ECO 751: Public Economics
Fall 2007
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Lecture 10: Welfare Measures and the Evaluation of Policy

1. *Introduction: Cost-Benefit Analysis as a Tool of Policy Analysis*

In general policy analysis addresses a number of issues including:

1) Justification for policy (normative);
2) Impact of the policy on individual budget spaces and subsequent predictions about the effects on behavior (consumption);
3) Modeling of the market and the impact of the policy on the market; and
4) Empirical estimates of the effects of a policy or policy change.
5) Some of the effects we might empirically estimate include:
    - Changes in income;
    - Changes in prices;
    - Changes in consumption.

For example, we might consider examining the impacts of an increase in the federal tax rate on tobacco products and empirically estimate the impact of the tax on the income of producers, the price of tobacco products, and the consumption of tobacco users. But none of these findings directly addresses the "bottom line" what effect the tax on the producers' and consumers' welfare.

2. *The Objective of Welfare Measures*

A. *Willingness-to-Pay*

The objective of welfare measures is to translate changes in prices and consumption into changes in "welfare". Our measure of the welfare effect of a price increase is based on the notions of"willingness-to-pay" (WTP) or "willingness-to-accept" (WTA). The WTP for a price decrease is the amount an individual would be willing to pay to receive the price decrease. The WTA for a price increase is the minimum an individual would be willing to accept in compensation for a price increase. More formally, this is the increase (decrease) in income that will keep the individual at the same level of utility given 'the price increase (decrease).

The value of the WTP and WTA measures is that they define changes in income that have the same affect on consumer well-being as the proposed change in price (or quantity). Thus these measures can be used to compare the welfare effects of two very different policies, for example, a 2% reduction in the personal income tax and an decrease of $0.10 on the federal excise tax on gasoline. Once the WTP is determined for the individual we can some understanding about which tax has the greatest impact on the individual. Further, we have a quantitative measure of the difference in the impact of the two taxes. Suppose that some individual had WTP of $500 for income tax reduction and $200 WTP for the gasoline tax reduction. Then for this individual the income tax reduction is equivalent to the gasoline tax reduction and an increase in income of $300.

Willingness-to-pay and accept measures are unique to the individual and depend on the individual preferences. However, determination of these measures for all members of an economy affected by the proposed policy change gives us an indication of the efficiency or inefficiency of a policy. For example, if one individual had WTP of $50 for a price decrease (a consumer of the good) and another had WTP of -$60 (a producer of it) then we would say that the price change in inefficient and the sum of the WTP's is a measure of the inefficiency.

B. Why and When to use WTP?

As we have already suggested, WTP measures are a means of standardizing in a meaningful way the impacts of changes in government policies on individuals. When evaluating policies, WTP allows us to translate the impact of radically different programs, taxes, transfers, in-kind programs into the equivalent change in income. This provides a base of comparison. Thus it makes it possible to compare the virtues (in terms of the equivalent impact on consumer income) of a food subsidy program that increase food consumption with a housing program that increase housing.

In addition since most taxes are not corrective but used strictly to collect revenue these taxes will distort prices and this leads to an **inefficiency**. Willingness to pay measures provide a means of quantifying and therefore comparing the inefficiency. The measure of the magnitude of the "inefficiency" is referred to as **deadweight loss** or **excess burden** where excess burden is the difference between the loss to consumers (WTA for tax the tax increase) and the tax revenue.

We use excess burden as a way of comparing alternative tax systems;, determining the appropriate tax systems (optimal taxation); and determining the efficient provision of government services.

3. The Traditional Approach

How do actually obtain measures of the impact of price and quantity changes on welfare? How do measure the inefficiency of taxes and in-kind programs. We begin by outlining the traditional approach of determining consumer and producer surplus.

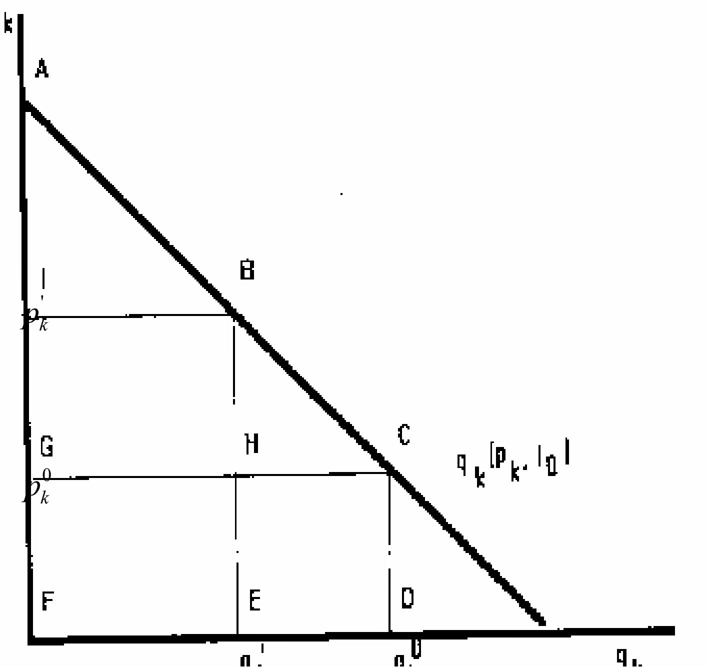
A. Marginal and Total Benefit

Consider the Marshallian demand curve, $q_k^i(\underline{p},I)$ that states a relationship between prices ($\underline{p}$) (where $\underline{p}$ is a vector) and the quantity of good *k* consumer i desires at that price $q_k^i(\underline{p},I)$ given income (I). Alternatively we can think of the **inverse** demand curve or **Marginal Benefit** function $q_k^i(\underline{p},I)^{-1} = p_k^i(q,I)$. The interpretation of the marginal benefit is simply the most the consumer is "willing to pay" for the q^{th} unit of good *k* given current income. Then if $p_k^i(q_k^o,I)$ is how much the consumer is willing to pay for the q_o^{th} unit it follows that the consumer would be willing to pay

$$TB_k^i(q_k^0, I) = \int_0^{q_k^0} p_k^i(q, I)\,dq \qquad (1)$$

for the entire q_k^0 units of good k which we refer to as total benefit. Graphically the marginal benefit is the height of the supply curve and the total benefit is the area under the demand curve. Then in *Figure 1* the total benefits from q_k^0 units is the area ACDF and the marginal benefit of the q_o^{th} unit is the distance CD.

Figure 1



B. Consumer Surplus

In a market consumption depends on a price. In Figure I, the consumer chooses *q,* when the price is *P**. Then the consumer does not receive ACDF since he has to pay for his purchases, instead he receives:

Consumer Surplus = Total Benefit - Cost of Purchases (PxQ)

$$CS_k(p_k^0, I) = \underbrace{\int_0^{q(p_k^0)} p_k(q_k, I)\,dq_k - p_k^0 q_k(p_k^0, I)}_{(a)} = \underbrace{\lim_{p_k' \to \infty} \int_{p_k^0}^{p_k'} q_k(p_k, I)\,dp_k}_{(b)} \qquad (2)$$

Consumer Surplus is defined in terms of prices because of the assumption that quantity demanded is chosen as a function of prices. Expression (a) is simply total benefit - cost of purchases using the inverse demand curve to determine total benefit. Equivalently we can simply use the demand curve and integrate over prices to obtain consumer surplus. In Figure I both expressions give the area ACG.

Our real interest is not in the absolute amount of consumer surplus but in how it changes as the result of changes in prices, say brought upon by a tax increase. Consider an increase in the price of good *k* from p_k^0 to $p_k^{'}$. In *Figure 1* the change in consumer surplus is the area IBCG. We can formally express this as

$$CS_k(p_k^0, I) - CS_k(p_k^{'}, I) = -\int_{p_k^0}^{p_k^{'}} q_k(p_k)dp_k \qquad (3)$$

This change in consumer surplus is interpreted (incorrectly as we shall demonstrate) as the WTP to avoid the price increase from p_k^0 to $p_k^{'}$.

C. Deadweight Loss

Suppose that the price increase from p_k^0 to $p_k^{'}$ was due to a tax of $\tau_k = p_k^{'} - p_k^0$ on good *k.* We shall wish to compare the loss in income (loosely speaking) to the consumer, the change in consumer surplus to the revenue collected from the tax. Revenue is, of course, equal to $\tau_k q_k(p_k^0+\tau_k, I)$ which in Figure 1 is the area *IBHG*. Then we define deadweight loss:

$$DWL(\tau_k, p_k^0, I) = -[CS_k(p_k^0+\tau_k, I) - CS_k(p_k^0, I)] + \tau_k q_k(p_k^0+\tau_k, I)$$
(4)

In *Figure 1* this is the triangle *BCH*. More formally,

$$\begin{aligned} DWL(\tau_k p_k^0, I) &= \int_{p_k^0}^{p_k^{'}} q_k(p_k, I)dp - \tau_k q_k(p_k^0+\tau_k) \approx \frac{1}{2}\tau_k\left[q_k(p_k^0+\tau_k) - q_k)p_k^0\right] \\ &\approx \frac{1}{2}\tau_k^2 \frac{\partial q_k}{\partial p_k} \end{aligned} \qquad (4')$$

For a linear demand curve deadweight loss is simply a triangle. For nonlinear demand curves we often approximate using a demand curve. The second expression is an approximation obtained by using $\tau_k(\partial q_k/\partial p_k)$ approximate the change in quantity. Using this expression we can also approximate deadweight loss by

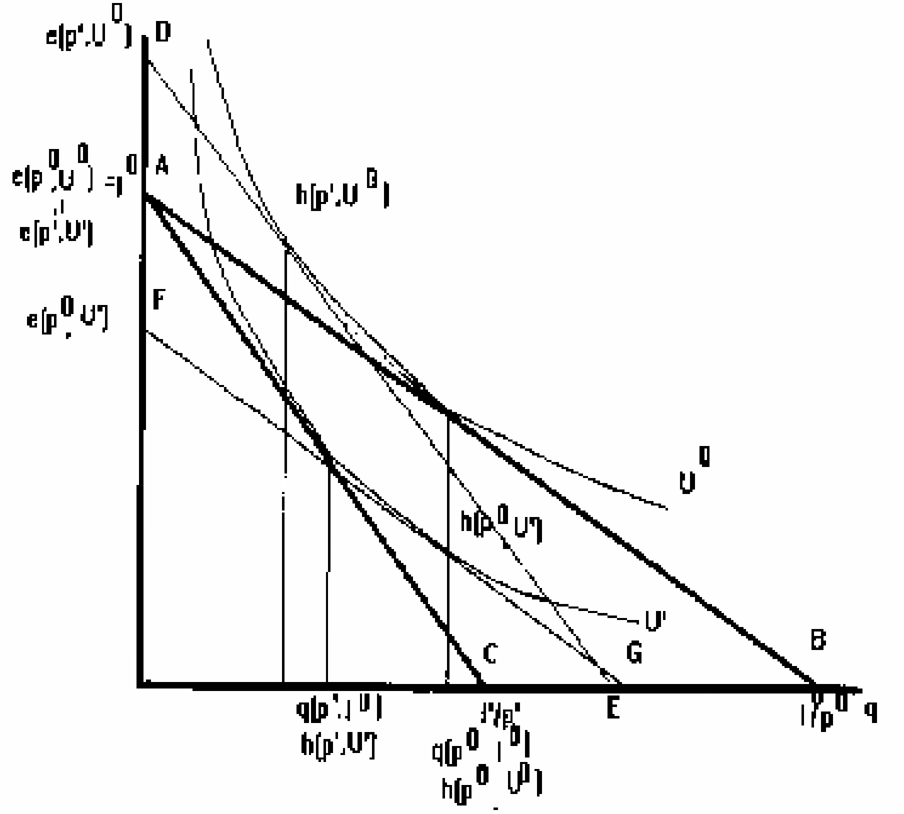
$$DWL(\tau_k p_k^0, I) \approx \frac{1}{2}\tau_k^2 \varepsilon_k \frac{q_k(p_k^0)}{p_k} \qquad (4'')$$

where ε_k is the price elasticity for good *k.* Thus deadweight loss increases by the square of the tax rate and with the price elasticity.

4. Exact Measures of Willingness to Pay

As we have suggested in the preceding section consumer surplus is not equivalent to the loss in income due to a price increase. Or, alternatively, consumer surplus does not accurately capture the willingness to pay or accept due to a price change. We can see this by considering the impact of a price increase from p_O to p' (suppressing notation for good and consumer) in *Figure 2*.

Figure 2



The effect of the price increase is to shift the consumer's budget constraint from AB to AC. This reduces consumption of q from $q(p^O,I^O)$ to $q(p',I^O)$ but it also reduces utility from U^O to U'. Because the change in price reduces utility and because consumer surplus is based on the change in consumption from $q(p^0,I^0)$ to $q(p',I^0)$, consumer surplus is not accurately measuring the willingness to pay to avoid a price decrease or, alternatively, the willingness to accept. Any measures based on changing utility will not be able to determine the amount of additional income a consumer needs to keep utility constant when a price change occurs.

What is a measure of the income loss from a price increase consistent with WTP or WTA? Suppose that we increased the price to p', a consumer would be willing to accept this price increase if he obtained enough income to keep the same level of utility as when price was p^O and income was I^0. In *Figure 2* this amount of income is given by DA. If the consumer receives an additional income of DA and the price increases from p^O to p' he still receives utility of u0. This additional income is referred to as **compensating variation.**

Compensating variation is the amount of income needed (or taken away) to keep the consumer at his original level of utility given the new price. Recall that the expenditure function $e(\underline{p},U)$ defines the (minimum) amount of income needed to obtain utility level U given prices p. Then *compensating variation* is formally defined as:

$$CV(\underline{p}', \underline{p}^O,U^O) = e(\underline{p}',U^0)-e(\underline{p}^O.U^0) = e(\underline{p}',U^0) - I,$$

(5)

the difference in expenditures at the new price and original utility level and the expenditures at the original price and utility level (which is equal to income). Note that the definition refers to the change in expenditures from all price changes and need not apply only to the change in price of a single good. Note that compensating variation in the case of a price increase gives the answer to willingness-to-accept. What measure is consistent with willingness-to-pay - what is the answer to the question "what is the most the consumer would pay to avoid a price increase?" In *Figure 2* the individual obtains utility of U' when the price increases to p' and his income is unchanged. Note that he obtains *V* when his budget constraint is FE which is parallel to the original budget constraint. This means that prices are unchanged and only income has changed. As the utility is the same as with the new prices, this means the difference in income between the budget constraints AB and FE, which is equal to AF, is the most an individual would pay to avoid a price increase. This is referred to as **equivalent variation.**

Equivalent Variation is the amount of income taken away (or needed) to make the consumer indifferent between his new price (and original income) and the original price. Then equivalent variation is formally given as

$$EV(\underline{p}',\underline{p}^0,U') = e(\underline{p}^0,U') - e(\underline{p}',U') = e(\underline{p}^0,U') - I \tag{6}$$

Note that equivalent variation is a negative number as income is taken away -- the consumer pays to avoid a price increase.

Recall that $\partial e(\underline{p},U)/\partial p_k = h_k(\underline{p},U)$ where $h_k(.)$ is the Hicksian demand for good k. Then

$$e(p',U^0) - e(p^0,U^0) = \int_{p^0}^{p'} h(p,U^0)dp \tag{7}$$

Then using (7) in (5) and (6) gives

$$CV(\underline{p}',\underline{p}^0,U^0) = \int_{p'}^{p^0} h(p,U^0)dp \tag{5'}$$

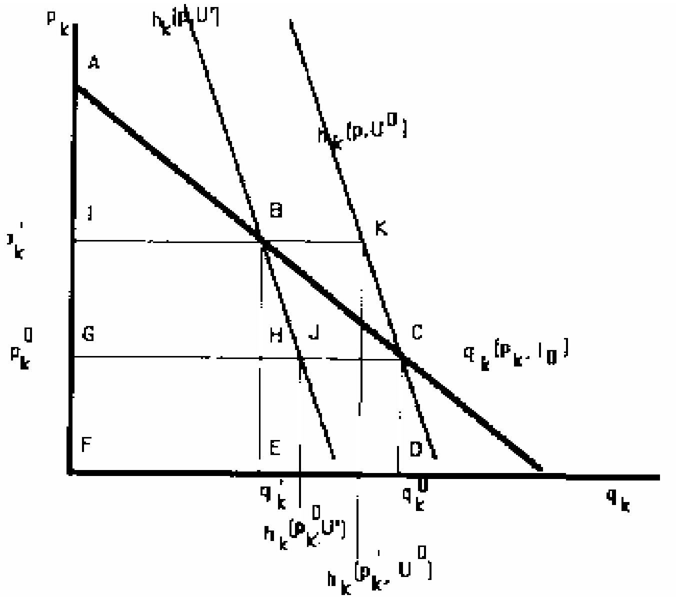
and

$$EV(\underline{p}',\underline{p}^0,U') = \int_{p^0}^{p'} h(p,U')dp \tag{6'}$$

Expressions (5') and (6') denote areas under the demand curve analogous to consumer surplus. In *Figure 3*, the Hicksian demands are given for the original utility level (given by p^O and I^0) an the post-price change utility level. The fact that $h_k(p,U^O) > h_k(p,U')$ is based on the assumption that good is a normal good. Compensating variation is given by the area IKCG, the area "under" the Hicksian demand curve hk(p,UO); equivalent variation is given by the area IBJG; and consumer surplus is the area IBCG. Note that for a price increase of a normal good we have CV > CS > EV; if it were a price decrease we would have EV > CS > CV.

Why the difference in measures? Income effects on demand. In the absence of any income effects on demand all three measures would be identical.

Figure 3



B. *Excess Burden with Exact Measures*

Because consumer surplus was an inaccurate measure of the income loss suffered from a tax increase, a measure of the inefficiency of the tax using consumer surplus will also be flawed. The alternative is simply to use compensating variation or equivalent variation in the place of consumer surplus. Generally, compensating variation is used as the basis of determining how much compensation is required to keep the consumer as well off. Using compensation variation as a base we have

$$EB(\tau,\ p_{0}.,U^{0}) = [e(p',\ U) - e(p^{0},U^{0})]\ - \sum_{k=1}^{n}\tau_{k}h_{k}(p,U^{0}) \approx \frac{1}{2}\sum_{k=1}^{n}\tau_{k}\left[h_{k}(\underline{p'},U^{0}) - h_{k}(\underline{p^{0}},U^{0})\right]$$

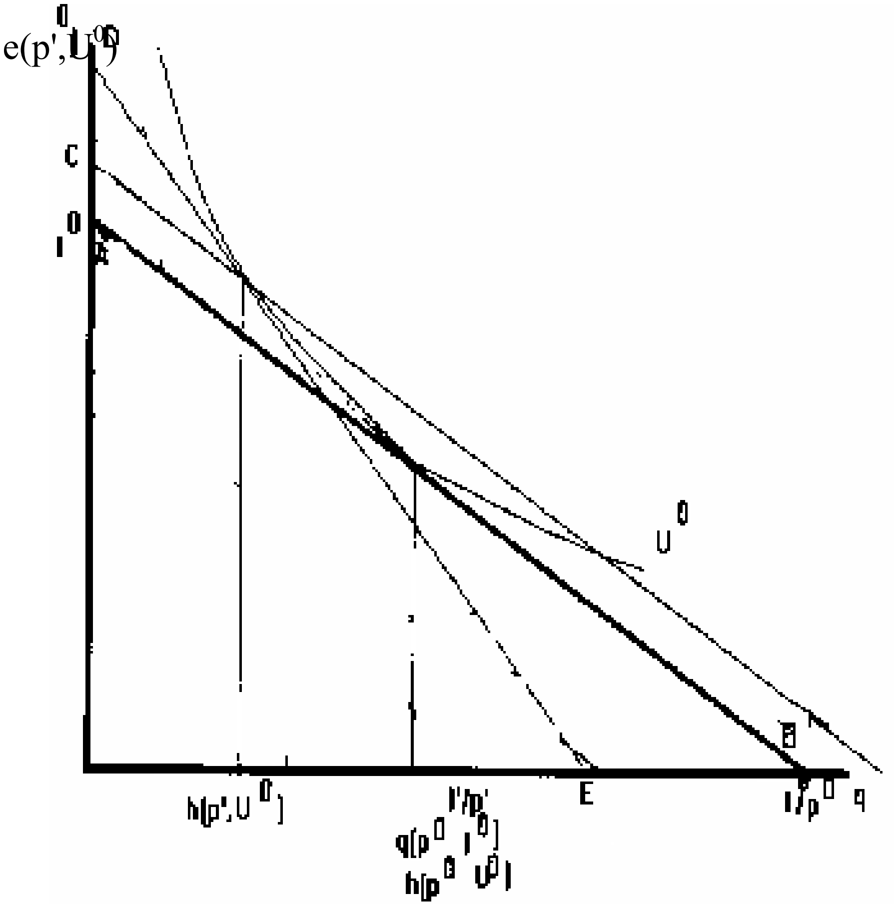
(8)

where excess burden is defined as a positive number the amount of income needed to obtain U^O at p' that exceeds revenue collected from the tax. Note that revenue is based on demand at p' and U^O not p' and U'. The definition of excess burden allows us to generalize the concept of excess burden to multiple taxes. The second expression for excess burden corresponds to the graphical depletion of a single tax increase. In *Figure 3* when utility is kept at U^O revenue is the area IKLG and excess burden is KCL. An alternative graphical depiction of excess burden is given in *Figure 4*.

In Figure 4, as in Figure 2, the compensating variation of a price increase from p^O to p' is the distance DA. Part of this loss in income is from tax revenues collected from the consumer. The tax revenues collected from the consumer are equal to the distance DC. Note that both line DE and AF intersect at $h(p',U^O)$, the quantity consumed. The line AF has a slope of p^O while DE has a slope of p'. Then the distance between the two lines at q = 0 is the additional expenses needed to purchase $h(p',U^O)$ when there is a tax on it. These additional expenditures are therefore equal to tax revenue. Finally if DA is compensating variation and DC is tax revenue, excess burden is

CA.

Figure 4



5. *Marginal Excess Burden and Second-Best Measures*

In this section we consider how a marginal increase in a tax rate affects excess burden when there are existing taxes. We first consider the impact of a marginal change in the tax rate on excess burden when only a single good is being taxed and we are increasing that tax. We then consider the case of existing taxes on other goods.

A. Marginal Excess Burden

We have thus far considered the implementation of a tax when no preexisting taxes existed. We now wish to consider increasing the amount of an existing tax and the impact the increase has. As we shall show the excess burden of any change in tax can only be determined when there is full information about the existing tax structure. Excess burden for a single tax is given by

$$EB(\tau^0,p^0,U^0)= [\ e(p^0+\tau,U^0)-e(p0,U^0)]\ -\ \tau h(p^0+\tau,U^0) \qquad (9)$$

Then differentiating (9) with respect to τ yields

$$\frac{\partial EB(\tau, p^0, U^0)}{\partial \tau} = \frac{\partial e(p^0 + \tau, U^0)}{\partial \tau} - \left[h(p^0 + \tau, U^0) + \tau \frac{\partial h(p^0 + \tau, U^0)}{\partial \tau} \right] \tag{10}$$

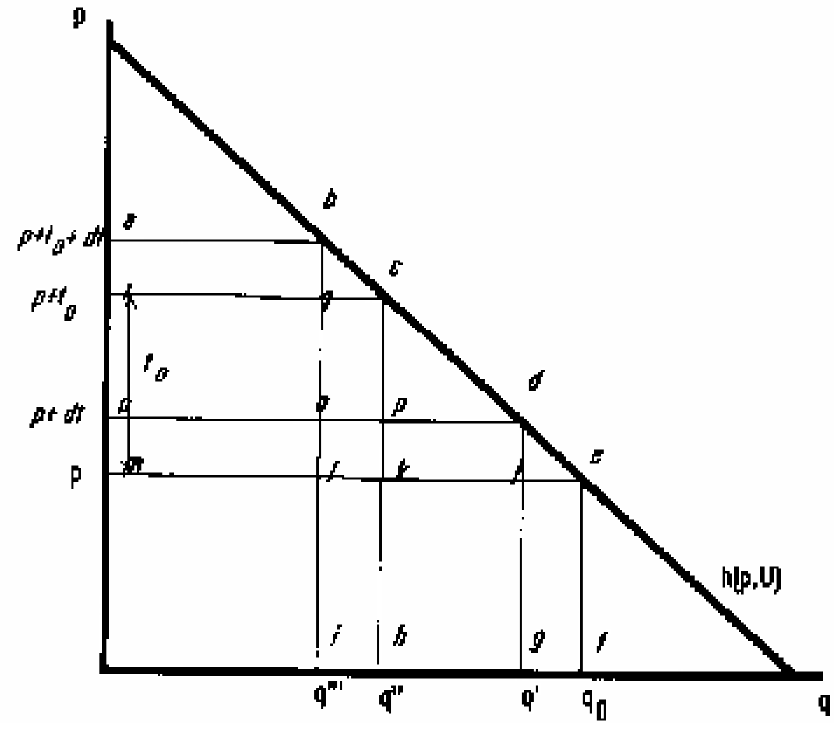
but by Shephard's Lemma $\partial e(p,U)/\partial p = h(p,U)$ so simplifying (10) gives

$$\frac{\partial EB(\tau, p^0, U^0)}{\partial \tau} = -\tau \frac{\partial h(p^0 + \tau, U^0)}{\partial \tau} = -\tau\varepsilon \frac{(p^0 + \tau)}{h(p^0 + \tau, U^0)} \tag{10'}$$

Marginal excess burden varies directly with the magnitude of the existing tax. This can also be seen by differentiating (4') with respect to τ. When τ = O, marginal excess burden is zero.

Figure 5 gives a graphical interpretation of (10') by comparing a small tax increase *(dt)* when there is no tax and when there is an existing tax of *t*. When there is no existing tax implementation of the increase *dt* causes a loss in income of *ndem* and tax revenue of *ndjm,* making the excess burden equal to *dej.* With a tax of fan increase of *dt* causes a loss of income of *abcr* and additional revenue of *abqr - qckl,* where *abqr* is the revenue raised from increasing the rate by *dt* on the *q'''* units that are purchased at the higher price while *qckl* is the loss from no longer selling units *q'''-q''*. Then the increase in excess burden is equal to *bel* - *cek* = *bckl.* Why the difference in excess burden for the same increase in tax rate and the same decrease in consumption *(q'''-q'' = q'-q^O)?* Because with an existing tax, the increase in tax rate will lead to a greater reduction in tax revenue.

Figure 5



B. Excess Burden Measures in a Second-Best World

We can now apply this same approach to consider the impact of a marginal increase on the tax on good i when there are existing taxes on other goods. Letting p^O *and* τ now represent vectors we have the change in excess burden from an increase in τ_i given by

$$\frac{\partial EB(\tau, p^0, U^0)}{\partial \tau_i} = \frac{\partial e(p^0 + \tau, U^0)}{\partial p_i} - \left[h_i(p^0 + \tau, U^0) + \sum_{k=1}^{n} \tau_k \frac{\partial h_k(p^0 + \tau, U^0)}{\partial p_i} \right] \tag{11}$$

Using Shephard's Lemma to cancel $h_i(p,U)$ with $\partial e(p,U)/\partial p_i$ gives us

$$\frac{\partial EB(\tau, p^0, U^0)}{\partial \tau_i} = \sum_{k=1}^{n} \tau_k \frac{\partial h_k(p^0 + \tau, U^0)}{\partial p_i} = \sum_{k=1}^{n} \tau_k \varepsilon_{ki} \frac{(p_i^0 + \tau_i)}{h_k(p^0 + \tau, U^0)} \tag{11'}$$

Here ε_{ki} is the compensated price elasticity of good *k* with respect to the price of good i. Then (11') simply states that the excess burden of any marginal tax change *(drj)* in one market depends on the changes in tax revenue in other markets due to the tax T]{[5hk(pO+T,UO)/5pi]. It is possible that an increase in the tax on good i has a negative excess burden. Why? Because the loss in income from the tax increase is less than the increase in revenue. When is this possible? When substitutes for good i are taxed heavily and complements are not. Consider a two good example with goods 1 and 2. As there are only two goods then it must be the case that I and 2 are Hicksian substitutes, that is, $\partial h_1/\partial p_2 > 0$. Let good 2 have an existing tax and impose a tax on good 1. Then in *Figure 6a* the excess burden in market I is *abc*. However the tax on good I increases demand for good 2. Because there is a tax of t2 on good 2 the increase in demand increases tax revenue by *defg*. As *defg > abc,* excess burden decreases.

Figure 6a

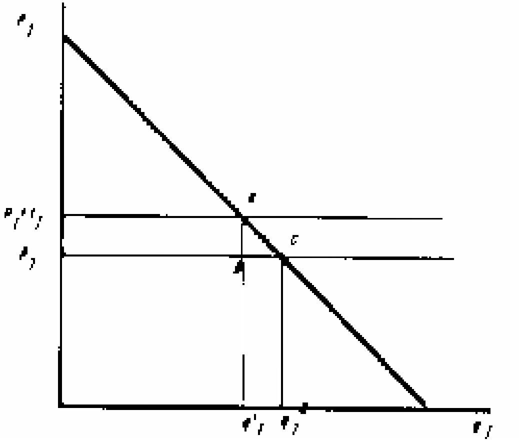
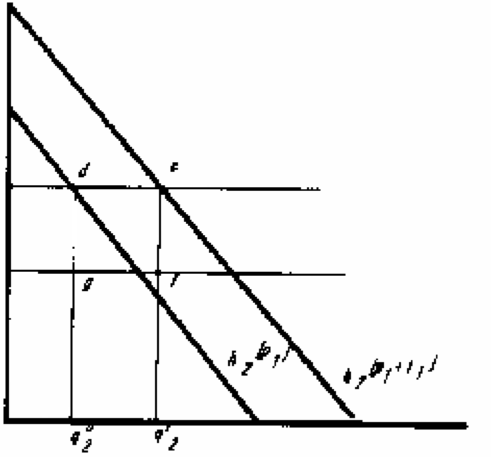


Figure 6b



6. From Theoretical Measures to Empirical Estimates: Approaches to Implementing Welfare Measures

It should be apparent that obtaining an empirical estimate of deadweight loss based upon consumer surplus is relatively straightforward. We simply estimate a Marshallian demand curve, demand based upon prices and income. What had been less apparent to the profession was how to obtain welfare measures such as compensating and equivalent variation. While these are also areas under demand curves, these demand curves are based on prices and utility something we do not observe. In this section we present two alternative approaches to obtain "exact" welfare measures: compensating variation, equivalent variation, and excess burden based upon these income measures.

The first approach starts with a utility function and solves for Marshallian demand functions. The second approach, from Hausman *(1981)* begins with Marshallian demand curves and recovers the utility function.

A. From Utility Function to Marshallian Demand

The first approach we take is simply to begin with a specific utility function. From there we can:

1. Solve for the Marshallian demand equations;
2. Estimate the parameters of the Marshallian demand equations;
3. Derive the parameters of the utility function from the parameters obtained from the Marshallian demand equations.
4. Derive the expenditure function and compensating and equivalent variations given the parameterization of the demand equation.

We give the Cobb-Douglas example. The *Appendix* to this lecture gives the specifications for the *Stone-Geary* and *CES* functions.

Example 1: Cobb-Douglas Function

Utility is given by $U = x^{\alpha} y^{1-\alpha}$ *and* $p_x x + y = I$
(12)

Note that x is the good whose price is changed and y refers to spending on all other goods.

a) Marshallian Demand Equation:

This is the stage where estimation is done. We need only estimate the demand for good x as the demand for y is implied by the demand for x.

$$x(p_x, I) = \frac{\alpha}{p_x} I \tag{13a}$$

and

$$y(p_x,) = (1-\alpha) I \tag{13b}$$

b) Derivation of the Indirect Utility Function:

We have *U(x(p*$_x$*,I),y(p*$_x$*,I))=V(p*$_x$*,I)* which gives

$$V(p_x, I) = \left(\frac{\alpha}{p_x} I \right)^{\alpha} \left((1-\alpha) I \right)^{1-\alpha} = \frac{\alpha^{\alpha} (1-\alpha)^{(1-\alpha)}}{p_x^{\alpha}} I \tag{14}$$

c) The Expenditure Function

Inverting V(p$_x$,I), gives e(P$_x$,U), the expenditure function, since *V(p*$_x$*,,e(p*$_x$*,,U)) = U.*

$$e(p_x, U) = \frac{p_x^{\alpha}}{\alpha^{\alpha} (1-\alpha)^{(1-\alpha)}} U \tag{15}$$

Note that this is not an operational expenditure function as it is in terms of the level of utility. However, from (14) we know initial utility is given by

$$U^0 = V(p_x^0) = \frac{\alpha^{\alpha} (1-\alpha)^{(1-\alpha)}}{\left(p_x^0\right)^{\alpha}} I^0 \tag{16}$$

Then substituting for U using (16) in (15) gives

$$e(p_x^{'}, V(p_x^0, I^0) = \left(\frac{p_x^{'}}{p_x^0} \right)^{\alpha} I^0 \tag{14'}$$

where we define the expenditures needed to maintain the original utility given the new price of p$_x$'.

d) Compensating and Equivalent Variation

$$CV(p_x^{'}, p_x^0, I^0) = e(p_x^{'}, V(p_x^0, I^0)) - e(p_x^0, V(p_x^0, I^0)) = \left(\frac{p_x^{'}}{p_x^0}\right)^{\alpha} I^0 - I^0 \qquad (15)$$

$$EV(p_x^{'}, p_x^0, I^0) = e(p_x^0, V(p_x^{'}, I^0)) - e(p_x^{'}, V(p_x^{'}, I^0)) = \left(\frac{p_x^0}{p_x^{'}}\right)^{\alpha} I^0 - I^0 \qquad (16)$$

B) Recoverable Utility Functions

Hausman, in "Exact Consumer's Surplus and Deadweight Loss" (*American Economic Review* (1981)) takes another approach. Beginning with a Marshallian demand function he recovers the underlying utility and expenditure functions. The equations are from Hausman with I instead of Y for income.

a) Specification of the Demand Equation

$$X(P,I)= \alpha P + \delta I + z\tau \qquad (17)$$

This is the stage where estimation occurs.

b) Roy's Identity and the Indirect Utility Function

Roy's identity states that

$$X(P,I)=-V_p(P,I)/V_I(P,I) \qquad (18)$$

where $V_p = \partial V/\partial p$ and $V_I = \partial V/\partial I$. If X(P,I) is linear, then the above equation is a linear partial differential equation. Welfare comparisons are made on the same indifference curve so V(P,I) = U. Totally differentiating yields

$$V_p(P,I)dP + V_I(P,I)dI=0 \qquad (19)$$

which implies

$$\frac{dI(P)}{dP} = -V_p(P,I)/V_I(P,I) \qquad (20)$$

when I is expressed as a function of P, using the implicit function theorem. Using Roy's Identity

$$\frac{dI(P)}{dP} = X(P,I) = \alpha P + \delta I + z\tau \qquad (21)$$

The next step is to solve for I as a function of P. We have a differential equation of the form

$$dI + [u(P)I+w(P)]dP = 0 \qquad (22)$$

where $u(P) = -\delta$ and $w(P) = -\alpha P - z\tau$.

From Chiang (2nd edition p484-486) the solution to this differential equation is

$$I(P) = e^{-udP}[c + w(P)e^{udP}dP] \tag{23}$$

so somewhat simplifying gives

$$I(P) = ce\delta P - e^{\delta P}\,[\int(\alpha P + z\tau)\,e^{-\delta P}\,dP] \tag{24}$$

Integrating by parts simplifies the second term to

$$du = e^{-\delta P},\ v = (\alpha P + z\tau) \text{ and } \int v\,du = uv - \int u\,dv = e^{\delta P}\left[-\frac{1}{\delta}e^{-\delta P}(\alpha p + z\tau) - \frac{\alpha}{\delta}e^{-\delta p}\right] \tag{25}$$

Then the differential equation can be solved to give

$$I(P) = ce^{\delta P} - \frac{1}{\delta}[\alpha P + \delta + z\tau] \tag{26}$$

c) The Indirect Utility Function

Let the second term be denoted by f(P,z) then the constant of intergration c can be set equal to the utility level, u, and inverting (26) we get

$$V(P,I) = c = e^{-\delta P}[I + f(P,z)]$$

(27)

d) *Expenditure Function and Compensating and Equivalent Varations*

The remaining steps, deriving the expenditure function, and the CV and EV measures are identical to the procedure in A. Hausman gets

$$CV(P_0,P_1,I_0) = \frac{1}{\delta}e^{\delta(P_1-P_0)}\left[X(P_0,I_0) + \frac{\alpha}{\delta}\right] - \left[X(P_1,I_0) + \frac{\alpha}{\delta}\right] \tag{28}$$

Why might we use one approach instead of another? Most estimation is based upon specification of the demand equations rather than the underlying utility functions because it is the demand equations that are actually estimated. Further the few utility functions for which we can obtain simple closed form solutions for demand equations often impose restrictive assumptions on the demand equations. For example, in the Cobb-Douglas case, the income and own-price elasticity are unity and all Marshallian cross-price elasticities equal zero. The demand equations that are simplest to estimate linear and log-linear specifications are not obtained from any simply direct utility function. Thus if you believe these are the forms of the demand equations you wish to estimate, it makes sense to recover the utility function from them.

7. *Empirical Differences in Exact and Approximate Welfare Measures*

Robert Willig in "Consumer Surplus without Apology" (*American Economic Review (*1976)) argues that in many cases we need not be concerned about the differences between consumer surplus and compensating or equivalent variation because they are likely to be very small. Specifically he proves that there are bounds in the differences in the measures:

$$\frac{\underline{\eta} - A}{2m} \le \frac{CV - A}{A} \le \frac{\bar{\eta} + A}{2m} \tag{29a}$$

and

$$\frac{\underline{\eta} - A}{2m} \le \frac{A - EV}{A} \le \frac{\bar{\eta} + A}{2m} \tag{29b}$$

when

$$\left|\frac{\bar{\eta} A}{2m}\right| \le 0.05, \left|\frac{\underline{\eta} A}{2m}\right| \le 0.05, \textit{ and } \left|\frac{A}{m}\right| \le 0.9 \tag{30}$$

where:

- A is the Marshallian change in consumer surplus from a change in the price (P);
- EV is equivalent variation;
- CV is compensating variation;
- m is base income;
- $\underline{\eta}$ and $\bar{\eta}$ are the lower and upper bounds for income elasticity.

Willig argues that:

1. A/m can be measured as the proportional change in real income from a price change and is likeh to be small; and
2. most income elasticities are cluster around 1.0 so we are likely to find that the conditions in (28) will be satisfied.

Thus, for example is $\eta = 0.8$ and A/m = 0.5 then using (29a) we have

$$\frac{0.8(0.05)}{2} = 0.02 \le \frac{CV - A}{|A|} \le \frac{0.8(0.05)}{2} = 0.02 \tag{31}$$

Consumer surplus (A) is within 2% of CV.

Hausman *(1981)* argues that while Willig's conditions are correct, his conclusions are objectionable on three major counts:

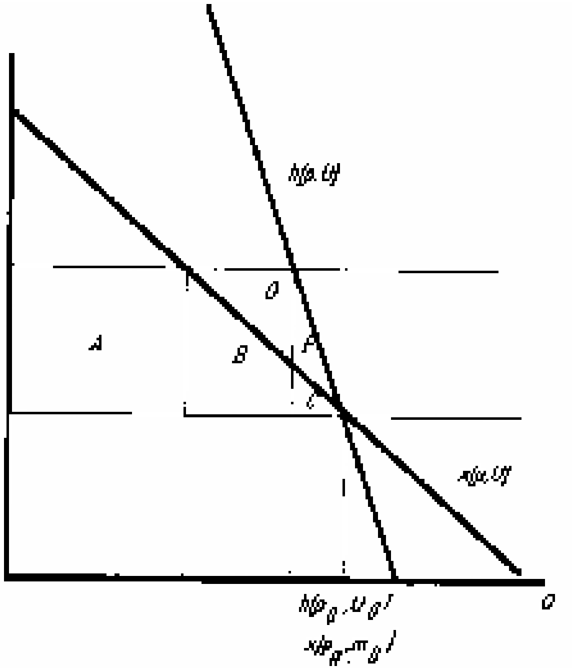
1. Often changes in A/m are very large if the good has a large budget share such as labor.
2. Even if A is close to CV, the excess burden measures calculated under both schemes may be very different; and
3. There is no need for approximation as the market demand curve can always be used to derive the unobserved compensated (Hicksian) demand curve.

We focus on the second of Hausman's objections. In *Figure* 7 we see both the compensated (h(p,U)) and uncompensated (x(p,m)) demand curves. Then from *Figure 7* we see that from the tax of *t* we have

$$\Delta \textit{Consumer Surplus (A)} = A + B + C$$
$$\textit{Compensating Variation (CV)} = A+B+C+D+E$$

Then $\frac{CV - A}{A} = \frac{D + E}{A + B + C}$

Figure 7



The excess burden using the Marshallian measures (DWL) is

$$DWL = B + C$$

and the excess burden using compensating variation (EB) is

$$EB = E + C$$

which means that

$$\frac{EB - DWL}{DWL} = \frac{E - B}{B + C}$$

Clearly the "error" in the excess burden, in percentage terms, is much greater than the error in measuring the income loss. Why?

Essentially we can decompose our measures into the following components:

$$A = \tau x(p_o,m_o) + (1/2)\tau\Delta x$$
$$CV=\tau h(p_o,U_o)+(1/2)\tau\Delta h$$
$$DWL = (1/2)\tau\Delta x$$
$$EB = (1/2)\tau\Delta h$$

The measures of the loss of income are composed of a first order effect, the product of the change in price and the original quantity, and a second order effect, one-half the product of the change in price and the change in quantity. The only error is in the estimation of the change in quantity, a second order effect, the first order effect is the same for both measures. Therefore, since DWL and EB are only second order effects, the base for the error is much smaller.

1. Two Examples

a) The Labor Supply of Married Women

The labor supply of married women is estimated to be:

$$h_j = 765.1 + 495.1lw_j - 0.1250y_j$$

where:

- j is the observation;
- h_j is annual hours worked;
- y_j is aftertax income of husband; and
- w_j is the hourly wage rate.

The mean wage is \$4.15 and the mean after-tax income of husband is \$8,236. Suppose we impose a 20% proportional tax on labor. A 20% tax reduces the after-tax wage to 3.32. Then consumer surplus is given
by

$$A = \int_{3.32}^{4.15} (495.1)w = \frac{1}{2}(495.1)\left[4.15^2 - 3{,}32^2\right] = 1{,}315$$

$$CV = \$2{,}056$$

a) 44% difference. Why? Because A/I is very large, 1315/8236 >> 0.05.

b) A Tax on Gasoline

Consider the hypothetical demand curve for gasoline:

$$q_j = 4.95 - 14.22p_j + 0.082y_j$$

where:

- q is gallons per month;
- p is price per gallon (assume p = 0.75);
- y is income/month (assume y = 720).

Then the income elasticity $\varepsilon_I = 1.1$ and the Marshallian price elasticity $\varepsilon_p = -0.2$. If a tax of t = \$0.75 is put on gasoline raising P to \$1.50 we obtain:

CV = \$37.17 per month and A = \$35.99/month, a 3.2% difference. But

EB = *\$2.88 and DWL = \$3.96,* which is a 31.7% difference.