Chapter 2
Theoretical Tools of Public Finance

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Main topics of this lesson

- ***Constrained Utility Maximization*** – preferences, budget constraints, utility functions, indifference curves, marginal utility, diminishing marginal utility, MRS, income/prices/shifting/rotating of budget constraint, income/substitution effects.
- ***Welfare economics*** – demand curves, elasticities, consumer & producer surplus, deadweight loss, welfare theorems, social welfare functions

Introduction
Theoretical Tools of Public Finance

- ***Theoretical tools*** are a set of tools used to understand economic decision making. They are primarily graphical and mathematical.
- ***Empirical tools*** allow you to examine the theory with data.

CONSTRAINED UTILITY MAXIMIZATION

- ***Constrained utility maximization*** means that all decisions are made in order to maximize the well-being of the individual, subject to his available resources.
- Utility maximization involves *preferences* and a *budget constraint*.
- One of the key assumptions about preferences is *non-satiation*–that "more is preferred to less."

Constrained Utility Maximization Preferences and indifference curves

- **Figure 1** illustrates some preferences over movies (on the x-axis) and CDs (on the y-axis).
- Because of non-satiation, bundles *A* and *B* are both inferior to bundle *C*.

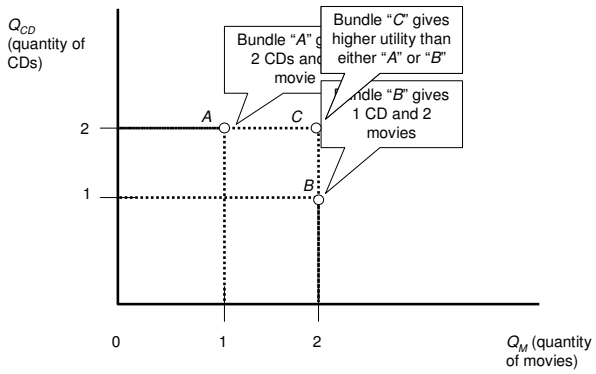

Figure 1 **Different Bundles of Goods**

Constrained Utility Maximization: Preferences and indifference curves

- A ***utility function*** is a mathematical representation $U = f(X_1, X_2, X_3, \ldots)$
  - Where X_1, X_2, X_3 and so on are the *goods* consumed by the individual,
  - And *f(•)* is some mathematical function.

Constrained Utility Maximization: Preferences and indifference curves

- One formulation of a utility function is $U(Q_M,Q_C) = Q_M Q_C$, where Q_M = quantity of movies and Q_C = quantity of CDs.
- The combinations {1, 2} (bundle *A*) and {2,1} (bundle *B*) both give 2 "utils."
- The combination {2, 2} (bundle *C*) gives 4 "utils."
- With these preferences, *indifferent* to *A* or *B*.
- **Figure 2** illustrates this.

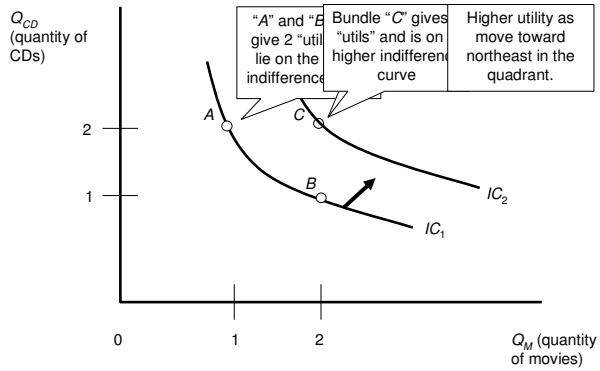

Figure 2 **Utility From Different Bundles**

Constrained Utility Maximization: Utility mapping of preferences

- How are indifference curves derived?
- Set utility equal to a constant level and figure out the bundles of goods that get that utility level.
- For $U = Q_M Q_C$, how would we find the bundles for the indifference curve associated with 25 utils?
  - Set $25 = Q_M Q_C$,
  - Yields $Q_C = 25/Q_M$,
  - Or bundles like {1,25}, {1.25,20}, {5,5}, etc.

Constrained Utility Maximization: Marginal utility

- ***Marginal utility*** is the additional increment to utility from consuming an additional unit of a good.
- ***Diminishing marginal utility*** means each additional unit makes the individual less happy than the previous unit.

Technical Constrained Utility Maximization: Marginal utility

- With the utility function given before, $U = Q_M Q_C$, the marginal utility is:

$$MU_{Q_M} = \frac{\partial U}{\partial Q_M} = Q_C$$

- Take the partial derivative of the utility function with respect to Q_M to get the marginal utility of movies.

Constrained Utility Maximization: Marginal utility

- Evaluating the utility function $U = (Q_M Q_C)^{1/2}$, at $Q_C = 2$ allows us to plot a relationship between marginal utility and movies consumed.
- **Figure 3** illustrates this.

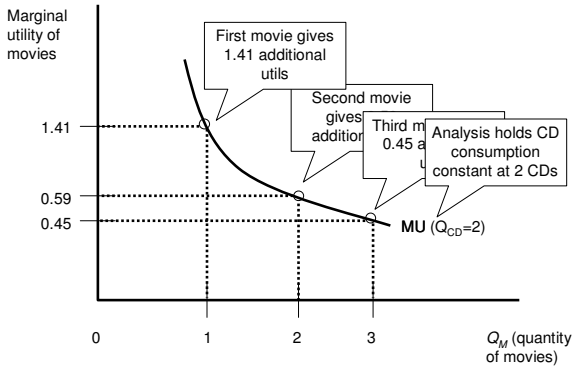

Figure 3 **Declining Marginal Utility From Movies**

Constrained Utility Maximization: Marginal utility

- Why does diminishing marginal utility make sense?
  - Most consumers order consumption of the goods with the highest utility first.

Constrained Utility Maximization: Marginal rate of substitution

- ***Marginal rate of substitution***—slope of the indifference curve is called the ***MRS***, and is the rate at which consumer is willing to trade off the two goods.
- Returning to the (CDs, movies) example.
- **Figure 4** illustrates this.

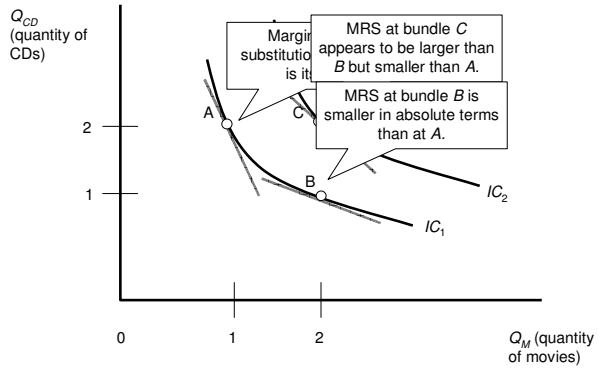

Figure 4 **Marginal Rate of Substitution At Different Bundles**

Constrained Utility Maximization: Marginal rate of substitution

- ***MRS*** is diminishing (in absolute terms) as we move along an indifference curve.
- This means that Andrea is willing to give up fewer CD's to get more movies when she has more movies (bundle *B*) than when she has less movies (bundle *A*).
- **Figure 5** illustrates this.

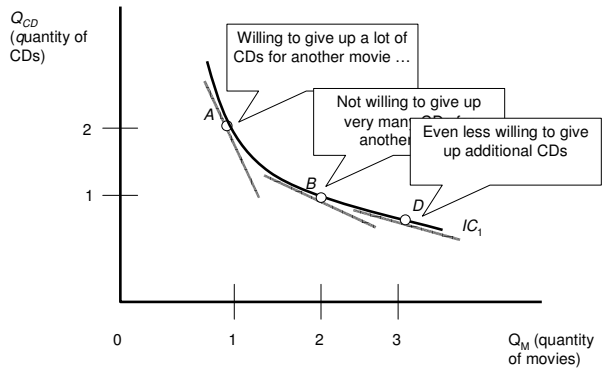

Figure 5 **Marginal Rate of Substitution is Diminishing**

Constrained Utility Maximization Marginal rate of substitution

- Direct relationship between *MRS* and marginal utility.

$$MRS = -\frac{MU_M}{MU_C}$$

- *MRS* shows how the relative marginal utilities evolve over the indifference curve.
- Straightforward to derive this relationship graphically, as well.
- Consider the movement from bundle *A* to bundle *B*. **Figure 6** illustrates this.

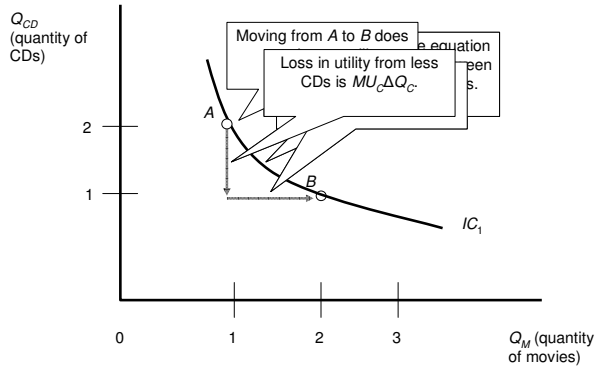

Figure 6 Relationship Between Marginal Utility and MRS

Constrained Utility Maximization: Budget constraints

- The ***budget constraint*** is a mathematical representation of the combination of goods the consumer can afford to buy with a given income.
- Assume there is no saving or borrowing.
- In the example, denote:
  - Y = Income level
  - P_M = Price of one movie
  - P_C = Price of one CD

Constrained Utility Maximization: Budget constraints

- The expenditure on movies is:

$$P_M Q_M$$

- While the expenditure on CDs is:

$$P_C Q_C$$

Constrained Utility Maximization: Budget constraints

- Thus, the total amount spent is:

$$P_M Q_M + P_C Q_C$$

- This must equal income, because of no saving or borrowing.

$$Y = P_M Q_M + P_C Q_C$$

Constrained Utility Maximization: Budget constraints

- This budget constraint is illustrated in the next figure.
- **Figure 7** illustrates this.

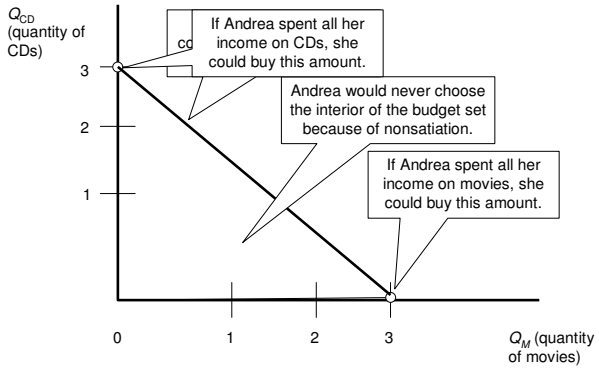

Figure 7 **The Budget Constraint**

Constrained Utility Maximization: Budget constraints

- The slope of the budget constraint is:

$$-\frac{P_M}{P_C}$$

- It is thought that government actions can change a consumer's budget constraint, but that a consumer's preferences are fixed.

Constrained Utility Maximization: Putting it together: Constrained choice

- What is the highest indifference curve that an individual can reach, given a budget constraint?
- Preferences tells us what a consumer wants, and the budget constraint tells us what a consumer can actually purchase.
- This leads to utility maximization, shown graphically, in **Figure 8**.

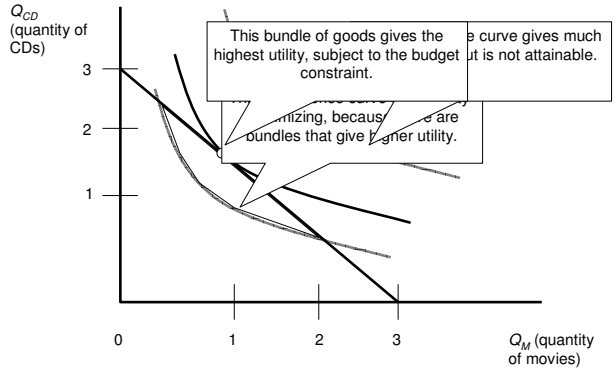

Figure 8 Utility Maximization

Constrained Utility Maximization: Putting it together: Constrained choice

- In this figure, the utility maximizing choice occurs where the indifference curve is *tangent* to the budget constraint.
- This implies that the slope of the indifference curve equals the slope of the budget constraint.

Constrained Utility Maximization: Putting it together: Constrained choice

- Thus, the marginal rate of substitution equals the ratio of prices:

$$MRS = -\frac{MU_M}{MU_C} = -\frac{P_M}{P_C}$$

- At the optimum, the ratio of the marginal utilities equals the ratio of prices. But this is not the only condition for utility maximization.
- **Figure 9** illustrates this.

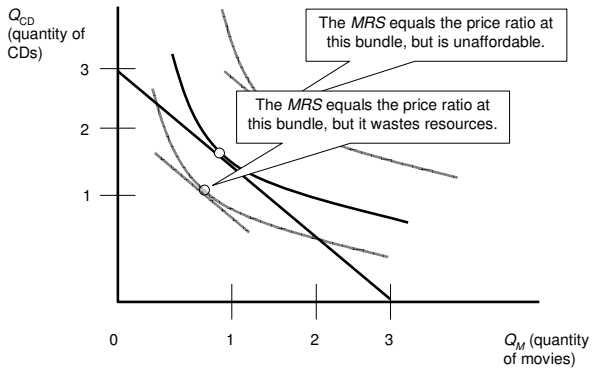

Figure 9 MRS Equal to Price Ratio is Insufficient

Constrained Utility Maximization: Putting it together: Constrained choice

- Thus, the second condition is that all of the consumer's money is spent:

$$Y = P_M Q_M + P_C Q_C$$

- These two conditions are used for utility maximization.

The Effects of Price Changes: Substitution and income effects

- Consider a typical price change in our framework:
- Increase the price of movies, P_M.
- This rotates the budget constraint inward along the x-axis.
- **Figure 10** illustrates this.

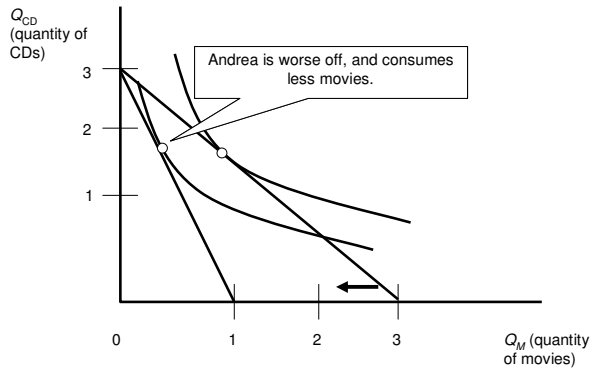

Figure 10 Increase in the Price of Movies

The Effects of Price Changes: Substitution and income effects

- A change in price consists of two effects:
- ***Substitution effect***–change in consumption due to change in relative prices, *holding utility constant.*
- ***Income effect***–change in consumption due to feeling "poorer" after price increase.
- **Figure 11** illustrates this.

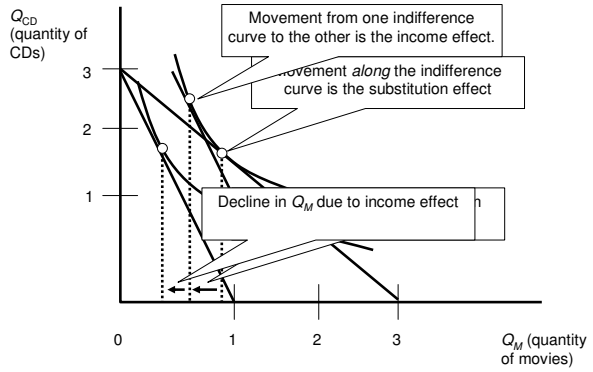

Figure 11 **Illustration of Income and Substitution Effects**

EQUILIBRIUM AND SOCIAL WELFARE

- ***Welfare economics*** is the study of the determinants of well-being, or welfare, in society. It depends on:
- Determinants of social efficiency, or size of the economic "pie."
- Redistribution.

EQUILIBRIUM AND SOCIAL WELFARE
Demand curves

- ***Demand curve*** is the relationship between the price of a good and the quantity demanded.
- Derive demand curve from utility maximization problem, as shown in **Figure 18**.

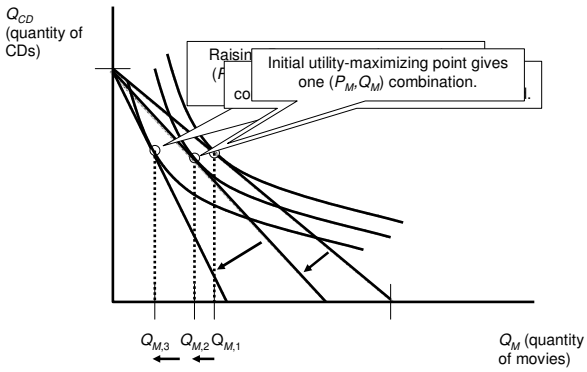

Figure 18 **Increase in the Price of Movies**

EQUILIBRIUM AND SOCIAL WELFARE

Demand curves

- This gives various (P_M,Q_M) combinations that can be mapped into price/quantity space.
- This gives us the demand curve for movies.
- **Figure 19** illustrates this.

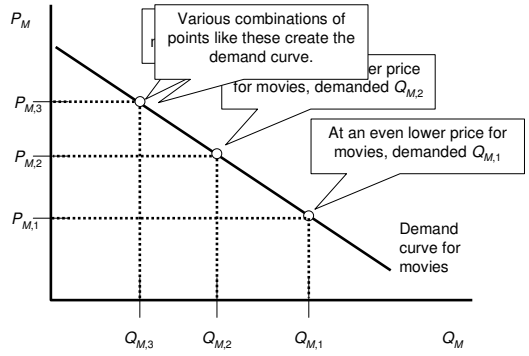

Figure 19 **Deriving the Demand Curve for Movies**

EQUILIBRIUM AND SOCIAL WELFARE

Elasticity of demand

- A key feature of demand analysis is the ***elasticity of demand***. It is defined as:

$$\varepsilon_D = \frac{\Delta Q_D / Q_D}{\Delta P / P}$$

- That is, the percent change in quantity demanded divided by the percent change in price.

EQUILIBRIUM AND SOCIAL WELFARE

Elasticity of demand

- For example, an increase in the price of movies from \$8 to \$12 is a 50% rise in price.
- If the number of movies purchased fell from 6 to 4, there is an associated 33% reduction in quantity demanded.
  - The demand elasticity is therefore -0.67.
- Demand elasticities features:
  - Typically negative number.
  - Not constant along the demand curve (for a linear demand curve).

EQUILIBRIUM AND SOCIAL WELFARE

Elasticity of demand

- For a vertical demand curve
  - Elasticity of demand is zero—quantity does not change as price goes up or down.
  - ***Perfectly inelastic***
- For a horizontal demand curve
  - Elasticity of demand is negative infinity—quantity changes infinitely for even a small change in price.
  - ***Perfectly elastic***
- **Figure 20** illustrates this.

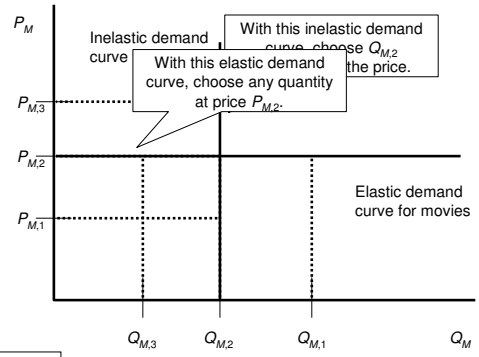

Figure 20 **Perfectly Elastic and Perfectly Inelastic Demand**

EQUILIBRIUM AND SOCIAL WELFARE

Elasticity of demand

- More generally, an ***elasticity*** divides the percent change in a dependent variable by the percent change in an independent variable:

$$\varepsilon = \frac{\Delta Y / Y}{\Delta X / X}$$

- For example, Y is often the quantity demanded or supplied, while X might be own-price, cross-price, or income.

EQUILIBRIUM AND SOCIAL WELFARE

Social efficiency

- Measuring *social efficiency* is computing the potential size of the economic pie. It represents the net gain from trade to consumers and producers.

EQUILIBRIUM AND SOCIAL WELFARE
Social efficiency

- ***Consumer surplus*** is the benefit that consumers derive from a good, beyond what they paid for it.
- Each point on the demand curve represents a "willingness-to-pay" for that quantity.
- **Figure 22** illustrates this.

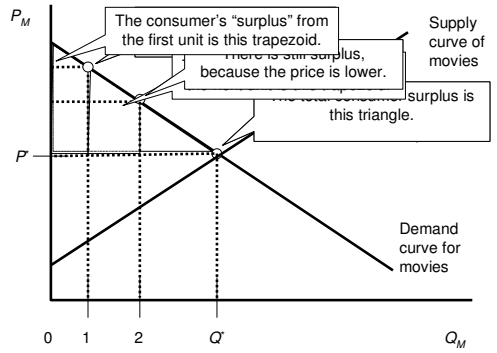

Figure 22 Deriving Consumer Surplus

EQUILIBRIUM AND SOCIAL WELFARE
Social efficiency

- Consumer surplus is determined by market price and the elasticity of demand:
  - With inelastic demand, demand curve is more vertical, so surplus is higher.
  - With elastic demand, surplus is lower.
- **Figure 23** illustrates this.

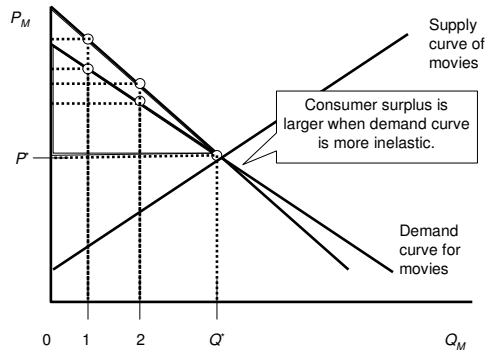

Figure 23 Consumer Surplus and Inelastic Demand

EQUILIBRIUM AND SOCIAL WELFARE
Social efficiency

- ***Producer surplus*** is the benefit derived by producers from the sale of a unit above and beyond their cost of producing it.
- Each point on the supply curve represents the marginal cost of producing it.
- **Figure 24** illustrates this.

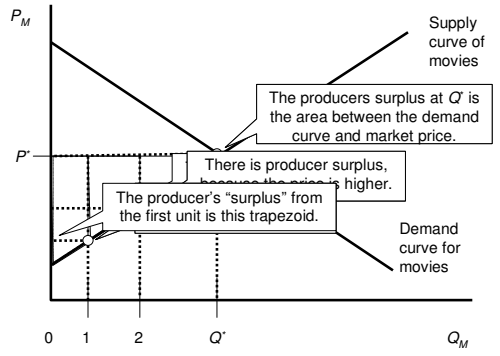

Figure 24 **Producer Surplus**

EQUILIBRIUM AND SOCIAL WELFARE
Social efficiency

- Similar to consumer surplus, producer surplus is determined by market price and the elasticity of supply:
  - With inelastic supply, supply curve is more vertical, so producer surplus is higher.
  - With elastic supply, producer surplus is lower.

EQUILIBRIUM AND SOCIAL WELFARE
Social efficiency

- The ***total social surplus,*** also known as "social efficiency," is the sum of the consumer's and producer's surplus.
- **Figure 25** illustrates this.

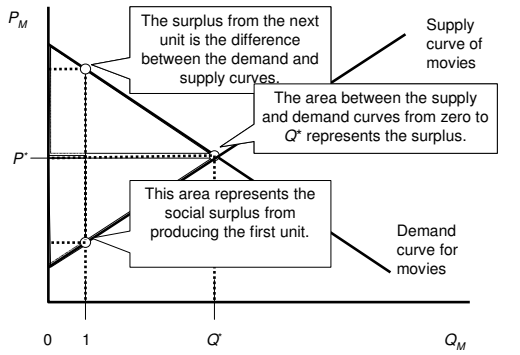

Figure 25 **Social Surplus**

EQUILIBRIUM AND SOCIAL WELFARE

Competitive equilibrium maximizes social efficiency

- The ***First Fundamental Theorem of Welfare Economics*** states that the competitive equilibrium, where supply equals demand, maximizes social efficiency.
- Any quantity other than Q^* reduces social efficiency, or the size of the "economic pie."
- Consider restricting the price of the good to $P' < P^*$.
- **Figure 26** illustrates this.

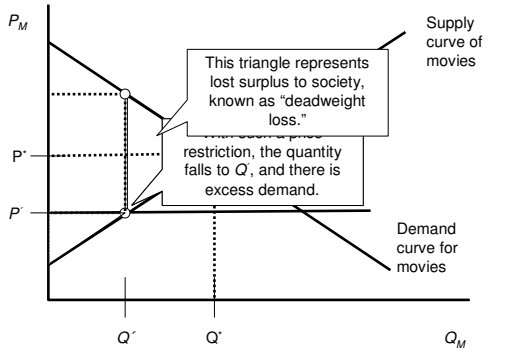

Figure 26 **Deadweight Loss from a Price Floor**

EQUILIBRIUM AND SOCIAL WELFARE

Competitive equilibrium maximizes social efficiency

- A policy like price controls creates ***deadweight loss***, the reduction in social efficiency by restricting quantity below the competitive equilibrium.

EQUILIBRIUM AND SOCIAL WELFARE
The role of equity

- Societies usually care not only about how much surplus there is, but also about how it is distributed among the population.
- ***Social welfare*** is determined by both criteria.
- The ***Second Fundamental Theorem of Welfare Economics*** states that society can attain any efficient outcome by a suitable redistribution of resources and free trade.
- In reality, society often faces an equity-efficiency tradeoff.

EQUILIBRIUM AND SOCIAL WELFARE
The role of equity

- Society's tradeoffs of equity and efficiency are models with a ***Social Welfare Function***.
- This maps individual utilities into an overall social utility function.

EQUILIBRIUM AND SOCIAL WELFARE
The role of equity

- The *utilitarian* social welfare function is:

$$SWF = \sum_i U_i$$

- The utilities of all individuals are given equal weight.
- Implies that government should transfer from person 1 to person 2 as long as person 2's gain is bigger than person 1's loss in utility.

EQUILIBRIUM AND SOCIAL WELFARE
The role of equity

- Utilitarian SWF is defined in terms of utility, not dollars.
- Society not indifferent between giving $1 of income to rich and poor; rather indifferent between one *util* to rich and one util to poor.

EQUILIBRIUM AND SOCIAL WELFARE
The role of equity

- Utilitarian SWF is maximized when the marginal utilities of everyone are equal:

$$MU_1 = MU_2 = \ldots = MU_i$$

- Thus, society should redistribute from rich to poor if the marginal utility of the next dollar is higher to the poor person than to the rich person.

EQUILIBRIUM AND SOCIAL WELFARE
The role of equity

- The *Rawlsian* social welfare function is:

$$SWF = \min(U_1, U_2, \ldots, U_N)$$

- Societal welfare is maximized by maximizing the well-being of the worst-off person in society.
- Generally suggests more redistribution than the utilitarian SWF.

WELFARE IMPLICATIONS OF BENEFIT REDUCTIONS: TANF continued

- Different policies involve different deadweight loss triangles, but also different levels of redistribution for the poor.
- SWF helps determine the right policy for society.

Recap of Theoretical Tools

- Utility maximization
- Labor supply example
- Efficiency
- Social welfare functions