Private Equity as Financial Intermediation

Working Paper

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Abstract

We present a model in which private equity emerges endogenously as a form of financial intermediation. The key to the model is twofold. First, investors have different levels of alignment with the manager. Second, trading in public markets results in little surplus for those with high alignment. Private equity investors solve a coordination problem created in the public market by competition. For each potential private equity investor with a high level of alignment with the manager, any trading creates a frenzy in which the marginal surplus is zero. Thus, forming a coalition of private equity investors and extracting benefit from the higher alignment provides more surplus than the cost of forming the coalition. The benefit is greatest when the public market has a low level of alignment and a large amount of volatility. The firm's benefit for issuing private equity is the gain from having investors with a higher level of alignment. If equity scrutiny is likely to result in an unfavorable opinion of the project, debt will be selected. Further, if the asset-substitution problem is large, then public debt is not an option and private debt will be selected in order to monitor project choice.

1 Introduction

Private equity is no longer a curiosity. *The Economist* reports that in 1991 investors committed roughly \$10 billion to private equity and by 2000 investors had committed about \$160 billion. The size of the largest fund, Kohlberg Kravis Roberts (KKR), in 1980 was \$135 million and as of November 2004, the largest firm, J.P. Morgan, is around \$6.5 billion. By the end of 2004, ten private equity firms each had (between 1987 and 2004) raised over \$5 billion.[1]

More surprisingly, private equity isn't constrained to *only* privately held firms. Gomes and Phillips (2005) examine 13,000 issues with more than half in the private market and 81% of small public firms issuing equity and convertibles in the private market. In addition, they show that from 2000 to 2003, the fraction of private issues by public companies has been increasing over time. Wu (2006) documents that of 1,596 IPOs between 1986 to 1996, 14.35% return to the private equity market. The private equity market is an important factor for public firms' issuance decisions.

This raises an interesting question, why would a public firm issue private equity given that the market isn't as liquid as the public equity market? Private equity firms are functioning as financial intermediators, but what is the benefit for the private equity firm? Such a theory provides an explanation for an increasingly important phenomenon.

We develop a simple model to address these questions. The value added by financial intermediation is solving the coordination problem created by competition in the public market. When private equity investors are aligned with the manager concerning project selection, trading in the public market results in little surplus do to stiff competition. However, a private equity coalition can bargain with the manager privately over the surplus and receive positive gains relative to public markets, as long as, the cost of forming the coalition is not excessive. In exchange for issuing a less liquid security, the manager is awarded with a higher level of alignment which increases the likely of choosing the project she wishes to pursue. While asymmetric information and agency problems are important in the choice of security issuance, our theory provides a model that requires neither while providing insights into the benefits of private equity.

The model provides insights into the public firms choice between debt/equity and public/private markets. If the additional scrutiny of equity investors is likely to result in their negative view of the

[1] *The Economist*, November 27^{th} 2004, pgs. 2 and 3

project, then the manager will choose debt instead of equity. When debt is issued the manager will choose the more liquid public debt market as long as the asset substitution moral hazard problem is not severe. If the asset substitution moral hazard problem is severe, private debt can control which project is selected thereby eliminating the problem.

The remainder of the paper is organized as follows: Section 2 presents the related literature, Section 3 gives a description of the model, Section 4 provides the details of the model, Section 5 presents the analysis, Section 6 contains empirical predictions, and Section 7 concludes.

2 Related Literature and Marginal Contribution

Several recent papers have shown that joint control of decisions between the manager and the investors may be optimal. Aghion and Tirole (1997), Burkhart, Gromb, and Panunzi (1997), and Thadden (1995) all present models in which there is joint control, but the ex-post allocation of control is not joint; it always rests with one party or the other.

Boot, Gopalan, and Thakor (2005a) present a model where ex-post control is optimally joint. In the model presented here, we use this as the building block of the theory. In a similar spirit, all of our control allocations are ex-post joint. The focus of their model is to determine when a firm may want to choose private ownership to customize the managerial control instead of accepting the exogenously imposed control of the public market. Our focus here is on what drives a public firm to issue private equity. The authors note that joint control is often found in practice. As in Boot, Gopalan, and Thakor (2005b), we allow the shareholder base in public markets to vary across time. Heterogenous shareholders that change throughout time allow for certain “aligned” investors to form a coalition and reap the benefits of the shared surplus from private equity.

Our paper is closely related to two other strands of literature: the financial intermediary literature and the private equity literature. Bhattacharya and Thakor (1993) review the contemporary banking literature which focuses primarily on moral hazard and adverse selection. Boyd and Prescott (1986) and Diamond (1984) argue that intermediaries, such as banks, have a cost advantage in producing information because a public offering to dispersed investors leads to a free rider problem or duplication of effort. Chemmanur and Fulghieri (1999) show that with significant asymmetric information, firms may prefer private to public since a private investor has better incentives to produce costly information than more dispersed public investors.

Much of the private equity literature focuses on asymmetric information and agency issues as well. Fulghieri and Lukin (2001) argue that the degree of asymmetric information is endogenous and depends on the information sensitivity of the security issued. They predict private equity to be issued when there is a high amount of asymmetric information. Gomes and Phillips (2005), Hertzel and Smith (1993), and Wu (2006) present empirical evidence that asymmetric information is important for private equity issuance.

Asquith and Mullins (1986), Allen and Phillips (2000), Chaplinsky and Haushalter (1993), and Brophy, Ouimet, and Sialm (1994) document that a firms stock price increases prior to equity issuance which is consistent with increased alignment between investors and the manager (See Smith (2005) and Faulkender, Milbourn, and Thakor (2005)). Our model predicts that when there is high alignment (through high stock price) public equity will be the issuance choice.

Several papers argue that private placement of debt is a solution to agency problems (See Blackwell and Kidwell (1988), Diamond (1991), or Ramakrishnan and Thakor (1984)). We assume that private debt will solve the asset-substitution moral hazard problem.

While we acknowledge that asymmetric information is important, it seems unclear why firms with low stock price and/or high volatility would choose private equity. Low growth immediately implies a low stock price and low growth should be the case when asymmetric information is lowest. In addition, high growth and high stock prices should also exacerbate asset substitution moral hazard which would imply greater benefit for private markets with high stock prices. The model presented here provides a clear benefit for the private equity firm without any complicating asymmetric information and/or agency issues.

3 Model Description

Our analysis begins with a publicly traded firm. The key question addressed is under what situation a firm should issue public equity, private equity, public debt, or private debt. This section describes the projects available, the source of disagreement (and the heterogeneity among agents), and trading. We conclude this section by summarizing the sequence of events.

3.1 Preferences and Projects Available

The economy consists of a publicly listed firm and many investors. All agents are risk-neutral and the risk-free interest rate is assumed to be zero. The manager has to determine the ownership stake in the firm which will imply a certain degree of managerial autonomy. Following Boot, Gopalan, and Thakor (2005a, 2005b), public markets dictate the amount of managerial autonomy whereas private markets can tailor the autonomy to the specific firm. There are five dates 0, 1, 2, 3, and 4. At t=0, the firm has assets in place that will yield a non-stochastic cash flow, $V > 0$, at t=4. At t=0, the manager decides which form of financing to pursue: public equity, private equity, public debt, or private debt.

All agents are aware that three mutually exclusive projects exists: risky innovative project, risky mundane project, and a lemon project. All projects require an investment of $\$I > 0$ at t=3. The risky innovative project will pay a random amount $\tilde{\Lambda} \in \{H^{\text{inn}} + I, L^{\text{inn}} + I\}$, where $L^{\text{inn}} < 0 < H^{\text{inn}} < \infty$. The risky mundane project will pay a random amount $\tilde{\Upsilon} \in \{H^{\text{mun}} + I, L^{\text{mun}} + I\}$, where $L^{\text{mun}} < 0 < H^{\text{mun}} < \infty$. We assume that $H^{\text{mun}} < H^{\text{inn}}$. Ex-ante $H^{\text{inn}} + I$ occurs with probability $p^{\text{inn}} \in (0, 1)$ and $H^{\text{mun}} + I$ occurs with probability $p^{\text{mun}} \in (0, 1)$ where $p^j H^j + (1 - p^j)L^j \equiv D^j \quad \forall j \in \{\text{inn}, \text{mun}\}$.[2] We assume that the ex-ante expected value of the mundane project is positive and the ex-ante expected value of the innovative project, in other words, $D^{\text{mun}} > 0 > D^{\text{inn}}$. Following Dittmar and Thakor (2005), the lemon project has a random payoff, $\tilde{\xi}$ with the following property: $\int_{-\infty}^{\infty} \xi f(\xi) d\xi + V < I$. This guarantees that even if the bondholder had claim to the entire lemon project and the firm's assets in place, the payoff will fall short of \$I.

We view the risky mundane project as an extension of the firm's operations and thus not subject to the same degree of disagreement as the risky innovative project. Everyone agrees the lemon project is bad and is aware of the potential asset-substitution moral hazard problem. We assume that all investors can distinguish between the innovative and mundane projects, but can not distinguish between the mundane and the lemon project.

[2] Henceforth, inn and mun, which identify the innovative and mundane projects will be omitted except when necessary to clarify the presentation.

3.2 Disagreement

Disagreement arises simply because the manager and the investors may disagree about the "success" of a project. Relatively innovative projects will not have sufficient history to form homogenous prior beliefs. Of course, if all agents are behaving rationally, Bayes rule dictates how posterior beliefs are formed. However, Morris (1995) shows that heterogenous priors are perfectly consistent with Bayesian rationality. Kreps (1990) views heterogenous priors as a *more general* specification than homogenous priors that should be viewed as primitives in the model similar to preferences and endowments.[3]

The rational learning literature asserts that agents can not disagree indefinitely (Aumann (1976) and Blackwell and Dubins (1962)). However, in order for convergence to occur agents must have either sufficient time to exchange information or a sufficient amount of objective data. Miller and Sanchirico (1997) show that convergence of heterogenous priors may never occur if the prior beliefs are drawn from distributions that are not absolutely continuous with respect to each other.[4]

Numerous papers have used heterogenous priors. For example, Allen and Gale (1999) examine how heterogenous priors affect new firm financing, Garmaise (2001) considers heterogenous priors in security design, and Coval and Thakor (2005) show how heterogenous priors can give rise to financial intermediation. Kandel and Pearson (1995) argue that the evidence of trading volume around public information announcements is consistent with agents interpreting the same information differently. Boot, Gopalan, and Thakor (2005a) use heterogenous priors between the manager and the investors to show when the manager will choose to be a public or private firm. Dittmar and Thakor (2005) use heterogenous priors to show that the manager is likely to issue equity when the market is most likely to be aligned with the managers view. Smith (2005) shows that disagreement between the manager and the shareholder leads to the market affecting the firm's costs.

The standard argument for the use of homogenous priors is that by limiting modelling flexibility a certain level of discipline is maintained (Samuelson (2004)). We agree that care must be taken

[3]Kreps (1990) (p. 370) notes, "First, it is conventionally assumed that all players share the same assessments over nature's actions. This convention follows from deeply held 'religious' beliefs of many game theorists. Of course, one hesitates to criticize another individual's religion, but to my own mind this convention has little basis in philosophy or logic. Accordingly, one might prefer, being more general, to have probability distributions ρ and ρ_t, which are indexed by i, reflecting the possibly different subjective beliefs of each player."

[4]Irrational explanations for indefinite divergence of priors include the the propensity to view problems as unique and ignore history (Kahneman and Lovallo (1993)) and ignoring information which contradicts earlier beliefs (White (1971)).

to avoid too much modelling flexibility. Our results provide insights that are not found with either asymmetric information or agency problems.

We are assuming rational beliefs as defined by Kurz (1994a, b) , who provides a theoretical foundation for heterogenous priors. Two key aspects of Kurz's theory are that agents have different priors and that all these priors are consistent with the data in the sense that none can be precluded by historical data. Innovative projects, with a paucity of hard historical data and non-stationary distributions, will *not* be able to *uniquely* derive the precision of the signal for historical data, leaving many potential different distributions of precision that are consistent with the data.

The extent that the investors are likely to agree with the probability of success of the innovative project is modeled by the random variable $\tilde{\rho}$, which has density $f(\tilde{\rho})$ and cumulative distribution $F(\tilde{\rho})$. ρ reflects the amount of disagreement over the probability of success. A common public signal, S, is is realized declaring whether or not the state of the world will be good or bad. Prior to the signal, each agent draws a belief, $\tilde{\theta} \in \{\text{informative}, \text{uninformative}\}$, which guides how the prior will updated when the common public signal is observed. More concretely, agreement is defined as $P(\theta^I = \text{informative}|\theta^M = \text{informative}, S = G) = \rho$ and $P(\theta^I = \text{uninformative}|\theta^M = \text{informative}, S = G) = 1 - \rho$. It is assumed that an uninformative signal does not cause the agent to update beliefs at all. In other words, if the S is good and θ is uninformative the NPV is still believed to be $D^{\text{inn}} < 0$. On the other hand, an informative signal is believed without question. If the signal is good and informative the belief is that that state will occur with certainty.

In other words, ρ represents how likely the investors will agree with the manager that the NPV will be H^{inn}. High ρ implies a high level of alignment between investors and the management whereas low ρ implies a low level of alignment.

Equityholders may investigate the high state payoff of either the innovative or mundane project. Bondholders do not investigate in such detail since they only receive a fixed payment and do not participate in the upside. Equityholders are the residual claimants and may be more concerned how good the upside is. However, during the course of the evaluation the equityholders may perceive bad news on either the risky innovative or risky mundane projects. The manager believes that with probability Ψ the equity investors will concur with H^{inn} and with probability $1 - \Psi$ the investors will believe the payoff is D^{inn} (for the innovative project). Gomes and Phillips (2005) also argue that learning from due diligence is more likely to affect equity than debt.

3.3 Investor Participation and Trading

Heterogeneity across investors is modeled assuming that the public market is comprised of N investors and that the ρ_i for each investor at any point in time is a random draw from a continuous probability distribution $F(\rho_i)$ with density $f(\rho_i)$ and support $[\rho_{li}, \rho_{hi}]$, where i represents the fact that different projects can and will have different supports. For instance, a slightly innovative project where there is little to disagree about and lower upside payoff will have a a different support than a volatile innovative project with large high payout potential. After the financing and project choice announcement trading occurs for liquidity reasons. This trading is exactly what reveals the noisy signal of disagreement, ρ_i, to the manager, revealing the mean and dispersion of the disagreement variable. In general, trading will reveal the maximal heterogenous investor. That is, the investors with the highest valuation. However, we will show that there exists times where a coalition of private equity investors optimally "sit out" and do not participate in the public market.

3.4 Sequence of Events

At t=0, the manager must decide what type of financing to raise debt or equity and whether or not to use public or private markets. The firm has assets in place that will pay off an agreed amount V at t=4. At t=1, the firm announces whether the risky innovative or risky mundane project will be undertaken. Everyone knows that there exists a lemon project. The private equity investors decide optimally to participate in trading or abstain at t=1.

At t=2, trading reveals a noisy signal of ρ_i and its dispersion. Funding is requested at t=3. If provided, investment is made and the payoffs are realized at t=4. Figure 1 contains the sequence of events.

[Insert Figure 1]

4 Model Details

A number of assumptions are necessary to isolate the cases considered. It is assumed that the common public signal is S=G and that the manager draws an informative prior, θ = informative, implying she believes the probability of H^{inn} occurring is one. This is the case to be analyzed. If the public signal is bad, everyone agrees no innovative investment should be made. If the investors believe that the signal is informative, while the manager believes that it is not, the assumption is that the project will not be undertaken.[5] The idea is that the manager is critical to project completion and realization of the NPV.

Assumption 1 below guarantees that an asset substitution moral hazard problem exists with debt.

Assumption 1.

$$\int_{I-V}^{\infty} [\xi + V - I] f(\xi) d(\xi) > p^M H^M + (1 - p^M) L^M + V - I \tag{1}$$

holds if $I - V$ is large enough and the variance of $\tilde{\xi}$ is great enough.

Intuitively, assumption 1 states that if the lemon project is sufficiently volatile and/or the initial investment is relatively large, then the manager will be willing to take the gamble at the expense of the debt holders with the lemon project.

4.1 Public Market

All of the analysis that follows is for the innovative project. Recall that the risky mundane is not subject to disagreement other than the equity investors discovering negative news about the upside. For the initial analysis of the public and private market it is assumed that the corresponding ρ is taken as given. When the manager raises $\$I$ in the public market, he offers a fraction of ownership, $1 - \alpha_{pub}$, of date 4 cash flows in return for the $\$I$. The expected value of the investors' $1 - \alpha_{pub}$ share evaluated at date 0 by investors is

$$\begin{aligned} V_{pub}^I(\alpha_{pub}) &= [1 - \alpha_{pub}] \{ p\theta\rho_{pub}\Psi H + p\theta(1 - \rho_{pub}\Psi)\eta_{pub} D + I \} - L \\ &= [1 - \alpha_{pub}] W_{pub}^I - L \end{aligned} \tag{2}$$

where $W_{pub}^I = p\theta\rho_{pub}\Psi H + p\theta(1 - \rho_{pub}\Psi)\eta_{pub} D + I$ is the investors's assessment of firm value. The first term applies to the case when the signal, S, is good and both the manager and the investors draw informative priors and agree that the NPV is H. Ψ represents the upside scrutiny that the equity

[5] See Boot, Gopalan, and Thakor (2005a) for the explicit characterization when the results are unchanged when the manager is not critical for the project.

investors will subject the either the risky mundane or the innovative project to. Our intention is that a completely innovative project will have large amount of total uncertainty modeled through larger variance in ρ. That is, both the upside and downside are very uncertain. However, with a mundane project everyone may agree that the downside case is not very uncertain, but equityholders may still uncover bad news about the upside payoff. The second term, $p\theta(1-\rho_{pub}\Psi)\eta_{pub}D$, is the expected value when the signal is good, the manager draws an informative prior, and the investors draw an uninformative prior. In this disagreement state, the manager decides to invest with probability η_{pub} which leads the investors to assess the NPV as D. With probability $1-\eta_{pub}$, the investors get to decide, in which case there is no investment and the NPV=0. The NPV is also zero when the signal is bad and no investment occurs. The third term in W^I_{pub} is the initial investment and when added to the NPV yields firm value. L represents any liquidity costs and/or costs associated with issuance. In the public equity market, L=0.

The manager's evaluation of the firm at date 0 is

$$\begin{aligned} V^M_{pub}(\alpha_{pub}) &= \alpha_{pub}\left\{p\theta\rho_{pub}\Psi H + p\theta(1-\rho_{pub}\Psi)\eta_{pub}H + I\right\} \\ &= \alpha_{pub}W^M_{pub} \end{aligned} \tag{3}$$

where $W^M_{pub} = p\theta\rho_{pub}\Psi H + p\theta(1-\rho_{pub}\Psi)\eta_{pub}H + I$ is the manager's assessment of firm value. This expression is similar to the expression for the value of the investors except that the manager believes the NPV will be H. The only question that remains is whether or not she will be able to choose to take on the project which occurs with probability, η_{pub}.

The manager solves the following maximization problem:

$$\max_{\eta_{pub},\alpha_{pub}} V^M_{pub}(\alpha_{pub}) \tag{4}$$

$$\text{s.t. } V^I_{pub}(\alpha_{pub}) \geq I \tag{5}$$

$$\text{and } \alpha_{pub}, \eta_{pub} \in [0,1] \tag{6}$$

The manager chooses both the portion of ownership and the degree of autonomy to maximize the objective function. Equation (5) is the individual rationality (I.R.) constraint for the investors and equation (16) is the feasibility requirement.

In equilibrium, the I.R. constraint is binding since both α_{pub} and η_{pub} increase improving the manager's objective function and lowering the value to the investor. This immediately implies

$$\alpha^*_{pub} = 1 - \frac{I+L}{W^I_{pub}} \tag{7}$$

Now we can formally state the direct implications of the optimal α^*_{pub} that satisfies (7).

Lemma 1. *When the manager finances in the public market, the ownership portion retained, α^*_{pub}, is increasing in the agreement parameter ρ_{pub} and decreasing in the autonomy parameter η_{pub} and decreasing in liquidity costs, L.*

Lemma (1) characterizes the trade-off between ownership retained and the managerial autonomy parameter. Managerial autonomy is desirable, but costly since the manager may make a decision that the investors believe is value destroying increasing their required return, $1 - \alpha^*_{pub}$.

Substituting α^*_{pub} into equation (4), the manager's objective becomes

$$\max_{\eta_{pub}} \left[1 - \frac{I+C}{W^I_{pub}}\right] W^M_{pub} \tag{8}$$

The following assumption guarantees that the optimal η_{pub} will not take extreme values.

Assumption 2.

$$K \in \left(1, 1 - \frac{p\theta(1-\rho_{pub}\Psi)DH}{t}\right) \tag{9}$$

where

$$K \equiv \sqrt{\frac{H(I + p\theta\rho_{pub}\Psi H)}{I(H-D)}}$$

and

$$t \equiv \sqrt{H(I+C)(H-D)(I+p\theta\rho_{pub}\Psi H)}.$$

K is a measure of the attractiveness of the project to investors; it is increasing in H, the NPV in the agreement state. If the project is sufficiently unattractive to investors, so much of the ownership will have to be sold to investors to raise the necessary capital that they will have claim to most of the project cash flows. It then becomes too costly to give autonomy to the manager and $\eta^*_{pub} = 0$. If the project is very attractive, it is not very costly to give control to the manager and $\eta^*_{pub} = 1$.

Assumption 2 rules out the boundary cases.

Proposition 1. *The unique globally optimal value of managerial autonomy with public ownership, η^*_{pub}, is*

$$\eta^*_{pub} = \frac{-H(I + p\theta\rho_{pub}\Psi H) + t}{p\theta(1 - \rho_{pub}\Psi)HD} \tag{10}$$

The corresponding optimal ownership retained by the manager, α^*_{pub} is

$$\alpha^*_{pub} = \frac{t - H(I + L)}{t} \tag{11}$$

Proposition 1 characterizes the optimal of both managerial autonomy and managerial ownership. Note that, when L=0, these are the optimal values in the public equity market with $\eta_{pub} = \eta^*_{pub}$. See Boot, Gopalan, and Thakor (2005a) for additional details. It is assumed here that public market governance is the amount chosen when this condition is satisfied.

Lemma 2. *The optimal autonomy parameter η^*_{pub} is increasing in the agreement parameter $\rho_{pub}\,\forall L \in [0, \infty)$ and decreasing in the expected cost $L\,\forall \rho_{pub} \in [0, 1)$.*

The more likely it is the manager and investors are to agree with project decisions the less costly it is give to manager autonomy. Similarly, the less liquid the shares are the more costly it is to give the manager autonomy.

4.2 Private Market

In this section, we analyze the benefit that can be provided to the manager when private equity investors have an agreement parameter greater than that in the public market, $\rho_{pe} \geq \rho_{pub}$. In the private equity market the private equity investor has all the bargaining power and thus solves a similar maximization problem. The expected value of the investors' $1 - \alpha_{pe}$ share of the firm at date 4 evaluated at date 0 by investors is given in the following:

$$\begin{aligned} V^I_{pe}(\alpha_{pe}) &= [1 - \alpha_{pe}]\,\{p\theta\rho_{pe}\Psi H + p\theta(1 - \rho_{pe}\Psi)\eta_{pe}D + I\} \\ &= [1 - \alpha_{pe}]\,W^I_{pe} \end{aligned} \tag{12}$$

Equation (12) is analogous to equation (2) for the public investor except the optimal ownership and control allocations will general be different.

The expected value of the manager's α_{pe} share evaluated at date 0 by the manager is

$$\begin{aligned} V_{pe}^{M}(\alpha_{pe}) &= \alpha_{pe}\left\{p\theta\rho_{pe}\Psi H + p\theta(1-\rho_{pe}\Psi)\eta_{pe}H + I\right\} \\ &= \alpha_{pe}W_{pe}^{M} \end{aligned} \tag{13}$$

The private equity investors solve the following maximization problem:

$$\max_{\eta_{pe},\alpha_{pe}} V_{pe}^{I}(\alpha_{pe}) - I - C \tag{14}$$

$$\text{s.t. } V_{pe}^{M}(\alpha_{pe}) \geq V_{pub}^{M}(\alpha_{pub}) \tag{15}$$

$$\text{and } \alpha_{pe}, \eta_{pe} \in [0,1] \tag{16}$$

The costs, C, to the private equity investors are those associated with finding a coalition of other private equity investors and any liquidity costs associated with holding private equity. Private equity investors choose the optimal ownership and managerial autonomy, but must guarantee that the manager does at least as well as she would do in the public market. Define $T \equiv \alpha_{pub}^{*}W_{pub}^{M}(\eta_{pub}^{*}) = V_{pub}^{M}(\alpha_{pub}^{*})$ as the manager's threat point. The threat point is nothing more than what the manager can get in the public market. If not at least as well off, the manager will raise funds in the public market. Since the private equity investors have all the bargaining power, the individual rationality (I.R.) condition given in equation (15) holds with equality implying the following simplified problem:

$$\max_{\eta_{pe}} \left(1 - \frac{T}{W_{pe}^{M}}\right) W_{pe}^{I} \tag{17}$$

This implies the following lemma:

Lemma 3. *With private equity financing the ownership portion the manager retains α_{pe}^{*} is decreasing in the agreement parameter ρ_{pe}, decreasing in the managerial autonomy parameter η_{pe}, and increasing in the threat point T.*

Lemma 3 is analogous to Lemma 1 in the public market. In this case, the higher the level of agreement the private equity investors have to offer the manager, the lower the ownership portion necessary to satisfy the I.R. constraint, ceteris paribus. Similarly, increasing the amount autonomy given to the manager implies that a smaller ownership portion will meet the constraint.

The following assumption guarantees that η^*_{pe} will not take extreme values.

Assumption 3.

$$\widehat{K} \in \left(1 + \frac{p\theta(1-\rho_{pe}\Psi)DH}{\hat{t}}, 1\right) \tag{18}$$

where

$$\widehat{K} \equiv \sqrt{\frac{-D(I + p\theta\rho_{pe}\Psi H)}{T(H-D)}}$$

and

$$\hat{t} \equiv \sqrt{-DT(H-D)(I + p\theta\rho_{pe}\Psi H)}.$$

Assumption 3 is analogous to assumption 2 for the public market and simply guarantees that parameters are such that the private equity will not set η^*_{pe} or α^*_{pe} to 0 or 1. Given later results, it is simple to show that when $\rho_{pe} = \rho_{pub}$ both assumption K and assumption $\widehat{K}$ are identical.

Proposition 2. *The unique globally optimal value of managerial autonomy with private equity, η^*_{pe}, is*

$$\eta^*_{pe} = \frac{-D(I + p\theta\rho_{pe}\Psi H) - \hat{t}}{p\theta(1-\rho_{pe}\Psi)HD} \tag{19}$$

The corresponding optimal ownership retained by the manager, α^*_{pe} is

$$\alpha^*_{pe} = \frac{-DT}{\hat{t}} \tag{20}$$

In the public market analysis, η_{pub} was shown to be increasing in ρ_{pub}. In the private equity market, η_{pe} is not so simply to analyze. The following lemma characterizes the marginal behavior of η_{pe}.

Lemma 4. *η_{pe} is increasing in ρ_{pe} if and only if the following condition holds:*

$$\frac{p\theta H\left[\frac{1+\rho_{pe}\psi}{2}\right] + I}{p\theta H + I} > \widehat{K}. \tag{21}$$

Otherwise, η_{pe} is decreasing in ρ_{pe}.

If the level of alignment between public investors and the manager is low, then the benefit of higher alignment is very large and the private equity investors can decrease *both* the manager's ownership and managerial autonomy. If the level of alignment is high in public markets, then the benefit of higher alignment is not as large and the private equity investors will decrease the manager's ownership portion, but must *increase* the autonomy.

In order to analyze if any parameters exist where η_{pe} is increasing in ρ_{pe} and if any parameters exist where η_{pe} is decreasing in ρ_{pe}, we ran simulations. Both cases are possible for plausible parameters. The intuition is fairly simple. The private equity investor must have at least as great an agreement parameter as the public market otherwise the manager can not be given her threat point. Lemma 5 below shows that the ownership portion is decreasing in the private equity agreement parameter. Thus, something must shift to get the manager to the threat point. If ρ_{pub} is low, then the added benefit of higher agreement is so great that private equity investors will actually *decrease* the managerial autonomy and lower the ownership portion. If ρ_{pub} is high, then the added benefit of higher agreement isn't so great and the private equity equity investors will increase the autonomy parameter to compensate for the reduction in ownership. The degree to which this statements remain true depend on all the parameters. In particular, the more valuable the high payout is relative to the ex-ante expected value, the lower ρ_{pub} can be with η_{pe} increasing in ρ_{pe}. If the payout is relatively low, then the level of ρ_{pub} necessary for η_{pe} to be increasing is much higher. Figure 2 presents four simulations with the following parameters: H=10, D=-5, I=1, p =0.5, $\psi = 1$, and $\theta = 0.5$, and various ρ_{pub}.

[Insert Figure 2]

Notice that panels A and B in figure 2 are decreasing in ρ_{pe} while panels C and D are increasing. Panels A and B both have lower public agreement while C and D have higher public agreement.

4.3 Private Equity Coalition

Private equity investors can choose to trade in the public market or wait and participate in the coalition of private equity investors. In other words, if they form a coalition they will cooperate with each other and if they trade in the public market they will compete. Ramakrishnan and Thakor (1984) analyze cooperation versus competition in an agency setting. The main result is that if the conditional correlation of outputs is low enough, cooperation is preferred, whereas, if the conditional correlation is high enough, competition is preferred. In our model, each private equity investor has a high degree of alignment with the manager and thus one could think that the payoffs are highly correlated. This would imply that competition is preferred by the principal (in this case the manager). By forming a coalition, competition is weakened providing rents to the coalition members. The manager would prefer competition since all the surplus is received. Liquidity, trading volume, and competition in the public equity market will leave each private equity investor with marginal surplus, which we assume to be zero. However, participating in the private equity coalition and personalizing the deal allows the coalition to retain the surplus. In other words, the private equity coalition has all the bargaining power in so much as the manager is willing to participate (receives at least her threat point). Thus, the private equity investors will form a coalition if and only if the surplus gained is greater than the cost of finding and forming the coalition plus any reduction in liquidity costs. In what follows, it is assumed that the total cost for private equity investors to form a coalition is C.

Holmström (1982) focuses on free riding and competition in teams. His main finding is that the principal must administer incentive schemes that do not balance the budget to eliminate free riding. Flat wages with bonuses is one example. These results do not hold if the principal participates in providing the inputs. In our model, each member of the coalition participates and thus there potentially exists a free rider problem. However, the repeated nature of the coalitions should eliminate most of this problem, as well as, stop market trading if the deal goes sour.

The private equity coalitions in our model could also be thought of as loan syndicates. There are two alternative hypotheses regarding the agency structure of loan syndicates. The specialization hypothesis claims that multiple co-agents arise in loan syndicates because of different competitive advantages in performing the administrative tasks. The monitoring hypothesis states that multiple co-agents in loan syndicates are the result of delegation of monitoring that mitigates information asymmetry problems (See Das and Nanda (1999) and Strauss (1997), respectively). Francois and Missonier-Piera (2005) find empirical support for the specialization hypothesis. Dennis and Mullineaux (2000) find that information and agency problems affect the extent to which a loan is "transaction oriented" rather than "relationship oriented" for debt claims. Although there may be other gains from forming the coalition, the premise of this paper is that the primary motivation is to avoid the competition in the public market and to extract the surplus from the manager consistent with the theory developed by Ramakrishnan and Thakor (1984).

Interestingly, the manager receives all the surplus if the high alignment private equity investors compete. With the coalition, the manager receives strictly less than competition provides.[6] This insight suggests that the manager is being forced into a more costly private equity situation by the coalition. Of course, the manager technically is optimally choosing to issue private equity, but in actuality her "back is against the wall".

5 Analysis

The analysis is completed using backward induction. Since only the payoffs are realized at t=4, the analysis will start with t=3. At t=3, the manager attempts to raise funding. Investors will either supply the funding or decline based on the amount of actual disagreement and payoffs. At time t=2, the manager must decide which project to pursue and the private investor must decide whether to participate in the public market or attempt to form a coalition of private equity investors. The following two results are necessary before considering actual participation and issuance decisions.

Proposition 3. *When the level of agreement in the private equity equity market equals the level in the public market, that is, $\rho_{pe} = \rho_{pub}$, then the private equity investor will optimally choose the exact amount of managerial ownership and control as the manager receives in the public market. In other words, $\eta^*_{pe} = \eta^*_{pub}$ and $\alpha^*_{pe} = \alpha^*_{pub}$.*

Proposition (3) allows us to make a fruitful analysis of the benefit of the increased level of agreement provided by the private equity investor. The comparative statics derived in the previous section do not allow for a comparison between the change from the public level of ownership and control to the private level of ownership and control. For example, with this result and lemma 5 below, it can be concluded that the increase in agreement lowers the ownership portion *relative* to public markets. This is in contrast to only being able to conclude that the ownership portion is less relative to what it would have been with a lower amount of private equity agreement.

Lemma (3) showed that when the ownership portion is constant an increase in agreement lowers the ownership portion that the manager retains. With the private equity investor controlling *both* the ownership portion and managerial control parameter and with the control parameter increasing

[6] In the paper, the coalition has all the bargaining power implying the manager only receives his threat point and nothing more. It is a simple exercise to show that as long as the coalition is willing to participate the manager will receive strictly less than in the competitive market since the coalition must cover formation costs.

for some parameters and decreasing for others, η_{pe} can not be considered as constant for a change in ρ_{pe}. The following lemma addresses this issue.

Lemma 5. *With private equity, an increase in ρ_{pe} decreases the ownership portion that the manager retains, α^*_{pe}. Furthermore, $\alpha^*_{pe} \leq \alpha^*_{pub}$.*

Intuitively, when the private equity investor can offer a higher level of agreement the manager is kept at the threat point, T, by either decreasing the ownership portion or decreasing the managerial control or some combination of the two. Lemma (5) shows that the private equity investor always finds it optimal to decrease the ownership percentage. Proposition (3) demonstrates that if the levels of agreement are equal the private equity investor will choose the same level of ownership as the manager receives in the public market. Since the private equity investor can never meet the threat point with a lower amount of agreement, it follows that when private equity is present the amount of agreement must be at least as large as public markets. The manager does not have the bargaining power and the cost paid to the private equity investor for the greater amount of agreement is always a smaller ownership portion. An interesting note here is that the manager is actual increasing the costs of capital in order to be able to maximize the value of the firm.

5.1 Investor Participation with Different Projects

Earlier ρ_i was discussed as being drawn from a distribution thereby creating heterogeneity amongst investors. Let's consider how the expected level of agreement, $E(\tilde{\rho}_i) \equiv \bar{\rho}_i$ is determined. When the manager requests funding at t= 3, the firm's shares will be held by the maximal investor unless a coalition of private equity investors are abstaining from participation. The reason why the maximal valuation investor will be holding the stock is that investors with lower valuations will sell there shares to those who value it more. It is assumed that at each class there is enough wealth to purchase the entire issue.

With ρ_i varying cross-sectionally from a probability distribution, the ρ_i of the maximal investor will be the highest draw of ρ_i from amongst the N investors. In other words, the N^{th} order statistics. If ordered from low to high, $\rho_{1i} \leq \cdots \leq \rho_{Ni}$, ρ_{Ni} is the maximal investor. Recall that ρ_i has cumulative distribution, $F(\rho_i)$, with density $f(\rho_i)$ and support $[\rho_{li}, \rho_{hi}]$, where i represents the fact that different projects can and will have different supports. Galton and Pearson (1902) show that the expected value of the N^{th} order statistic is given by the following:

$$E(\rho_{Ni}) = \frac{Ni!}{(Ni-1)!} \int_{\rho_{li}}^{\rho_{hi}} F(\rho_{Ni})^{Ni-1} f(\rho_{Ni}) \rho_{Ni} d(\rho_{Ni}) \tag{22}$$

Lemma 6 follows from equation (22).

Lemma 6. *For each group i, the expected value of the N^{th} order statistic is increasing in N, that is, $\frac{\partial \bar{\rho}_i}{\partial Ni} > 0$. The variance of ρ_{Ni}, σ^2_{Ni}, is decreasing in Ni as long as $T(Ni) = \left(\frac{f\left[F^{-1}\left(\frac{1}{Ni+1}\right)\right]}{Ni} \right)^{-1}$ is non-increasing in Ni.*

Generally speaking we assume that different projects may have different expected levels of agreement and different levels of dispersion. This is accomplished by assumed that there are low agreement and high agreement projects as well as low dispersion and high dispersion projects. Statistically, this is accomplished simply by varying the support of the draw from distribution. Intuitively, a project that everyone is familiar with and has large amounts of history to guide in the prior formation should have much less disagreement than a completely innovative new idea that no one has seen before. Similarly, there can be different levels of uncertainty in both high or low agreement projects. Our purpose here is to analyze the preferred issuance choice for the manager under each possible scenario, as well as, when the coalition of private equity investors would participate.

Ex-ante the manager faces three different scenarios:

1. High expected value of $\overline{\rho}_{_{N1}}$ with any amount of dispersion, $\sigma^2_{_{N1}}$.
2. Low expected value of $\overline{\rho}_{_{N2}}$ with a large amount of dispersion, $\sigma^2_{_{N2}}$.
3. Low expected value of $\overline{\rho}_{_{N3}}$ with a small amount of dispersion, $\sigma^2_{_{N3}}$.

Let's look at each of the scenarios in order beginning with scenario (1).

5.2 Scenario 1: High expected agreement

In this instance, $\overline{\rho}_{_{N1}}$, is characterized by a high amount of expected agreement with a large amount of volatility. The manager finds it very likely that the public equity market will agree with the decision to proceed with the innovative project. Thus, we have the following result:

Proposition 4. *Given* $\overline{\rho}_{N1}$, σ^2_{N1}, *and* ψ *close enough to 1, there exist* $\widehat{C}_{N1}$ *such that for all* $C > \widehat{C}_{N1}$, *public equity will be chosen.*

Intuitively, we have by assumption that $H^{\text{mun}} < H^{\text{inn}}$ implying that the manager would prefer the innovative project. Since $\bar{\rho}_{N1}$ is relatively large it becomes to costly for the private equity investors to absorb the cost, C, of forming a coalition. Note that this cost is decreasing in dispersion. This follows since it becomes less costly to find other investors to form the coalition. Public markets will not be preferred unless it is very likely that the upside investigation results in a ψ value that is large enough.

5.3 Scenario 2: Low expected agreement with a large amount of dispersion

$\overline{\rho}_{N2}$ is the case most likely to see a private equity coalition formed and the manager issue private equity shares. A low agreement parameter is bad for the manager, lowering both the ownership portion and managerial autonomy parameter in the public markets. However, in this case the dispersion is great making it very likely that a private equity investor exists and can provide a higher level of agreement allowing the manager to make the decision believed correct much more frequently.

Proposition 5. *Given* $\overline{\rho}_{N2}$, σ^2_{N2}, *and* ψ *close enough to 1, there exist* $\widehat{C}_{N2}$ *such that for all* $C < \widehat{C}_{N2}$, *private equity will be chosen. In addition,* $\widehat{C}_{N2} > \widehat{C}_{N1}$, *which immediately implies that private equity will be more likely in this scenario. If this is violated, then the manager will pursue the mundane project.*

When $\overline{\rho}_{N2}$ is low, it is very unlikely that the public equity market will provide funding. In other words, it is very likely that the investors will disagree with the decision to invest, but with a high amount of variance it is very likely that the a coalition of private equity investors will exist. In addition, since the dispersion is great, it should be less costly to form the coalition. As long as the costs of forming the coalition and the liquidity costs are low enough then the manager will choose private equity.

5.4 Scenario 3: Low expected agreement with a small amount of dispersion

$\overline{\rho}_{N3}$ is low indicating that the public equity market will likely not agree with the decision to proceed with the project and not provide funding. Further, with a small amount of dispersion there isn't likely to be a private equity investor in the market who can make enough to cover the costs of forming a coalition. In this scenario, the mundane project will be selected. Further, if ψ is low, public debt will be used unless the asset substitution moral hazard potential is too great then the private debt market will be used since monitoring can occur more easily.

Proposition 6. *Given $\overline{\rho}_{N3}$, σ^2_{N3}, and ψ close enough to 1, there exist $\widehat{C}_{N3}$ such that for all $C > \widehat{C}_{N3}$ the mundane project will be selected. If ψ is low, then debt markets will be accessed. Further, provided the asset-substitution moral hazard is large, then private debt markets will be used. Otherwise, public debt markets will be used.*

6 Empirical Predictions

There are several empirical predictions that follow from the model. As mentioned in the previous work with joint control and private equity in Boot, Gopolan, and Thakor (2005a, 2005b), finding proxies for agreement between shareholders and management, as well as, proxies for managerial autonomy are difficult. Nevertheless, several recent papers have taken on the task of using agreement proxies. For instance, Dittmar and Thakor (2005), Faulkender, Milbourn, and Thakor (2005), and Smith (2005).

1. Proposition 6 predicts that when agreement is low and that there is lot of uncertainty, then private equity will be the most likely choice of financing for the innovative project. Smith (2005) and Faulkender, Milbourn, and Thakor (2005) both document the link between low stock prices and low agreement. Thus, when stock prices are low, private equity should be used more frequently. This would occur either because the coalition of private equity investors has a higher agreement parameter or has the ability to screen the true ability of the manager.

2. During times when private equity issuance is most prevalent, the cost of capital should be relatively large. In other words, since the ownership portion retained is small, the cost

of capital is relatively large. This will be manifested through the returns private equity intermediaries expect to receive.

3. Proposition 4 demonstrates that public equity will be the choice security when expected agreement is high. This provides an alternative to the market timing literature and lends additional theoretical support to Dittmar and Thakor (2005) who find that agreement provides incremental support above that of other issuance theories.

4. Private equity will be chosen provided that the costs are not too large. The search costs should be smaller the larger the number of potential private equity investors that exist. In particular, the anecdotal increase in private equity may be due to a cycle of increased private equity and lower search costs. Lowering the costs of forming the private equity coalition increases the number of private equity coalitions which lowers the costs of forming a coalition. This should continue until the marginal gains are surpassed by the marginal costs.

5. Proposition 6 shows that private equity is optimal when agreement is low. Thus, we should expect for public firms to use private equity more frequently for innovative projects rather than mundane projects. Incidentally, this is at precisely the time when the market price is low and the public market is skeptical of the existence of an innovative project. Private equity intermediaries may reduce the trough of business cycles. Without private equity intermediaries these innovative projects would be rejected thereby exacerbating the downside of the business cycle and slowing the recovery of the economy.

7 Conclusion

We present a model that highlights the choice between debt, private equity, and public equity. We are addressing two questions:

1. How do public firms choose from among these three alternatives?

2. How and why do private equity funds emerge as intermediaries?

Private equity serves a valuable function in financial markets as a financial intermediary. Private equity investors offer a higher level of agreement in exchange for the surplus the manager would enjoy in the public market. If the costs of forming the coalition and reduction in liquidity are less than the surplus the private equity coalition will gain. In some instances, it may allow the manager to invest in the more lucrative innovative project, when the alternative would have been the mundane project.

Private equity is most likely to occur when the public equity (more liquid) has a low level of alignment with the manager. In this state, private equity intermediaries with high levels of alignment have a lot to offer to the firm in exchange for the surplus created.

Appendix

Proof of Lemma 1. Simplifying equation (7), we have

$$\alpha^*_{pub} = 1 - \frac{I + L}{p\theta\rho_{pub}\Psi H + p\theta(1 - \rho_{pub}\Psi)\eta_{pub}D + I} \tag{23}$$

Since $D < 0$, it immediately follows that α^*_{pub} is decreasing in η_{pub} and increasing in ρ_{pub}. Further inspection reveals α_{pub} is decreasing in the cost incurred to the investors, L. □

Proof of Proposition 1. Recall the manager's reduced problem from equation (8) is

$$\max_{\eta_{pub}} \left[1 - \frac{I + L}{W^I_{pub}}\right] W^M_{pub}$$

We initially ignore the boundary conditions on η_{pub} and take the first order conditions with respect to η_{pub} which yields

$$\frac{(I + L)p\theta(1 - \rho_{pub}\Psi)DW^M_{pub}}{\left(W^I_{pub}\right)^2} + p\theta(1 - \rho_{pub}\Psi)H\left[\frac{W^I_{pub} - (I + L)}{W^I_{pub}}\right] = 0. \tag{24}$$

This implies

$$(I + L)DW^M_{pub} + HW^I_{pub}\left[W^I_{pub} - (I + L)\right] = 0 \tag{25}$$

Expanding the equation gives a quadratic equation with two real positive roots. The smaller root corresponds to the maximum given in equation (10). The larger root yields $W^I_{pub} < 0$ and is thus not feasible. Assumption 2 guarantees that η^*_{pub} and α^*_{pub} are not boundary solutions.

Now consider the second-order condition to establish the objective function is globally concave yielding a unique maximum. Differentiating (24) yields

$$\begin{aligned}&\frac{(I + L)p^2\theta^2(1 - \rho_{pub}\Psi)^2DH\left(W^I_{pub}\right)^2 - 2(I + L)W^I_{pub}W^M_{pub}p^2\theta^2(1 - \rho_{pub}\Psi)^2D^2}{\left(W^I_{pub}\right)^4} \\ &+ p^2\theta^2(1 - \rho_{pub}\Psi)^2DH\left[\frac{I + L}{\left(W^I_{pub}\right)^2}\right]\end{aligned} \tag{26}$$

since $D < 0$ and $W^I_{pub} > I$ and $W^M_{pub} > 0$ it immediately follows that $\frac{d^2 V^M_{pub}}{d\eta^2_{pub}} < 0$. □

Proof of Lemma 2. Differentiate η_{pub} in (10) with respect to ρ_{pub} yields the following:

$$\frac{\partial \eta^*_{pub}}{\partial \rho_{pub}} = \frac{\left(-Hp\theta\Psi H + \frac{\partial t}{\partial \rho_{pub}}\right) p\theta(1-\rho_{pub}\Psi)HD + p\theta\Psi HD\left[-H(I + p\theta\rho_{pub}\Psi H) + t\right]}{\left(p\theta(1-\rho_{pub}\Psi)HD\right)^2} \tag{27}$$

Since $D < 0$ the above equation is greater than zero if and only if

$$\left(Hp\theta\Psi H - \frac{\partial t}{\partial \rho_{pub}}\right)(1-\rho_{pub}\Psi) + \Psi[H(I + p\theta\rho_{pub}\Psi H) - t] > 0 \tag{28}$$

Substituting $\frac{\partial t}{\partial \rho_{pub}}$ from the definition in Assumption 2, we have

$$\left(Hp\theta\Psi H - \frac{p\theta\Psi H^2(I+L)(H-D)}{2t}\right)(1-\rho_{pub}\Psi) + \Psi[H(I + p\theta\rho_{pub}\Psi H) - t] > 0 \tag{29}$$

Simplifying equation (29) implies the following

$$2tH[I + p\theta H] > 2t^2 + p\theta H^2(I+L)(H-D)(1-\rho_{pub}\Psi) \tag{30}$$

Substituting t^2 and simplifying yields

$$t[2I + 2p\theta H] > (I+L)(H-D)[2I + p\theta H(1+\rho_{pub}\Psi)] \tag{31}$$

Since $\rho_{pub}\Psi < 1$ it is sufficient to show that

$$t > (I+L)(H-D) \tag{32}$$

Substituting for t from Assumption 2 we see that equation (32) will hold if $K^2 > 1$, which we know is true from Assumption 2. Thus, it follows that $\frac{\partial \eta^*_{pub}}{\partial \rho_{pub}} > 0$. Furthermore, since t is increasing in L and $D < 0$, we have $\frac{\partial \eta^*_{pub}}{\partial L} < 0$. □

Proof of Lemma 3. The binding individual rationality constraint implies the following, we have

$$\alpha^*_{pe} = \frac{T}{p\theta\rho_{pe}\Psi H + p\theta(1-\rho_{pe}\Psi)\eta_{pe}H + I} \tag{33}$$

Since W_{pe}^M (the denominator above) is increasing in both η_{pe} and ρ_{pe}, it immediately follows that α_{pe}^* is decreasing in η_{pe} and decreasing in ρ_{pe}. Further inspection reveals α_{pe} is increasing in the threat point, T. $\square$

Proof of Proposition 2. Recall the manager's reduced problem from equation (17) is

$$\max_{\eta_{pe}} \left(1 - \frac{T}{W_{pe}^M}\right) W_{pe}^I$$

We initially ignore the boundary conditions on η_{pe} and take the first order conditions with respect to η_{pe} which yields

$$\frac{Tp\theta(1-\rho_{pe}\Psi)HW_{pe}^I}{\left(W_{pe}^M\right)^2} + p\theta(1-\rho_{pe}\Psi)D\left[\frac{W_{pe}^M - T}{W_{pe}^M}\right] = 0. \tag{34}$$

This implies

$$THW_{pe}^I + DW_{pe}^M\left[W_{pe}^M - T\right] = 0 \tag{35}$$

Expanding the equation gives a quadratic equation with two real positive roots. The larger root corresponds to the maximum given in equation (19). The smaller root yields $W_{pe}^M < 0$ and is thus not feasible. Assumption 3 guarantees that η_{pe}^* and α_{pe}^* are not boundary solutions.

Now consider the second-order condition to establish the objective function is globally concave yielding a unique maximum. Differentiating (34) yields

$$\begin{aligned} &\frac{Tp^2\theta^2(1-\rho_{pe}\Psi)^2DH\left(W_{pe}^M\right)^2 - 2W_{pe}^IW_{pe}^MTp^2\theta^2(1-\rho_{pe}\Psi)^2H^2}{\left(W_{pe}^M\right)^4} \\ &+ p^2\theta^2(1-\rho_{pe}\Psi)^2DH\left[\frac{T}{\left(W_{pub}^I\right)^2}\right] \end{aligned} \tag{36}$$

since $D < 0$ and $W_{pe}^I > I$ and $W_{pe}^M > T$ it immediately follows that $\frac{d^2V_{pe}^M}{d\eta_{pe}^2} < 0$. $\square$

Proof of Lemma 4. Consider η_{pe}:

$$\eta_{pe}^* = \frac{-D(I + p\theta\rho_{pe}\Psi H) - \hat{t}}{p\theta(1-\rho_{pe}\Psi)HD}$$

For simplicity consider the following variation:

$$\eta_{pe}^{*} = \frac{C_1 + C_2\rho_{pe} - \sqrt{C_3 + C_4\rho_{pe}}}{C_5 + C_6} \tag{37}$$

Thus,

$$\frac{\partial \eta_{pe}^{*}}{\partial \rho_{pe}} = \frac{\left(C_2 - \frac{C_4}{2\sqrt{C_3 + C_4\rho_{pe}}}\right)(C_5 + C_6\rho_{pe}) - C_6\left(C_1 + C_2\rho_{pe} - \sqrt{C_3 + C_4\rho_{pe}}\right)}{(C_5 + C_6\rho_{pe})^2} \tag{38}$$

Simplifying and considering only the numerator we see that this first derivative is positive if and only if

$$C_4C_6\rho_{pe} - C_4C_5 + 2C_6C_3 > 2\hat{t}[C_6C_1 - C_2C_5] \tag{39}$$

Substituting for C_i and simplifying yields the following equation

$$T(H - D)\left[p\theta H(1 + \rho_{pe}\Psi) + 2I\right] > \hat{t}[2I + 2p\theta H] \tag{40}$$

Manipulating equation (7) implies the following

$$\frac{p\theta H\left[\frac{1+\rho_{pe}\psi}{2}\right] + I}{p\theta H + I} > \widehat{K} \tag{41}$$

□

Proof of Proposition 3. First, note that by assumption $\rho_{pub} = \rho_{pe}$ which we will simply denote ρ for the remainder of the proof. Note that $\eta_{pe}^{*} = \eta_{pub}^{*}$ if and only if

$$-H(I + p\theta\rho\psi H) + t = -D(I + p\theta\rho\psi H) - \hat{t}. \tag{42}$$

Since the left and right hand side above are negative multiplying by -1 yields the following positive equation

$$\hat{t} = (H - D)(I + p\theta\rho\psi H) - t \tag{43}$$

which will be true if and only if

$$\hat{t}^2 = \left[(H - D)(I + p\theta\rho\psi H) - t\right]^2. \tag{44}$$

Substituting for $\hat{t}$ and simplifying yields the following

$$-DT = (H - D)(I + p\theta\rho\psi H) - 2t + HI \tag{45}$$

Recall that

$$\begin{aligned} T &= \alpha^*_{pub} W^M_{pub}(\eta^*_{pub}) \\ &= \left[1 - \frac{HI}{t}\right]\left[p\theta\rho\psi H - \frac{H}{D}(I + p\theta\rho\psi H) + \frac{t}{D} + I\right] \end{aligned} \tag{46}$$

□

multiplying through by -D and simplifying yields

$$-DT = (H - D)(I + p\theta\rho\psi H) - \left[\frac{HI\,[H - D]\,[I + p\theta\rho\psi H]}{t}\right] + HI - t \tag{47}$$

Last note that,

$$\left[\frac{HI\,[H - D]\,[I + p\theta\rho\psi H]}{t}\right] = t. \tag{48}$$

Substituting yields

$$-DT = (H - D)(I + p\theta\rho\psi H) - 2t + HI \tag{49}$$

Since equation (49) is equal to equation (45), it follows that $\eta^*_{pe} = \eta^*_{pub}$.

Now let's consider the ownership portions. $\alpha^*_{pub} = \alpha^*_{pe}$ is true if and only if

$$\frac{t - HI}{t} = \frac{-DT}{\hat{t}} \tag{50}$$

Manipulating the above equation and using the definitions of t and $\hat{t}$ imply

$$-DT = \frac{(t - HI)^2}{HI} \tag{51}$$

Simplifying the above equation implies

$$-DT = (H - D)(I + p\theta\rho\psi H) - 2t + HI \tag{52}$$

which is equal to -DT by equation (45).

Proof of Lemma 5. Recall that $\alpha^*_{pe} = \frac{-DT}{\hat{t}}$ implying

$$\begin{aligned} \frac{\partial \alpha^*_{pe}}{\partial \rho_{pe}} &= \frac{DT}{\hat{t}^2}\left[\frac{\partial \hat{t}}{\partial \rho_{pe}}\right] \\ &= \frac{DT}{\hat{t}^4}[-DT(H-D)p\theta\psi H] < 0. \end{aligned} \tag{53}$$

□

Proof of Lemma 6. The proof presented here follows from Boot, Gopalan, and Thakor (2005b). Suppressing i, the distribution function of ρ_{Ni}, $Q(\rho_{Ni})$ can be written as $Q(\rho_N) = F^N(\rho)$ where $F(\rho)$ is the distribution function of ρ. Since $F(\rho) \leq 1$, it immediately follows that for any two values of N, say N_1 and N_2, such that $N_1 < N_2$, $Q(\rho_{N_2})$ first order stochastically dominates $Q(\rho_{N_1 1})$. First order stochastic dominance implies $\bar{\rho}_2 > \bar{\rho}_1$ proving the first part of the lemma.

To prove the second part, first note that ρ_N has the following asymptotic distribution with zero variance:

$$F(\rho_N) = \begin{cases} 1, & \rho_N = 1 \\ 0, & \rho_N \neq 1 \end{cases}$$

The last item that needs to be shown is that for finite N the variance of ρ_N is decreasing in N. For additional details, see Gibbons (1971). First use an approximate inverse Taylor series expansion of the variance of ρ_N. That is, the variance of ρ_N is given and $var(\rho_N) = E(\rho_N^2) - E(\rho_N)^2$, where

$$E(\rho_N) = \int_0^1 F(\rho)^{N-1} f(\rho)\rho d(\rho). \tag{54}$$

We know from the probability integral transformation that $F(\rho_N) = u_N$, where u_N is the N^{th} order statistic from the uniform distribution. Thus, we can express any function of the N^{th} order statistic of any continuous distribution as a function of the N^{th} order statistic from the uniform distribution. Using this and performing a Taylor series expansion of the variance of ρ_N,we have the following

$$Var(\rho_N) \approx \frac{N}{[N+1]^2[N+2]}\left(f\left[F^{-1}\left(\frac{1}{N+1}\right)\right]\right)^{-2} \tag{55}$$

Note that $F^{-1}\left(\frac{1}{N+1}\right)$ is decreasing in N. $f\left[F^{-1}\left(\frac{1}{N+1}\right)\right]^{-2}$ may be increasing or decreasing for different distributions. the condition required to ensure that the variance is increasing is $T(N) \equiv \left(\frac{f[F^{-1}(\frac{1}{N+1})]}{N} \right)^{-1}$ is not increasing in N at an order of magnitude greater than the rate at which $\frac{N}{[N+1]^2[N+2]}$ decreases with N. $\frac{N}{[N+1]^2[N+2]}$ decreases at an order of magnitude equal to N^2. Therefore, $T(N)$ should note increase at an order of magnitude greater than N^2. $T(N)$ non-increasing in N is sufficient to guarantee this is true. □

Proof of Proposition 4. Since $\overline{\rho}_N$ is high, then Lemma 1 and Lemma 2 imply that both α^*_{pub} and η^*_{pub} are large implying that the manager has a very high threat point, T. Proposition 3 shows that if the private equity investors have a level of agreement that is equal to that in the public market, then ownership portion and managerial autonomy are unchanged. Since the investors get no surplus with these ownership structures, increasing ρ_{pe} above $\overline{\rho}_{N1}$ adds surplus for he private equity investor. Thus, define the following:

$$(1-\alpha^*_{pe})W^I_{pe} - (1-\alpha^*_{pub})W^I_{pub} \equiv \widehat{C}_{N1} \tag{56}$$

It immediately follows that if C is greater than $\widehat{C}_{N1}$ the private equity investors will not participate implying the manager will not attempt to issue private equity. Since the public equity market is assumed to have no liquidity cost, $L = 0$, the manager will prefer public equity to debt so long as ψ is close enough to one to satisfy the following condition:

$$\alpha^*_{pub}W^*_{pub} \geq D^{\text{mun}} \tag{57}$$

This just guarantees that potential disagreement over the upside of either the innovative or mundane project is not so large that the manager wishes to choose the risky mundane project. □

Proof of Proposition 5. Since $\overline{\rho}_N$ is low, then Lemma 1 and Lemma 2 imply that both α^*_{pub} and η^*_{pub} are small implying that the manager has a very low threat point, T. Proposition 3 shows that if the private equity investors have a level of agreement that is equal to that in the public market, then ownership portion and managerial autonomy are unchanged. Since the investors get no surplus with these ownership structures, increasing ρ_{pe} above $\overline{\rho}_{N2}$ adds surplus for he private equity investor. Further, lemma 4 shows that in this scenario, $\frac{\partial \eta^*_{pe}}{\partial \rho_{pe}} < 0$ and lemma 5 shows that

$\frac{\partial \alpha^*_{pe}}{\partial \rho_{pe}}$ is decreasing, it immediately follows that

$$\frac{\partial V^I_{pe}}{\partial \rho_{pe}} = -\frac{\partial \alpha^*_{pe}}{\partial \rho_{pe}} \left[W^I_{pe}\right] + (1-\alpha^*_{pe}) \left(p\theta\psi H + \frac{\partial \eta^*_{pe}}{\partial \rho_{pe}} \left[p\theta(1-\rho_{pe}\psi)D\right] - p\theta\psi\eta^*_{pe}D \right) > 0. \tag{58}$$

Thus, the investors utility is increasing in ρ_{pe}. Careful inspection of equation (58) reveals that in addition it is increasing at an even greater rate than when a high $\overline{\rho}_N$ results in $\frac{\partial \eta^*_{pe}}{\partial \rho_{pe}} > 0$ (See Figure 2 for examples). Thus, private equity will be chosen if and only if

$$(1-\alpha^*_{pe})W^I_{pe} - (1-\alpha^*_{pub})W^I_{pub} \equiv \widehat{C}_{N2} \geq C. \tag{59}$$

Again this is true as long as ψ is close enough to 1 to guarantee

$$\alpha^*_{pub}W^*_{pub} \geq D^{\text{mun}}. \tag{60}$$

□

Proof of Proposition 6. Since $\overline{\rho}_{N3}$ is low, it follows that the threat point, T, is also low. This provides great benefit to a high type private equity investor which will be very difficult to find or may not even exist at all given the low dispersion. Thus, the innovative project will not be pursued. If ψ is large enough, then public equity will be chosen. It is obvious with everyone agreeing on the probabilities of the mundane project that private equity will not be used. If ψ is low, then debt will be the funding of choice. If assumption 1 holds, then public debt markets will not lend and private debt markets must be used. If assumption 1 does not hold, then public debt will be used. □

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Figure 1: The following timeline reveals the sequence of events in the model.

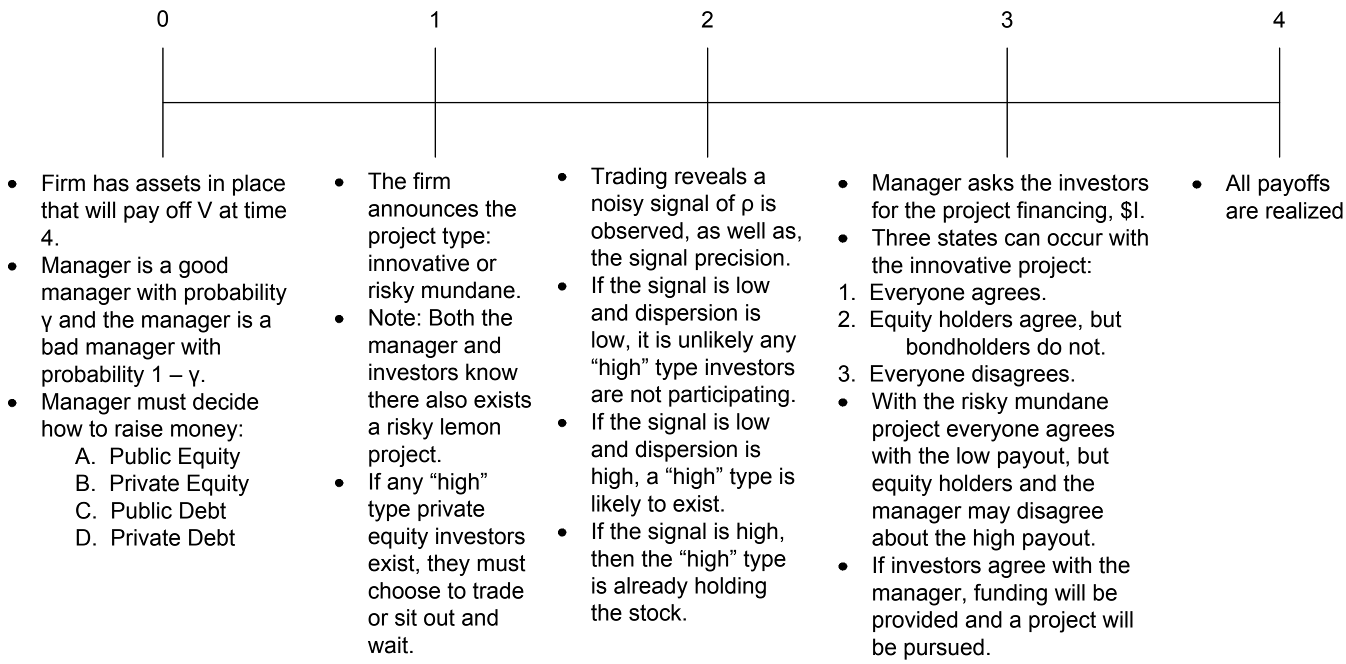

Figure 2: Plots of η_{pe} with the following common parameters: H = 10, D = -5, I = 1, p = 0.5, $\theta = 0.5$, $\psi = 1$, and ρ_{pub} as listed above each graph. Recall Lemma 5 guarantees that an increase in ρ_{pe} *always* decreases ownership portion. Graphs A and B give examples where the managerial control with private equity is also decreasing. Graphs C and D give examples of parameters where the managerial control is increasing.