

# Multi-attribute Procurement Auctions with Risk Averse Suppliers

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## **Abstract**

This article extends the analysis of multi-attribute procurement auctions to the risk averse suppliers case. In our model, suppliers submit bids in the form of price-quality pairs which are ranked by the buyer's preference. We obtain the equilibrium bidding strategies for both the first-score and the second-score auctions. Under the first-score auction, the suppliers' equilibrium bidding price decreases in both the number and the risk aversion of the suppliers. Under the second-score auction, however, neither affects the supplier's equilibrium bidding price. Furthermore, the buyer prefers the first-score auction to the second-score auction when the suppliers are risk averse.

**Keywords:** risk averse; multi-attribute procurement auctions; first-score auction; second-score auction

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# 1 Introduction

Procurement auctions are widely applied in government procurements and private enterprise purchases. Unlike traditional forward auctions, a bid in procurement auctions often involves price and non-price attributes, such as quality, time of delivery and service levels. Therefore, researchers consider procurement auctions as *multi-attribute*. Thus far, extensive research has been done on risk neutral procurement auctions (Branco, 1997; Che, 1993; Samuelson, 1986). But as pointed out by Maskin and Riley (1984), the marginal utility of income if a bidder wins is often not the same as that if he loses. Indeed, suppliers of procurement contracts are shown to be risk averse (Campo, 2009). Unfortunately, the existing literature on risk averse procurement is quite limited. Holt (1980) considered risk averse suppliers in a single-attribute auction setting. Baron and Besanko (1987) characterized the optimal procurement contract for a monopsonistic buyer who contracts with a single risk averse supplier. There has not been a systematic analysis of the role of supplier risk aversion in multi-attribute procurement auctions.

The goal of this article is to examine a multi-attribute procurement auction model with risk averse suppliers. In particular, we are interested in how risk aversion and number of the suppliers affect auction outcomes and which payment rule (first- or second-score) the buyer should use. Our contribution lies in extending the analysis of multi-attribute procurement auctions to the risk averse case. We analytically establish that buyer procurement cost decreases with risk aversion and number of suppliers. Finally, we show that the first-score auction is preferred to the second-score auction with risk averse suppliers.

## 2 The model

A buyer wishes to acquire a single good (or service) from one of  $n$  suppliers. The buyer cares about the cost of the acquisition and the *quality* of the acquired good, such as build quality, time of delivery, and other non-monetary attributes. For simplicity, we assume that quality is a single-dimensional variable, denoted as  $q$ . It shall be clear later (see footnote 3) that changing to multi-dimensional quality does not alter the qualitative nature of our findings.

The buyer uses a procurement auction to solicit bids from the  $n$  suppliers. Each bid is a *pair*  $(p, q)$  that consists of a price  $p$  and a quality  $q$ . A supplier incurs a cost of  $c(q, \theta)$  to

produce quality  $q$ . As in procurement auction literature (Che, 1993), the cost parameter  $\theta$  is the supplier's private information and is independently and identically distributed on  $[\underline{\theta}, \bar{\theta}]$  according to a distribution function  $F(\cdot)$  with a positive and continuously differentiable density  $f(\cdot)$ . We assume that supplier's cost is increasing in quality  $q$  and cost parameter  $\theta$  and convex in  $q$ , i.e.,  $c_q(\cdot, \cdot) > 0$ ,  $c_\theta(\cdot, \cdot) > 0$ , and  $c_{qq}(\cdot, \cdot) \geq 0$ . An example of such a cost function is  $c(q, \theta) = q\theta$ .

Let  $u(\cdot)$  denote supplier's von Neumann-Morgenstern utility for income. A supplier's expected utility is

$$U(p, q) = u(\text{payment} - c(q, \theta)) \times \text{Prob}\{\text{win}|p, q\} \quad (1)$$

where  $\text{Prob}\{\text{win}|p, q\}$  is the winning probability of bid  $(p, q)$ . We assume  $u(0) = 0$  and  $u'(x) > 0$  for all  $x \geq 0$ .  $u(\cdot)$  can be linear or concave, which models *risk neutral* and *risk averse* suppliers respectively.

The buyer is risk neutral and his utility is given by a quasi-linear function

$$M(\text{payment}, q) = m(q) - \text{payment} \quad (2)$$

where  $m'(\cdot) > 0$  and  $m''(\cdot) < 0$ . We assume  $m'(0) > c_q(0, \cdot)$  and  $m'(\infty) < c_q(\infty, \cdot)$  to ensure that each supplier would choose a finite positive quality upon winning.

The buyer ranks bids using a publically announced scoring rule that is consistent with his utility.<sup>2</sup> Namely, the *score* of bid  $(p, q)$  is given by,

$$S(p, q) = M(p, q) = m(q) - p \quad (3)$$

The supplier with the highest score wins the auction (ties are broken by coin toss). As in the literature (Che, 1993; Branco, 1997), the winner with a bid of  $(p, q)$  receives a payment of  $p$  and delivers quality  $q$  in the first-score auction. The same winner needs to "fulfill" the second highest score in the second-score auction in the sense that the winner is free to choose any

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<sup>2</sup>The buyer may not creditably commit to other scoring rules because it is optimal for him to select the bids based on his true utility after receiving all the bids.

quality and payment combination as long as the score of his chosen combination equals the second highest score.

### 3 Results

#### 3.1 Supplier's Quality Choice

**Lemma 1.** *In the first- and second-score auctions, supplier with cost parameter  $\theta$  will choose the equilibrium quality  $Q(\theta)$  according to*

$$Q(\theta) \equiv \arg \max_q [m(q) - c(q, \theta)] \quad (4)$$

*Proof.* See Che (1993) for the proof of a similar result for the first-score auction. For the second-score auction, suppose the contrary that the supplier with cost parameter  $\theta$  chooses to bid  $(p, q)$  in equilibrium, where  $q \neq Q(\theta)$ . We show that the supplier is strictly better off with an alternative bid  $(p^*, Q(\theta))$  where  $p^* = p + m(Q(\theta)) - m(q)$ . In fact, it follows from (3) that  $S(p^*, Q(\theta)) = S(p, q)$  and then  $\text{Prob}\{\text{win}|p^*, Q(\theta)\} = \text{Prob}\{\text{win}|p, q\}$ . Moreover, we have  $m(q) - S_2 - c(q, \theta) < m(Q(\theta)) - S_2 - c(Q(\theta), \theta)$ , where  $S_2$  is the second highest score and the inequality is true because  $m(q) - c(q, \theta)$  has unique interior maximum at  $Q(\theta)$  by assumptions about  $m(\cdot)$  and  $c(\cdot, \cdot)$ . Now the expected utilities for the two bids have the following relationship,

$$\begin{aligned} EU(p^*, Q(\theta)) &= u(m(Q(\theta)) - S_2 - c(Q(\theta), \theta)) \times \text{Prob}\{\text{win}|p^*, Q(\theta)\} \\ &> u(m(q) - S_2 - c(q, \theta)) \times \text{Prob}\{\text{win}|p, q\} \\ &= EU(p, q) \end{aligned}$$

which suggests that  $(p, q)$  cannot be an equilibrium bid.

□

Lemma 1 implies that in both first-score and second-score auctions, the supplier's equilibrium quality choice is a function of his cost parameter  $\theta$  and the scoring rule, but is independent of his equilibrium price. Furthermore, a supplier chooses quality to maximize the total surplus created by (4). Lemma 1 allows us to transform the multi-attribute procurement auction into a single attribute one.

### 3.2 Supplier's Equilibrium Price Choice

Denote

$$v = V(\theta) \equiv m(Q(\theta)) - c(Q(\theta), \theta) \quad (5)$$

as the "valuation" of a supplier with cost parameter  $\theta$ . By the envelope theorem,  $V'(\theta) = -c_\theta(Q(\theta), \theta) < 0$ . So  $V(\theta)$  is strictly decreasing in  $\theta$ . Let  $\bar{v} = V(\underline{\theta})$  and  $\underline{v} = V(\bar{\theta})$ , then  $v$  is distributed on  $[\underline{v}, \bar{v}]$  according to

$$H(v) \equiv 1 - F(V^{-1}(v)) \quad (6)$$

with density  $h(v) = H'(v)$ . We further denote the "bid score" of  $(p, Q(\theta))$  as

$$b \equiv m(Q(\theta)) - p \quad (7)$$

It is straightforward to verify that with transformations (5) and (7), the first- and second-score auctions correspond to standard first- and second-price auctions respectively with valuation  $v$  and bid  $b(v)$ .<sup>3</sup>  $v$  corresponds to the surplus created in the original auctions,  $b(v)$  corresponds to the equilibrium bidding score, and the equilibrium bid prices can be easily derived from (7) and (4).

**Theorem 1.** *In the first-score auction, the equilibrium bidding price is given by*

$$P_{FS}(\theta) = m(Q(\theta)) - b(V(\theta)) \quad (8)$$

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<sup>3</sup>In the case of multi-dimensional quality, say,  $q = (q_1, q_2, \dots, q_k)$ , we can similarly obtain an optimal quality vector  $Q(\theta) = \arg \max_q [m(q) - c(q, \theta)]$ . Once again, we can transform the multi-attribute procurement auction to a standard auction using (5) and (7). Our subsequent results are generalizable to multi-dimensional quality.

where equilibrium score  $b(\cdot)$  is determined by the following differential equation

$$b'(v) = \frac{(n-1)h(v)u(v-b(v))}{H(v)u'(v-b(v))} \quad (9)$$

with a boundary condition  $b(\underline{v}) = \underline{v}$ .

*Proof.* Following the existing methods (e.g, Riley and Samuelson 1981) for solving standard first auctions with risk averse bidders, we can obtain (9) in the transformed first-price auction. (8) follows directly from (7).  $\square$

**Theorem 2.** *In the second-score auction, it is a weakly dominant strategy for the suppliers to quote costs as prices where*

$$P_{SS}(\theta) = c(Q(\theta), \theta) \quad (10)$$

*Proof.* Directly follow from (7) and the known result that it is a weakly dominant strategy for risk averse bidders to bid truthfully in standard second-price auctions (e.g., McAfee and McMillan 1987).  $\square$

It is clear from Theorem 2 that the risk aversion and number of bidders have no effect on suppliers' bids in the second-score auction. But the same may not be true in the first-score auction.

### 3.3 The Impact of the Number of Suppliers in First-Score Auctions

**Theorem 3.** *In the first-score auction, supplier's equilibrium bidding price and the buyer's acquisition cost decrease in the number of suppliers.*

*Proof.* We first show that  $\frac{\partial P_{FS}(\theta)}{\partial n} < 0$  for all  $\theta < \bar{\theta}$ . Note that  $b(\underline{v}) = \underline{v}$ . Differentiating (9) with respect to  $n$  yields

$$\begin{aligned} \frac{\partial}{\partial n} \left( \frac{\partial b(v)}{\partial v} \right) + \left[ \frac{(n-1)h(v)}{H(v)} \frac{u'(v-b(v))^2 - u(v-b(v))u''(v-b(v))}{u'(v-b(v))^2} \right] \frac{\partial b(v)}{\partial n} \\ = \frac{h(v)}{H(v)} \frac{u(v-b(v))}{u'(v-b(v))} \end{aligned}$$

Denote

$$\begin{aligned} \left[ \frac{(n-1)h(v)}{H(v)} \frac{u'(v-b(v))^2 - u(v-b(v))u''(v-b(v))}{u'(v-b(v))^2} \right] = A(v, n) \\ \frac{h(v)}{H(v)} \frac{u(v-b(v))}{u'(v-b(v))} = B(v, n) \end{aligned}$$

then

$$\frac{\partial}{\partial v} \left( \frac{\partial b(v)}{\partial n} \right) + A(v, n) \frac{\partial b(v)}{\partial n} = B(v, n)$$

Multiplying both sides of the above expression by  $e^{\int_{\underline{v}}^v A(t, n) dt}$ , we get

$$\frac{\partial}{\partial v} \left[ \frac{\partial b(v)}{\partial n} e^{\int_{\underline{v}}^v A(t, n) dt} \right] = B(v, n) e^{\int_{\underline{v}}^v A(t, n) dt}$$

Taking definite integral over  $[\underline{v}, v]$ , we have

$$\int_{\underline{v}}^v \frac{\partial}{\partial s} \left[ \frac{\partial b(s)}{\partial n} e^{\int_{\underline{v}}^s A(t, n) dt} \right] ds = \int_{\underline{v}}^v B(s, n) e^{\int_{\underline{v}}^s A(t, n) dt} ds$$

i.e.,

$$\frac{\partial b(v)}{\partial n} e^{\int_{\underline{v}}^v A(t, n) dt} - \frac{\partial b(s)}{\partial n} \Big|_{s=\underline{v}} = \int_{\underline{v}}^v B(s, n) e^{\int_{\underline{v}}^s A(t, n) dt} ds$$

Because  $b(\underline{v}) = \underline{v}$  for all  $n$ , thus  $\frac{\partial b(s)}{\partial n} \Big|_{s=\underline{v}} = 0$ , then

$$\frac{\partial b(v)}{\partial n} e^{\int_{\underline{v}}^v A(t, n) dt} = \int_{\underline{v}}^v B(s, n) e^{\int_{\underline{v}}^s A(t, n) dt} ds$$

For all  $\theta < \bar{\theta}$  (i.e.,  $v > \underline{v}$ ), we have  $B(v, n) = \frac{h(v)}{H(v)} \frac{u(v-b(v))}{u'(v-b(v))} > 0$  and  $\int_{\underline{v}}^v B(s, n) e^{\int_{\underline{v}}^s A(t, n) dt} ds > 0$ ,

thus  $\frac{\partial b(v)}{\partial n} > 0$ . Because  $P_{FS}(\theta) = m(Q(\theta)) - b(v)$  and  $m(Q(\theta))$  does not depend on  $n$ , therefore  $\frac{\partial P_{FS}(\theta)}{\partial n} = -\frac{\partial b(v)}{\partial n} < 0$  for all  $\theta < \bar{\theta}$ . Obviously, as equilibrium bid prices decrease, the buyer's acquisition cost is lower. □

To our knowledge, this is the first article to *analytically* establish the impact of the number of suppliers in multi-attribute procurement auctions with risk averse suppliers. Holt (1980) drew similar conclusions for single-attribute procurement auctions through the analysis of supplier's expected utility.

Theorem 3 provides useful theoretical predictions on the bidding behaviors of risk averse suppliers and buyer revenues in first-score procurement auctions.

### 3.4 The Impact of Risk Preference in First-Score Auctions

**Theorem 4.** *In the first-score auction, supplier's equilibrium bidding price and the buyer's acquisition cost decrease with suppliers' risk aversion.*

*Proof.* It is sufficient to show that if  $u_2(\cdot)$  is a concave transformation of  $u_1(\cdot)$  then  $P_{FS}(\theta|u_1) > P_{FS}(\theta|u_2)$  for all  $\theta < \bar{\theta}$ . If  $u_2$  is a concave transformation of  $u_1$ , we have  $-\frac{u_1''}{u_1} < -\frac{u_2''}{u_2}$ . Similar to Riley and Samuelson (1981), we can show that  $b(v|u_1) < b(v|u_2)$  for all  $v > \underline{v}$ . Because  $P_{FS}(\theta) = m(Q(\theta)) - b(v)$  and  $m(Q(\theta))$  does not depend on  $u_i, i = 1, 2$ , it follows that  $P_{FS}(\theta|u_1) > P_{FS}(\theta|u_2)$  for all  $\theta < \bar{\theta}$ . □

Theorem 4 implies that suppliers bid lower prices in the first-score auctions as they become more risk averse. This theoretical finding is supported by empirical evidence presented in Campo (2009). Intuitively, as suppliers become more risk averse, they will bid lower prices to reduce the risk of losing the contract, which leads to lower buyer acquisition cost.

### 3.5 Buyer's Preference for Auction Formats

In this section, we compare the buyer's utility in first and second-score auctions in risk averse settings. Let  $\theta_{[1]}$  and  $\theta_{[2]}$  be the smallest and second-smallest cost parameters. Note that the supplier with  $\theta_{[1]}$  wins in both auction formats because equilibrium bidding scores  $b(v)$  increases in  $v$  and  $v = V(\theta)$  decreases in the cost parameter  $\theta$ . By (2), the buyer's expected utility in the first-score auction is

$$EU_{FS}(u) = E_{\theta_{[1]}}[m(Q(\theta_{[1]})) - P_{FS}(\theta_{[1]})] \quad (11)$$

In the second score auction, the winner with cost parameter  $\theta_{[1]}$  receives a payment of

$$payment = m(Q(\theta_{[1]})) - b(v_{[2]}) = m(Q(\theta_{[1]})) - m(Q(\theta_{[2]})) + c(Q(\theta_{[2]}), \theta_{[2]})$$

where  $b(v_{[2]})$  is the second highest score. So the buyer's expected utility in the second-score auction is

$$EU_{SS}(u) = E_{\theta_{[1], \theta_{[2]}}}[m(Q(\theta_{[1]})) - payment] = E_{\theta_{[2]}}[m(Q(\theta_{[2]})) - c(Q(\theta_{[2]}), \theta_{[2]})] \quad (12)$$

**Theorem 5.** *The expected utilities of the buyer in the first- and second-score auctions have the following relationship*

$$EU_{FS}(u_1) > EU_{FS}(u_0) = EU_{SS}(u_0) = EU_{SS}(u_1) \quad (13)$$

for any concave utility function  $u_1$  and linear utility function  $u_0$ .

*Proof.* Clearly  $u_1$  is a concave transformation of  $u_0$  (suppose  $u_0(x) = ax$ , we can define a concave function  $\phi(x) = u_1(\frac{1}{a}x)$  such that  $\phi(u_0(x)) = u_1(x)$ ). By Theorem 4 we have  $P_{FS}(\theta_{[1]}|u_1) < P_{FS}(\theta_{[1]}|u_0)$  when  $\theta_{[1]} < \bar{\theta}$ . It follows from (11) that  $EU_{FS}(u_1) > EU_{FS}(u_0)$ .  $EU_{SS}(u_0) = EU_{SS}(u_1)$  holds by Theorem 2. By the revenue equivalence theorem (Riley and Samuelson 1981) in the risk neutral setting, we obtain  $EU_{FS}(u_0) = EU_{SS}(u_0)$ .

□

Theorem 5 implies that the buyer is better off when facing risk averse suppliers than when facing risk neutral ones in the first-score auction. When suppliers are risk neutral, the revenue equivalence theorem holds and two auction formats generate identical utility for the buyer. However, because the second-score auction is not affected by risk aversion, the buyer obtains higher utility in first-score auctions with risk averse suppliers. This finding parallels the findings in standard (forward) auctions with risk averse bidders (e.g., Maskin and Riley 1984).

## 4 Conclusions

This article generalizes the existing multi-attribute procurement auction model to risk averse suppliers. We show that, in equilibrium, supplier's quality choice depends only on his private cost parameter and the scoring rule. This enables us to transform multi-attribute procurement auctions into standard auctions. Despite that the equilibrium bidding strategies in first-score auctions cannot be explicitly solved, we still prove that, for the first time, supplier equilibrium prices decrease with the number of suppliers and supplier risk aversion, so does the buyer's acquisition cost. Moreover, with risk averse suppliers, the buyer prefers the first-score auction to the second-score auction, which parallels the findings that high-bid auctions generate higher expected revenues than English auctions (e.g., Maskin and Riley 1984).

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