300\}$
$\frac{G}{P_C}$
$\{300,250\}$
$\{190,550\} = \left\{T - \frac{\frac{G}{\tau} + D}{w}, \frac{\frac{G}{\tau} + D}{P_C}\right\}$
$\{300,0\}$
0
T
Leisure axis
$T - \frac{D}{w}$

Suppose the government doubles the services that Medicaid provides (all the services are valued equally). Thus, Medicaid is now valued at $400.

13. Compared to the initial case in question 12, will government Medicaid spending exactly double, more than double, or less than double? Why?

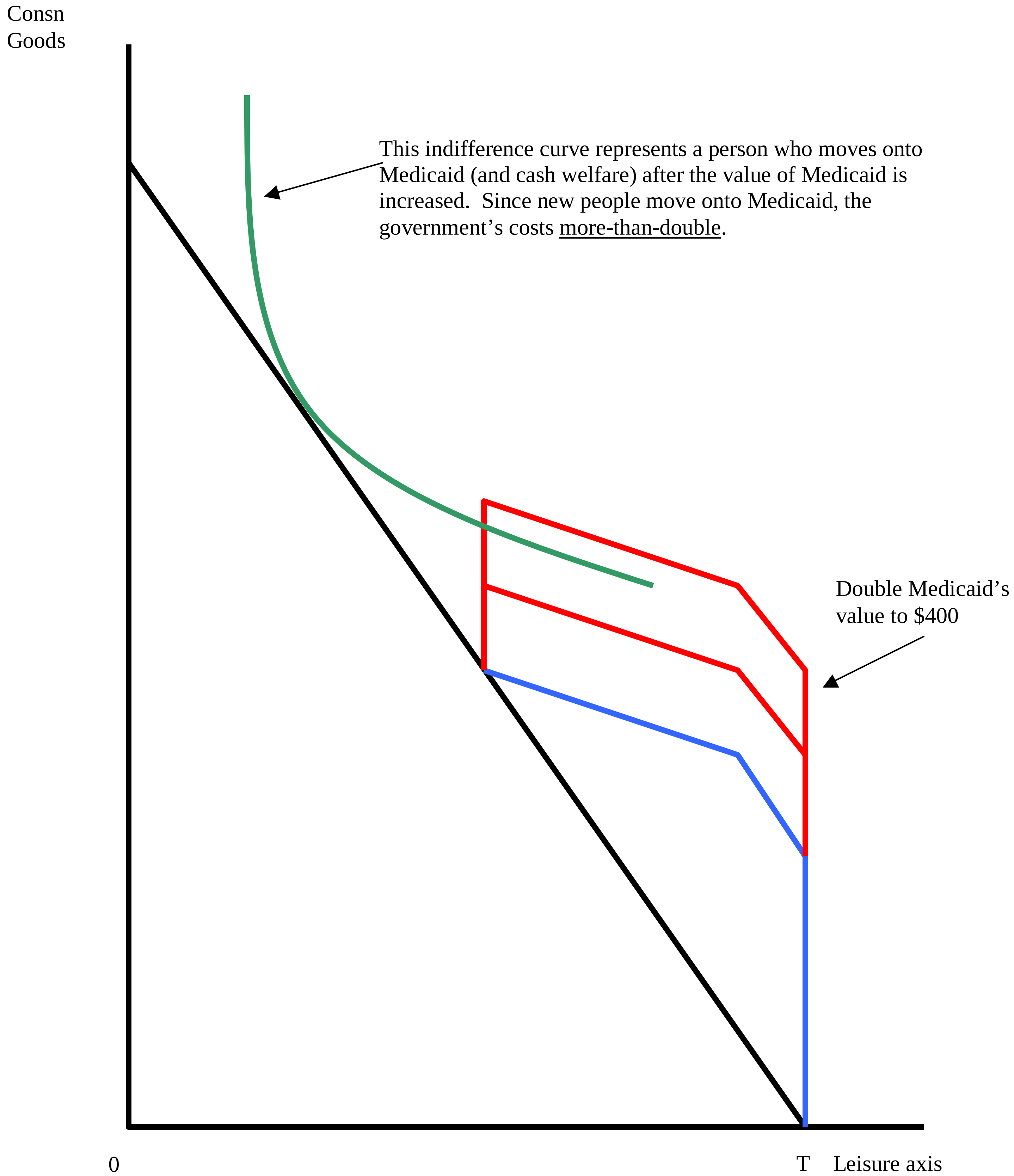
See FIGURE 3 for an illustration of how the budget constraint changes when Medicaid's value is doubled. When Medicaid's value is doubled, and nothing else is changed about the welfare system, then:

- **Everyone who is on cash welfare (and on Medicaid) will stay on, although they may adjust their hours of work/leisure and consumption. Hence, their Medicaid expenditure exactly doubles, going from $200 to $400.**
- **Some people who were initially off cash welfare (and off Medicaid) will now move onto cash welfare (and Medicaid) by reducing their hours of work. The green indifference curve in Figure 2 shows this possibility. These people were initially not collecting Medicaid, so Medicaid expenditure more-than-doubles for them, going from $0 to $400.**
- **Some people who were initially off cash welfare (and off Medicaid) continue to stay off, and stay at the initial point that they chose. Medicaid expenditure remains constant, at $0 for them.**

Altogether, government costs <u>more-than-double</u> because no one who was initially on Medicaid leaves, and some people who were initially off enter.

1 point for the complete argument.

FIGURE 3


Consn
Goods
This indifference curve represents a person who moves onto Medicaid (and cash welfare) after the value of Medicaid is increased. Since new people move onto Medicaid, the government's costs more-than-double.
Double Medicaid's value to $400
0
T
Leisure axis

Suppose the government charges Jane for Medicaid: the charge is equal to 5% of any earnings above the first $100 that Jane receives while on cash welfare. Medicaid eligibility is still contingent on cash welfare eligibility. Medicaid is valued at $200.

14. Draw the new budget constraint facing Jane, clearly labeling the axes, intercepts and any kink points. Compared with question 12, clearly explain (and justify) what predictions can be made about hours of work and cash welfare participation.

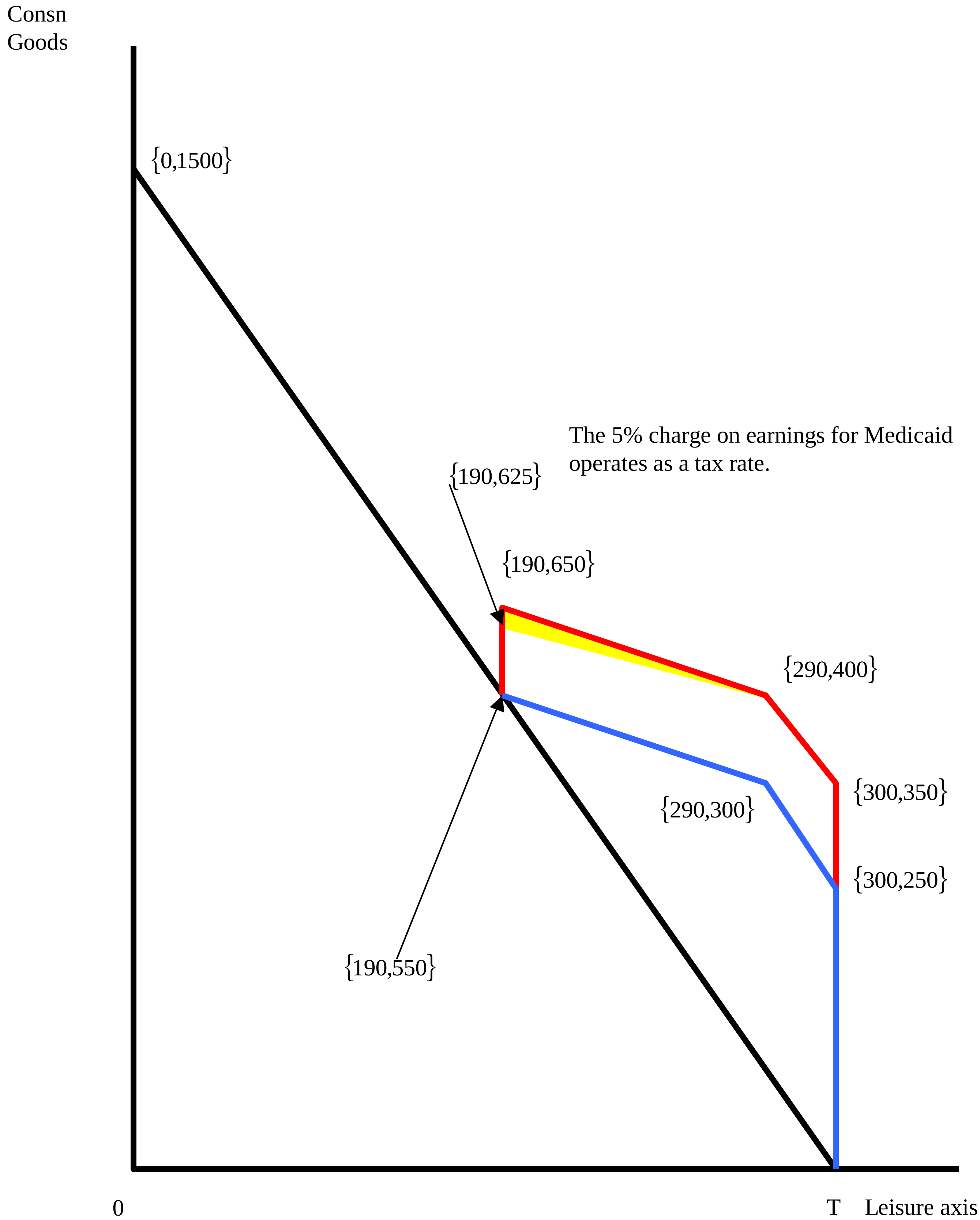
See FIGURE 4 for the new budget constraint after the charge is introduced. The charge operates as a tax rate – now each dollar earned on welfare above the first $100 is taxed at a cumulative rate of 55% rather than 50%. At $1100 of earnings, where the recipient would become ineligible for cash welfare and Medicaid, the charge for Medicaid is equal to $\tau_{MEDICAID}\left(earnings - D_{MEDICAID}\right) = .05(1100 - 100) = \$50$ **. Thus, the total income at the cash welfare eligibility limit (corresponding to 110 hours of work or 190 hours of leisure) is $1100 of earnings plus $200 of Medicaid's value minus the $50 charge, or $1250 of total income. This translates into 625 units of the consumption good.**

As shown in FIGURE 4, the yellow portion of the budget constraint is taken away after the charge on Medicaid is implemented. Thus,

> **- Anyone who was initially off welfare will stay off, and will not change their behavior at all. Those people preferred their initial point to the welfare system, and the only thing the charge on Medicaid does is make the welfare system even less desirable.**
>
> **- Anyone who was initially on the part of the budget constraint where the welfare standard deduction is operating will continue to stay there. They preferred that bundle to the bundles that were taken away.**
>
> **- Those who were initially located in the yellow area on the budget constraint (where the tax rate was initially 50%) must change their behavior, because those bundles are no longer feasible. All of those bundles were where the person was in the labor force, working between 10-110 hours (consuming leisure between 190 and 290 hours), and was on cash welfare. Depending on the person's preferences, she could either increase or decrease hours of work, so the prediction there is ambiguous. On the other hand, the person either remains on cash welfare (resulting in no change) or leaves cash welfare (resulting in a decrease). No one who was initially off cash welfare goes on. So we can predict that for the population as a whole, cash welfare participation falls.**

1 point for correctly drawing and labeling the budget constraint <u>AND</u> correctly arguing the predictions for hours of work and cash welfare.

FIGURE 4


Consn
Goods
{0,1500}
The 5% charge on earnings for Medicaid operates as a tax rate.
{190,625}
{190,650}
{290,400}
{300,350}
{290,300}
{300,250}
{190,550}
0
T
Leisure axis

Suppose the cigarette market, which is assumed to be perfectly competitive, is characterized by the following demand and supply curves: $Q^D = 200 - 4P$ and $Q^S = \frac{1}{5}P - 4$

15. Suppose the government imposes a $25 *per-unit* tax on the demanders of cigarettes. What price do demanders pay after the tax? How much money does the government raise?

The per-unit tax leads to $P^D - 25 = P^S$ **. Thus,**

$$200 - 4P^D = \frac{1}{5}\left(P^D - 25\right) - 4 \Rightarrow 209 = \frac{21}{5}P^D \Rightarrow P^D \approx 49.76.$$

Substituting this price into the demand curve gives $Q^D = 200 - 4(49.76) \approx 0.95$**, and therefore revenue equals** $25 * 0.95 \approx 23.75$**.**

1 point total - for getting the price demanders pay <u>AND</u> for getting government revenue.

16. What is the statutory incidence of the tax in question 15? What is its economic incidence?

The statutory incidence falls 100% on the consumers. The economic incidence falls 4.7% on consumers, and 95.3% on producers. Before taxes the price was $48.57. After, consumers pay approximately $1.19 more than before, and $\frac{\$1.19}{\$25} = 4.7\%$ **.**

1 point total - for getting the economic incidence <u>AND</u> for getting the statutory incidence.

17. Evaluate the following statement: "If the government imposed a $15 per unit tax on demanders <u>and</u> a $10 per unit tax on suppliers, this combination would produce exactly the same deadweight loss as the tax proposed in question 15."

True, both lead to the same deadweight loss because with the equation $P^D - \tau^D = P^S + \tau^S$**, setting J^S=10 and J^D=15 gives the same answer as if J^S=0 and J^D=25.**
1 point total for correct answer. Must explain reasoning.

18. Instead, suppose the government imposes a 20% *ad-valorem* tax on the demanders of cigarettes. What price do suppliers receive after this tax? What price do demanders pay after this tax?

Since $(1-\tau^D)P^D = P^S \Rightarrow \frac{4}{5}P^D = P^S$ **, then substituting into the previous equations gives**

$$200 - 4P^D = \frac{1}{5}\left(\frac{4}{5}P^D\right) - 4 \Rightarrow P^D \approx 49.03, P^S \approx 39.23$$

1 point total - for supplier's price AND for demander's price.

Instead of a perfectly competitive structure, suppose that there was a monopolist who faced the demand curve for cigarettes of $Q^D = 200 - 4P$ and had a marginal cost curve of $MC = 20 + 5Q$.

19. **(2 points)** Again, suppose the government imposes a 20% *ad-valorem* tax on the demanders of cigarettes. What quantity of cigarettes is now provided? What price to demanders pay after the tax is imposed?

The original demand curve can be rewritten as $P = 50 - \frac{1}{4}Q$**. After the 20% tax is imposed, the demand curve rotates downward, so it is now** $P = 40 - \frac{1}{5}Q$**. Thus, total revenue equals** $40Q - \frac{1}{5}Q^2$**. Therefore marginal revenue equals** $MR = 40 - \frac{2}{5}Q$**, and** $MC = 20 + 5Q$ **.**

Hence the monopolist produces $MR = MC \Rightarrow 40 - \frac{2}{5}Q = 20 + 5Q \Rightarrow Q \approx 3.7$**. Substituting this quantity back into the original demand curves gives** $P^D \approx 49.08$ **.**

2 points total - 1 point for quantity, 1 point for price.