Notes from ECO 479, Lecture 2, 8/28/2007 & 8/30/2007

1. Preferences, Indifference Curves
2. Calculus Review
3. Marginal Utility, Marginal Rate of Substitution
4. Budget Constraints – shifting and rotating
5. Utility Maximization
6. Demand Curves

1. Preferences and Indifference Curves (*note: see the attached powerpoint slides from last semester for pictures, etc.)*

a. We use theoretical tools understand economic decision making. They are primarily graphical and mathematical.
b. Constrained utility maximization means all decisions are made in order to maximize the utility of the individual, subject to his or her available resources. This involves preferences and budget constraints.
c. As illustrated in Figure 1 – with movies and CD's – different "bundles" of goods might be available for the person to consume. Bundle C in the figure – which gives at least as many movies and CD's ad Bundles A or B – must be preferred to those bundles because of non-satiation (more is preferred to less).
    i. Almost all goods satisfy this property. Perhaps you can drink so much beer or eat so much pizza, however, that you do feel satiated – meaning you are worse off if you consume more of the good.
d. A utility function is a mathematical representation that translates the consumption of goods into utility.
    i. Note that the utility number itself simply ranks the different bundles – it doesn't say how much better one bundle is than another.
    ii. There are ways for representing the same preferences with (what looks like) different utility functions. For example U=XY and U=10XY give the same rankings for all bundles. This is called a *monotonic transformation*.
    iii. We derive indifference curves from utility functions. Simply hold utility constant at some amount, and the different combinations of goods that give this utility emerges. For example, with the following utility function:

$$U = Q_M^{\frac{1}{3}} Q_{CD}^{\frac{1}{2}}$$

We could write the indifference curve as:

$$Q_{CD} = \overline{U}^2 Q_M^{-\frac{2}{3}}$$

For any fixed level of utility, $\overline{U}$. If you can't figure out how this indifference curve was derived, we simply moved Q_{CD} to the other side. If you are comfortable with exponents, this isn't hard.

2. Calculus Review

Specific Examples	General Case
$y = x^2$ $\frac{dy}{dx} = 2x$	$y = x^n$ $\frac{dy}{dx} = nx^{n-1}$
$y = 10x^3$ $\frac{dy}{dx} = 30x^2$	$y = kx^n$ $\frac{dy}{dx} = knx^{n-1}$
Partial Derivative $y = x^2 z^4$ $\frac{\partial y}{\partial x} = z^4 2x$ $\frac{\partial y}{\partial z} = 4z^3 x^2$	$y = x^{\alpha} z^{\beta}$ $\frac{\partial y}{\partial x} = z^{\beta} \alpha x^{\alpha-1}$
We often use partial derivatives in our micro problems. Here we are computing marginal utility. $U = Q_M^{\frac{1}{3}} Q_{CD}^{\frac{1}{2}}$ $MU_M = Q_{CD}^{\frac{1}{2}} \left(\frac{1}{3}\right) Q_M^{-\frac{2}{3}}$	

3. Marginal Utility and Marginal Rate of Substitution

a. Marginal Utility is the additional utility from consuming an infinitesimal amount more of some good. We take the partial derivative of the utility function to compute it. If the marginal utility of movies was equal to 5, then if we consume two additional movies, our total utility goes up by 10.
b. Figure 3 in the powerpoints shows *diminishing marginal utility* – as we consume more of a good, the additional happiness we get falls. Note that utility continues to rise, but just at a slower rate.

c. The Marginal Rate of Substitution (MRS) is the slope of the indifference curve. We can derive a relationship between the MRS and Marginal Utility as illustrated in Figure 6 of the powerpoints.
d. In general:

$$\Delta U = MU_M \Delta Q_M + MU_{CD} \Delta Q_{CD}$$

The first term is the change in utility level. The second term is the change in utility due to changing movie consumption. And the third term is the change in utility due to changing CD consumption. That is, $MU_M \Delta Q_M$ is multiplying the change in quantity by the utility change per unit. Going back to the above example, if the marginal utility of movies was equal to 5, and we consume two additional movies, our total utility goes up by 10.

Along an indifference curve, $\Delta U = 0$, by definition.

So consider small changes in movies and CD's that keep us on the same indifference curve. Then we have:

$$0 = \Delta U = MU_M \Delta Q_M + MU_{CD} \Delta Q_{CD}$$

$$\frac{\Delta Q_{CD}}{\Delta Q_M} = -\frac{MU_M}{MU_{CD}}$$

The term on the left side is the slope – the MRS – and is related to the ratio of the marginal utilities.

e. For the utility function $U = Q_M^{\frac{1}{3}} Q_{CD}^{\frac{1}{2}}$, the marginal utility of movies is $MU_M = Q_{CD}^{\frac{1}{2}} \left(\frac{1}{3}\right) Q_M^{-\frac{2}{3}}$ and the marginal utility of CD's is $MU_{CD} = Q_M^{\frac{1}{3}} \left(\frac{1}{2}\right) Q_{CD}^{-\frac{1}{2}}$. The MRS is therefore:

$$MRS = \frac{dCD}{dM} = \frac{Q_{CD}^{\frac{1}{2}} \left(\frac{1}{3}\right) Q_M^{-\frac{2}{3}}}{Q_M^{\frac{1}{3}} \left(\frac{1}{2}\right) Q_{CD}^{-\frac{1}{2}}}$$

Which simplifies to:

$$MRS = \frac{2Q_{CD}}{3Q_M}$$

f. Exponents Review: In order to simply the above expression, the following rules for manipulating exponents are helpful:

$$x^a x^b = x^{a+b}$$

$$x^{-a} = \frac{1}{x^a}$$

4. Budget Constraints – shifting and rotating

The budget constraint is represented by: $Y = P_M Q_M + P_{CD} Q_{CD}$, where the first term is total income, the second term is expenditure on movies, and the third term is expenditure on CD's.

Figure 7 in the powerpoint slides shows the mechanics of the budget constraint.

5. Utility Maximization

We put preferences and budget constraints together for utility maximization and deriving demand curves. 2 conditions must be met for utility maximization: first, the indifference curve is tangent to the budget constraint (meaning MRS=price ratio), and second, that we spend all of our money.

In class, we reviewed a "bang-per-buck" argument. If $MU_M < MU_{CD}$, does that tell us that we should reallocate from movies to CD's? No, because we don't know how much movies cost and how much CD's cost. What we need to do is normalize by price. If

$$\frac{MU_M}{P_M} < \frac{MU_{CD}}{P_{CD}}$$

Then reallocating $1 from movies to CD's will raise utility (and is feasible, given our budget). Because of diminishing marginal utility, as we spend more on CD's, its marginal utility will fall, while the marginal utility of movies will rise as we spend less on it.

When:

$$\frac{MU_M}{P_M} = \frac{MU_{CD}}{P_{CD}}$$

There is no possible reallocation of spending from movies to CD's (or vice-versa) that can raise utility. Note that the expression above is simply another way of writing the MRS equal to the price ratio.

6. Demand Curves

Assume that $U = Q_M^{\frac{1}{3}} Q_{CD}^{\frac{1}{2}}$, and that the price of movies is \$1, the price of CD's is \$7, and income is \$1000.

First, note that $MRS = \frac{MU_M}{MU_{CD}} = \frac{2Q_{CD}}{3Q_M} = \frac{P_M}{P_{CD}} = \frac{\$1}{\$7}$, or $14Q_{CD} = 3Q_M$

In addition, the budget constraint is: $Q_M + 7Q_{CD} = 1000$

By substituting, we get: $\frac{14}{3}Q_{CD} + 7Q_{CD} = 1000$ or $Q_{CD} = \frac{3000}{35}$. Solving for movies is also straightforward.